Project Report1
Capacity and Delay Tradeoffs in MANETs with Multicast

Group 2
Xiaoyu Chu, Jia Guo,Zhang Li, Chaomin Ye

I. Introduction

To start with our project, we lucky got a classical paper about capacity and delay tradeoffs for ad-hoc mobile networks. After careful reading of this paper, we abstracted the main chain and in every part we added some ideas of our own. Thanks to our senior who provided us a possible direction of study and suggested another paper using multicast. It is similar to the first one, so we just grasped the differences and initiative part. After these two we initially came up with the short-term and long-term goals of this projct. Then we decided to reread the first paper to get some useful details about calculation. Moreover, in order to find the specific long-term goal, we generally survey the history of the study in wireless ad-hoc network and some recent works, which helped us better understanding this area and find some possible directions for later choosing.

II. Comprehension on Neely's Paper

After reading Prof. Neely's paper "Capacity and Delay Tradeoffs for Ad-Hoc Mobile Networks", we get lots of information on the topic of Capacity and Delay. By building several typical models and deducing some theorems and lemmas, this paper opens the window of Ad-Hoc Mobile Networks for us so that we could see the vista of this field. According to our humble opinion, Neely's paper brings us a comprehensive view on our topic, and our research would be based upon this paper. Therefore, a brief comprehension is needed.

A. Introduction of Basic model

The basic model that is raised in Neely's paper is *cell partitioned* network model: The network is partitioned into C non-overlapping cells of equal size. There are N mobile users distributed upon the network according to i.i.d., which stands for *independent and identically distributed* . Moreover, each cell can support exactly one packet transfer per timeslot, and users within different cells cannot communicate during the slot. In addition, once a packet has been received by a user, it can be stored in memory and transmitted again and again if so desired. To interpret the model better, d, which equals to N/C, is defined to represent the density of users in network.

B. Capacity Calculation of Basic model

Having built *cell partitioned* network model, the author began to consider capacity. λ represent the exogenous arrivals rate of packets to user i, and the capacity of the network is the maximum rate λ that the network can stably support. Prof. Neely raises theorem 1 about capacity:

$$\mu = (p+q)/2d$$

(Where

$$p = 1-(1-\frac{1}{C})^N - \frac{N}{C}(1-\frac{1}{C})^{N-1}$$

$$q = 1-(1-\frac{1}{C^2})^{N/2}$$

p represents the probability of finding at least two users in a particular cell, and q represents the probability of finding source-destination pair within a cell.) Taking limits as $n \to \infty$, we find the network capacity tends to the fixed value $(1-e^{-d}-de^{-d})/2d$. Regarding d as a parameter, a figure of this function value shows capacity tends to zero as d tends either to zero or infinity, and when d=1.7933, capacity attains its maximum: 0.1492.

Based on above calculations and Theorem 1, Prof. Neely gives us Corollary1 that the use of redundant packet transfers, multi-user reception, or perfect feedback cannot increase network capacity. Then he considers Heterogeneous Demands. λ_{ij} represents the rate user i receives exogenous data intended for user j and K represents the maximum number of destination users to which a source transmits. Theorem 2 follows: The symmetric capacity region of the network has the form:

$$\sum_j \lambda_{ij} \leq \frac{(1-e^{-d}-de^{-d})}{2d} + O(K/N)\forall i$$

$$\sum_i \lambda_{ij} \leq \frac{(1-e^{-d}-de^{-d})}{2d} + O(K/N)\forall j$$

In the rest of Neely's paper, K is set as 1.

C. An Algorithm with Capacity Achievement

Prof. Neely raises an algorithm which is capacity achieving with a bounded average delay. It is *Cell Partitioned Relay Algorithm* :
Every timeslot and for each cell containing at least two users:

1) If there exists a source-destination pair within the cell, randomly choose such a pair (uniformly over all such pairs in the cell). If the source contains a new packet intended for that destination, transmit. Else remain idle.
2) If there is no source-destination pair in the cell, designate a random user within the cell as sender. Independently choose another user as receiver among the remaining users within the cell. With equal probability, randomly choose one of the two options:

- *Send a Relay packet to its Destination* : If the designated transmitter has a packet destined for the designated receiver, send that packet to the receiver. Else remain idle.
- *Send a New Relay Packet* : If the designated transmitter has a new packet (one that has never before been transmitted), relay that packet to the designated receiver. Else remain idle.

This algorithm is simple and classic. However, we think randomly choosing a source and a destination is not proper enough because it is entirely possible that the two users we choose do not have conditions to transmit a packet where waste happens.

Theorem 3 points out that under 2-hop relay algorithm, assuming that users change cells i.i.d. and exogenous input stream to user i is a Bernoulli stream of rate λ_i, then the total network delay

$$E\{W_i\} = \frac{N-1-\lambda_i}{\mu-\lambda_i}$$

This equation shows that delay of this algorithm is $O(N)$, which cannot satisfy our demands. Therefore, we may need a new model for a better condition of delay. And from now on, emphases of this paper convert to delay rather than capacity.

D. Redundancy model for delay improvement

1) Theoretical Calculation: Prof. Neely offers us a pretty nice thought way which probably conduces to our following research and extension. He just considers the situation that there is only one packet in the network and finds the delay T(N). Then he supposes the exogenous input stream to user i is a stream with some probability distribution, such as Bernoulli, Poisson and so forth. He builds the relationship between T(N) and W. We think in the future research we can also inherit this fantastic thought way.

When considering sending a single packet, the author divided it into three models:

- Scheduling Without Redundancy
- Scheduling With Redundancy
- Multi-User

Reception He provide Theorem 4 and 5 calculate the delay of A and B. In addition, in Appendix E, the author shows that $O(\sqrt{N})$ cannot be overcome by introducing multi-user reception. The equation (34) in Appendix E is the key of this proof, which amazes us much. Deserving to say, inequality (7) of the process of theorem 5 is technical because it calculates the probability in another way rather than the most exact way.

2) In-Cell Feedback Scheme with $\sqrt{N}$ Redundancy: Following Theorem 5, another model is built to show $O(\sqrt{N})$ can be realized. In-Cell Feedback Scheme with $\sqrt{N}$ Redundancy: In every cell with at least two users, a random sender and a random receiver are selected, with uniform probability over all users in the cell. With probability 1/2, the sender is scheduled to operate in either 'source-to-relay' mode, or 'relay-to-destination' mode, described as follows:
1) *Source-to-Relay Mode* : The sender transmits packet SN, and does so upon every transmission opportunity until N replicas have been delivered to distinct users, or until the sender transmits SN directly to the destination. After such a time, the send number is incremented to SN +1. If the sender does not have a new packet to send, remain idle.
2) *Relay-to-Destination Mode* : When a user is scheduled to transmit a relay packet to its destination, the following handshake is performed:

- The receiver delivers its current RN number for the packet it desires.
- The transmitter deletes all packets in its buffer destined for this receiver which have SN numbers lower than RN.
- The transmitter sends packet RN to the receiver. If the transmitter does not have the requested packet RN, it remains idle for that slot.

Theorem 6 shows the delay and capacity of this model.

E. Multi-Hop Scheduling for delay improvement

Then we convert our focus on Multi-Hop Scheduling and Logarithmic Delay. To achieve the $O(\log(N))$delay, Fair Packet Flooding Protocol is raised.
Fair Packet Flooding Protocol: Every timeslot and in each cell, users perform the following: Among all packets contained in at least one user of the cell but which have never been received by some other user in the same cell, choose the packet p which arrived earliest (i.e., it has the smallest timestamp t_p). If there are ties, choose the packet from the session i which maximizes $(t_p + i)$ mod N. Transmit this packet to all other users in the cell. If no such packet exists, remain idle. Theorem 7 followed this model present its delay.

tips : This model confirms the fairness by choosing packet according to the formula $(t_p + i)$ mod N. As time passes, t_p increases, and the value $(t_p + i)$ mod N for particular i goes like a circle. Thus, no number i can guarantee a large probability to be transmitted.

We should note that the flooding algorithm easily allows for multicast sessions, where data of λ is delivered from each source to all other users. Therefore, considering more about multicast session in this field is meaningless.

F. Tradeoff of Delay and Capacity

After calculating delay and capacity of several models and algorithms, the author focuses on fundamental delay/rate tradeoffs. From no redundancy scheme to redundancy 2-hop scheme, and then to redundancy multi-hop scheme, delay improves by sacrificing capacity. Theorem 8 with its complicated proof show

$$\frac{W}{\lambda} \geq O(N)$$

G. Markovian Model

Now, main part of this paper is illustrated. Last problem is about Non-I.I.D. Mobility Models. Stimulations show similar performance for both i.i.d. and non-i.i.d. mobility. This paper points out a possible proving way, but it says such questions should be left in future. We think Markovian can be one of our cut-in point.

III. Another Paper Focus on Multicast

Prof Neely's paper, "Capacity and Delay Tradeoffs for Ad-Hoc Mobile Networks"[1] equips us with the basic skills to analyse capacity and delay for a MANET. However, our project focus more on multicast which is seldom mentioned in [1]. As a result, in order to get deeper insight to the capacity delay tradeoff analysis, we read Chen Hui's paper: MotionCast: On the Capacity and Delay Tradeoffs"[2].

A. Why multicast?

This paper lists some reasons for taking multicast into our consideration, such as group communications in military networks and disaster alarming in sensor networks. The author also demonstrate the vital importance of multicast in current mobile multimedia services. Actually multicast is not only practically valuable, but also theoretically.

Reaserches involving muticast in MANET were firstly leading by Liet [3],Jacquet[4], Shakkottai[5], for their respective contributions on static networks multicast. Followed by Grossglauser and Tse, for demonstration on preventing capacity vanishing as the size of network grows by implementing 2-hop relay algorithm and finally ends with Neely and Modiano [1].

B. How multicast is analysed?

To begin with, Chen introduces the key feature of multicast in MANETs ,explains the network model, mobility model, defines similar concepts as capacity, redundancy and cooperative. Most of these model and concepts are similar to those in Neely's work[1], respectively. Having all these foundation laid, Chen carrys on with the deeper analysis for multicast.

The analysis sections can mainly be divided into two parts, without redundancy or with redundancy. The former part can also be separated to two scenarios, non-cooperative mode or cooperative mode, as an expansion to Neely's works. Theorem 1 is derived in the former scenario, confirming that the average delay for the traffic of node i satisfies $E(W_i) = O(nlogk)$ if $k = O(n^{\xi})(0 \leq \xi < 1)$. To lose constraint on k to the extend that $k \leq n$, cooperative mode is exploited in the later situation and this leads to Theorem 2, restricting the capacity and delay to $\Omega(1/k)$ and $O(nlogk)$ respectively for all $k \leq n$. As a comparison with [1], here just lists a 2-hop relay algorithm without redundancy in cooperative mode:

With equal probability, the sender is scheduled to operate in the two options below:

1) Source-to-Relay Transmission: If the sender has a new packet one that has never been transmitted before, send the packet to the receiver and delete it from the buffer. Otherwise, stay idle.
2) Relay-to-Destination Transmission: If the sender has packets received from other nodes which are destined for the receiver and have not been transmitted to the receiver yet, then choose the latest one, transmit. If all the destinations who want to get this packet have received it, it will be dropped from the buffer in the sender. Otherwise, stay idle.

Although the above algorithm are different with Neely's works, the ideas for proving theorem 1 and theorem 2 are closely related.

And what is the maximum capacity and minimum delay? Answers are provided and proved in the next subsection as can be seen from Theorem 3, the multicast capacity of a cell partitioned network is $O(1/k)$ if only a pair of sender and receiver is active in each cell per timeslot and Theorem 4, algorithm permitting at most one transmission in a cell at each timeslot without redundancy cannot achieve an average delay of $O(nlogk)$.

Although capacity shown above is quite satisfying, the delay without redundancy is really intolerable, and the desire to improve delay leads to the employment of redundancy. The author first consider the minimum delay of 2-hop relay algorithms with redundancy and then design a protocol to achieve the minimum delay. As can be seen from Theorem 5, if only one transimission from a sender to a receiver is permitted in a cell, no 2-hop algorithm can provide an average delay lower than $O(\sqrt{nlogk})$. He proves this by considering the optimal transmission scheme and using the multi-destination reception style. A careful designed scheduling scheme is then shown following the lower delay bound. After examing the performance of the scheme , as is stated in Theorem 6, this 2-hop relay algorithm with redundancy achieves the $O(\sqrt{nlogk})$ with a capacity of $\Omega(1/(k\sqrt{nlogk})$.

C. More on Chen's multicast analysis

As can be seen above, many parts of Chen's works are originated from are similar to [1], which has already been carefully analysed in section II. There are innovating and creative ideas, however. The following lemma are taken from Chen's work, which are utilised to prove the corresponding theorems and lemma 2 within Chen's article.

- Theorem 3 in calculating the multicast capacity $O(1/N)$ of a cell partitioned network.
- Theorem 5 in proving the lower bound $O(\sqrt{N \log k})$ of delay in 2-hop algorithm with redundancy in multicast.
- Appendix II in calculating the expectation $\mathbb{E}\{\max\{X_1, X_2, \dots X_k\}\}$ when X_i has the same continuous distribution.

IV. Project Goals for Short-term and Long-term

After reading these two papers, we initially decided our project interest - multicast.

- *Short-term Goal*: Considering the incomplete part in [2], we first made our short-term goal to perfect it, i.e. discussing the capacity and delay tradeoff for the multi-hop scheme with multicast. In order to realize it, we came back to [1], and made some deeper understandings, which can be seen in Section V.
- *Long-term Goal*: Further more, we hope to investigate more about our interest to avoid repetition. And more importantly, trying to find something undeveloped but meaningful to go deep in. These work will be done in Section VI.

V. Further Study on Neely's Paper

In order to get some ideas to solve the short-term goal, we came back to the first paper and this time looked deeper into the detail proofs. By doing this we find something interesting.

A. Why Choose $\sqrt{N \log k}$ Redundancy for k Multicast

To explain this, we should mention the equation used to calculate the delay: $T_N = S_1 + S_2$, where S_1 represents the time required for the source to send out N^β (β is any number within [0,1)) replicas of the packet, and S_2 represents the time required to reach the destination given that N^β users have the packet.

- *The* $\mathbb{E}\{S_1\}$ *scale*: Use lemma 6 in [1], we get that $\mathbb{E}\{S_1\} \leq O(N^\beta)$. Add that each timeslot, at most one replica can be made, so $\mathbb{E}\{S_1\} \geq N^\beta$. Then we get the scale $\mathbb{E}\{S_1\} = O(N^\beta)$. $\square$
- *The* $\mathbb{E}\{S_2\}$ *scale*: Similar to the $\mathbb{E}\{S_2\}$ bounding part of lemma 3 in [1]. We have to change the 'M' to 'k'. And ψ becomes to the exact value $1-(1-1/C)^{N^\beta}$, which equals to $1-e^{-dN^{\beta-1}}$ when N becomes infinite. Then we directly get the higher bound:

$$\begin{aligned} \mathbb{E}\{S_2\} &\leq 1+\frac{1+\log k}{\log(1/(1-\psi))} \\ &\rightarrow 1+\frac{N^{1-\beta(1+\log k)}}{d} \\ &= O(N^{1-\beta}\log k) \end{aligned}$$

  On the other hand, the new variables $\{Y_1, Y_2, \ldots, Y_k\}$ used in lemma 3 are stochastically less than $\{X_1, X_2, \ldots, X_k\}$. So

$$\begin{aligned} \mathbb{E}\{S_2\} &= \mathbb{E}\{\max\{X_1, X_2, \ldots, X_k\}\} \\ &\geq \mathbb{E}\{\max\{Y_1, Y_2, \ldots, Y_k\}\} \end{aligned}$$

  From Appendix A, we have the following result

$$\begin{aligned} \mathbb{E}\{S_2\} &\geq \frac{\log(k+1)}{\log(1/(1-\psi))} \\ &\rightarrow \frac{N^{1-\beta\log(k+1)}}{d} \\ &= O(N^{1-\beta}\log k) \end{aligned}$$

  Hence, we get the final scale
  $\mathbb{E}\{S_2\} = O(N^{1-\beta}\log k)$. $\square$

Considering $T_N = S_1 + S_2$, the order of $\mathbb{E}\{T_N\}$ is the larger one between $\mathbb{E}\{S_1\}$ and $\mathbb{E}\{S_2\}$. Since

$$N^{1-\beta}\log k \times N^\beta = N\log k(independentof\beta)$$

We get the optimal result

$$\mathbb{E}\{T_N\} \geq O(\sqrt{N\log k})$$

where this lower bound can be realized by letting $N^\beta = \sqrt{N\log k}$, yielding the result. $\square$

B. Multi-hop in Multicast

When we looked back into the first paper, we found that the capacity and delay under multi-hop and multicast have already been included in it. For the lemma 3 calculates the total time T_N for a packet to reach all users. Then we came up with two questions.

1) Would this delay decrease as we only want k destinations receive the packet in stead of all the N users?
2) Could this capacity increase some degree without largely affecting the delay?

The first question seems to have a false answer, as the order $O(\log N)$ is almost the lower bound of all conditions. And the second one is not worthy either, for that if we increase the capacity, the interference caused from the simultaneous usage of the network would largely affect the delay.

So, our focus turns to the last part of the paper, to find the ratio of delay and capacity in general conditions.

C. Chain of Thought in Proving $\overline{W}/\lambda$

Fist of all, these following equations are the basic ones

$$\overline{W} = \frac{1}{N}\sum_i \overline{W_i} \tag{1}$$

$$\lambda\sum_{i=1}^N \overline{R_i} \leq N \tag{2}$$

In order to use the only known conditions described in (2), the author thought about a conditional expectation of $\overline{W_i}$ dependent of R_i. Undeniably, the choose of condition $R_i \leq 2\overline{R_i}$ is really a wise one. By using the property of $P_r[R_i \leq 2\overline{R_i}] \geq \frac{1}{2}$ for any nonnegative random variable R_i, the proof really simplified a lot. Though other ratios could be used, this mid-split value give a strong symmetry in the whole process. Thus, we now have

$$\overline{W_i} \geq \mathbb{E}\{W_i | R_i \leq \overline{R_i}\}\frac{1}{2} \tag{3}$$

Considering this condition, the author thought about a *restricted scheduling policy* which directly restricts the redundancy to the upper bound $2\overline{R_i}$, and let W_i^{rest} represent the corresponding delay. Furthermore, he cut the delay to a virtual system where $2\overline{R_i}$ redundancy has already exist and let Z represent the time required for one of these users to enter the same cell as the destination. Thus, Z has a clear geometric distribution with $\mathbb{E}\{Z\} = 1/\phi$, where $\phi = 1-(1-\frac{1}{C})^{2\overline{R_i}}$. A rigorous proof that W_i^{rest} is stochastically greater than Z is presented in Appendix B. Then we come to the Claim 1 result

$$\mathbb{E}\{W_i|R_i \leq 2\overline{R_i}\} \geq \inf_{\Theta} \mathbb{E}\{Z|\Theta\}^1 \tag{4}$$

Now our task is to solve this Z. But this geometric variable can not be easily calculated, and for the result of lemma 8 would be simpler to $\mathbb{E}\{X|X \leq w\}$ when the variable X is continuous. So the author made a couple variable $\tilde{Z}$, which is stochastically less than Z. And the result concerning $\tilde{Z}$ can be directly obtained in Appendix C. Here come to the Claim 2 result

$$\inf_{\Theta} \mathbb{E}\{Z|\Theta\} \geq \inf_{\tilde{\Theta}} \mathbb{E}\{\tilde{Z}|\tilde{\Theta}\}^2 = \frac{1-\log 2}{\gamma} \tag{5}$$

The following part is much simple and only mentioned about the Jensen's Inequality, which makes the combination between $\sum_{i=1}^{N} \frac{1}{R_i}$ and $\sum_{i=1}^{N} \overline{R_i}$. □

VI. History and Done

As we have initially came up with our interests, we would like to do some further study of this field, so we searched for papers concerning this area and got some useful information about its history and the recent productions.

A. Ad-Hoc Network History

First, Gupta and Kumar [5] initiated the investigation on how the throughput of static wireless networks scales with N. They did this work under assumption of common transmission range and fluid model, in which the packets are allowed to be arbitrarily small as $N \to \infty$. And the result is $\Theta(1/\sqrt{N \log N})$ of throughput. Later, [7] consolidated this result but with an explicit constant packet size model.

Then, in [8], with percolation theory, throughput increased to $\Theta(1/\sqrt{N})$ under the model that each node can adjust its transmission range instead of having a common one. However, the throughput vanishing problem for large-scale($N \to \infty$) static wireless networks still remains.

[4] overcomes this problem by exploiting the mobility of nodes. Specifically, the 2-hop relaying scheme they proposed achieves a constant throughput (i.e. $\gamma = O(1)$) at the cost of a large delay = $O(N)$ [6], [1]. This result reveals the possibility of trading larger delay for higher throughput or lower throughput for smaller delay in MANETs. Since then, a flurry of research activities have tried to characterize the throughput-delay relationship with respect to node mobility, e.g., [6],[1][9]-[16].

Among these study, there are generally two ways to trade throughput for delay.

1) Reduce delay by *increasing the transmission radius* of each relay node, which may be first found in [**?**] and implicitly under fluid model. This method would reduce the number os simultaneous transmissions the network can support, which lead to a lower throughput. Similar study can be seen in [6],[10]-[16].
2) Improve delay via *redundant packet transfers*, considered in [1],[17]. This method, in comparison, under the constant packet size model. Which is prefered in reality. And network coding is operated in this model.

B. Works about Multicast Having Done

As far as we know, the system model can be classified by four kinds of standards, by static or mobile, by numbers of hops, by i.i.d. or Markovian walking method, and by relaying or redundancy saving method. Till now, the static multicast has been fully studied. However, there remain plenty of opening questions in the mobile multicast system.

Here, our multicast system means multiple multicast. The one-hop and two-hop multicast with relaying or redundancy have been considered a lot, but there are still some questions left. Also, the above models are all with i.i.d. walking method.

In [18], the writers presents an overlay multicast protocol in mobile Ad Hoc network. However, his calculation method is still based on the unicast model. [19] talked about topology design of network-coding-based multicast networks, but obviously his model is a static multicast model as shown in his paper. [20] designed multicast protocols for non-cooperative networks. In this paper, the multicast by one-hop transmission method is discussed. Performance analysis for overlay multicast on tree and M-D mesh topologies are presented in [21]. Although multicast is studied, this paper is still limited in the static system. In [22], the author provided the consideration for security in multicast networks. After reading the related studies, we can easily find that, the multicast area has been focused on these years, and some results have been provided. However, with respect to our present research direction, the tradeoff of capacity and delay in multicast networks hasn't been provided yet. Although in [1], Neely considered the tradeoff in multicast system a little, a general conclusion is still an opening question, which is obviously very hard to achieve.

For short, there are two main opening questions now. One is the multicast multi-hop networks. Another one is the Markovian random walk in any system permutation.

In fact, the definition of relaying, redundancy and flooding varies from different papers. Each writer can have his own definition, which makes the classification harder here. For instance, the flooding defined by Neely in [1] is actually the redundancy multi-hop. As far as we know, the multi-hop is just getting started recently, and there lacks a general model which can indicate the general tradeoff performance of the system.

VII. Conclusion and Further Study

After repeatedly read the initial paper written by Neely, we got a lot of ideas about calculating the delay and capacity through probability theory and queueing theory, and made our

[1]The reason why he use Θ instead of $R \leq \overline{R_i}$ is that, despite two variables like X and Y, which have the same distribution, we can not get $\mathbb{E}\{X|\Phi\} = \mathbb{E}\{Y|\Phi\}$, because it is very likely that the condition Θ in X space has no meaning in Y space.

[2]this equation can be proved by using the property of stochastically less than relationship presented in Appendix D

short-term goal to search the tradeoff between throughput and delay with multicast in Ad-hoc Mobile Networks' models.

Additionally, based on the further and expander survey of history and recent works, we came up with some possible directions for choosing.

- *Converge-cast*, in which there are multiple sources transmitting packets to a single destination node.
- Assume a *traffic function*, say F(X), which assigns a permutation of uni-cast, multi-cast and broadcast within the system.
- From the respect of *walking method*, since the Markovian random walk has not been fully discussed, we could also develop this area to achieve some exciting discoveries.

APPENDIX A – ORDER OF $\sum_{k=1}^{n} \frac{1}{i}$

Solving this problem, we have to prove the following bound

Lemma 1:

$$\ln(n+1) < \sum_{k=1}^{n} \frac{1}{k} < \ln n + 1$$

Proof: Here we use the integral way as follows.

$$\begin{aligned} \frac{1}{k+1} &< \int_{k}^{k+1} \frac{1}{x} dx < \frac{1}{k} \\ \Rightarrow \sum_{k=1}^{n} \frac{1}{k+1} &< \sum_{k=1}^{n} \int_{k}^{k+1} \frac{1}{x} dx < \sum_{k=1}^{n} \frac{1}{k} \\ \Rightarrow \sum_{k=1}^{n} \frac{1}{k+1} &< \int_{1}^{n+1} \frac{1}{x} dx < \sum_{k=1}^{n} \frac{1}{k} \\ \Rightarrow \sum_{k=1}^{n} \frac{1}{k+1} &< \ln(n+1) < \sum_{k=1}^{n} \frac{1}{k} \end{aligned}$$

■

Hence we get the both bounds of $\sum_{k=1}^{n} \frac{1}{i}$ from above, and can easily obtain the order.

$$\sum_{k=1}^{n} \frac{1}{i} = O(\ln n) = O(\log n)$$

APPENDIX B – RELATIONSHIP BETWEEN W_i^{rest} AND Z

We let S_k and P_k represent the time required when redundancy is k and the probability of this condition, respectively. Thus $k \leq 2\overline{R_i}$ Separate S_k to X_k and Y_k, which represent the time to reach k users and the time to encounter the destination given i users holding this packet, respectively. Then we have

$$\begin{aligned} \mathbb{E}\{W_i^{rest}\} &= \sum_k \mathbb{E}\{S_k\} P_k \\ &= \sum_k \mathbb{E}\{(X_k + Y_k)\} P_k \\ &> \sum_k \mathbb{E}\{(Y_k)\} P_k \\ &= \sum_k [1 - (1 - \frac{1}{C})^k]^{-1} P_k \\ &\geq \sum_k [1 - (1 - \frac{1}{C})^{2\overline{R_i}}]^{-1} P_k \\ &= \mathbb{E}\{Z\} \sum_k P_k \\ &= \mathbb{E}\{Z\} \end{aligned}$$

From the property in [3] we have the result that W_i^{rest} is stochastically larger than Z.

APPENDIX C – $\mathbb{E}\{X|X \leq w\}$ FOR CONTINUOUS VARIABLE X

For w is the value which $P_r[X > w] = e^{-\gamma w} = \frac{1}{2}$, we have that $w = \frac{\log 2}{\gamma}$. And

$$\begin{aligned} \mathbb{E}\{X|X \leq w\} &= \frac{\mathbb{E}\{X\} - \mathbb{E}\{X|X > w\} P_r[X > w]}{P_r[X \leq w]} \\ &= \frac{\frac{1}{\gamma} - \mathbb{E}\{X|X > w\}\frac{1}{2}}{\frac{1}{2}} \end{aligned}$$

Then we have to calculate $\mathbb{E}\{X|X > w\}$. Considering this

$$P_r[X = w + y] = \gamma e^{-\gamma(w+y)} = \gamma e^{-\gamma w} e^{-\gamma y} = \frac{1}{2}\gamma e^{-\gamma y}$$

We have

$$\begin{aligned} \mathbb{E}\{X|X > w\} &= \int_w^{\infty} x \frac{P_r[X = x]}{P_r[X > w]} dx \\ &= \int_0^{\infty} (w + y) \frac{\frac{1}{2}\gamma e^{-\gamma y}}{\frac{1}{2}} dy \\ &= w \int_0^{\infty} \gamma e^{-\gamma y} dy + \int_0^{\infty} y P_r[X = y] dy \\ &= w + \mathbb{E}\{X\} \\ &= w + \frac{1}{\gamma} \end{aligned}$$

So we can back to the original equation and get

$$\mathbb{E}\{X|X \leq w\} = \frac{\frac{1}{\gamma} - (w + \frac{1}{\gamma}\frac{1}{2})}{\frac{1}{2}} = \frac{1 - \log 2}{\gamma}$$

APPENDIX D – COUPLING VARIABLE

Lemma 2: Given that two nonnegative distributions G and F satisfied that G is stochastically less than F, i.e. for any variables X and Y having distributions G and F respectively, $P_r[X > w] \leq P_r[Y > w], \forall w \geq 0$.Then there must exist two variables X and Y having G and F distributions respectively, and $P_r[X \leq Y] = 1$

Proof: For a certain variable Y having distribution F, create a variable X satisfied $X = G^{-1}[F(Y)]$. Then

$$\begin{aligned} P_r[X \leq x] &= P_r\{G^{-1}[F(Y)] \leq x\} \\ &= P_r\{F(Y) \leq G(x)\} \\ &= P_r\{Y \leq F^{-1}[G(x)]\} \\ &= F\{F^{-1}[G(x)]\} \\ &= G(x) \end{aligned}$$

Thus X has the distribution of G. Furthermore, from the definition, $1 - G(x) \leq 1 - F(x)$, we have $G^{-1}(x) \leq F^{-1}(x)$ for any nonnegative value x.Then we have

$$X = G^{-1}[F(Y)] \leq F^{-1}[F(Y)] = Y$$

∎

This is one of the properties with stochastic relationship. Another one should be mentioned is that like the above distributions G and F, and the corresponding variables X and Y, we have

$$\mathbb{E}\{X\} \leq \mathbb{E}\{Y\}$$

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