The Capacity Gain of Ad Hoc Wireless Network Employing Network Coding(Report I)

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Abstract

Wireless Ad Hoc network suffers from a lot of problems such as low throughput and hot and dead spots. Gupta and Kumar have established the throughput of ad hoc networks only for the unicast pattern which is not effective in application. And they did not take the possible network coding and broadcast and multicast pattern into consideration. There has been interest in applying network coding in ad hoc networks. In this paper, we explore the upper bound of capacity of ad hoc networks for different network models.

1 Introduction

Wireless networks consist of a number of nodes communicating with each other over wireless channels. The architecture can be roughly divided into two categories, the cellular paradigm and ad hoc paradigm.

In recent years most of the wireless communication relies on base stations which is responsible of controlling all the transmissions in the certain region and forwarding data to the prospective receivers. Base stations restrict the area of communication service so people are studying a new technology that is Ad Hoc, in which all the nodes have the same responsibility in transmissions. Basically there are two major advantages of this kind of network. First, no base station is needed. Second, in this network there is no centralized control node so if part of the network break down, the system can still work. In Ad Hoc network, two mobile nodes can reach each other in the range of communication. But if they are far from each other, they must use another node between them to help communicate.Since the range of communication is limited, route consists of many hops.

The characteristics of Ad Hoc network are shown in the following.

1. Independence
   No basic hardware devices are necessary to support this network. So it is easy to setup this mobile network. That is meaningful for rescue or communication in remote places.
2. Changeable topological structure
   Mobile nodes can travel in this network freely. Thus, links between nodes will absolutely change.
3. Limited bandwidth
   Due to the physical characteristics of wireless channels, it is impossible to provide large bandwidth.
4. Limited node energy supply
   In this networks, nodes are almost some mobile devices such as PDA or portable computers whose energy is from battery.
5. Distribution characteristic
   Once some nodes in the network break down, the rest nodes can still work well.
6. Short period of existence
   Ad Hoc is used for temporary communication so its period of existence is short.
7. Limited physical safety
   Mobile network is easier to be attacked in ways such as interception.

Multi-hop wireless network has been intensively studied recently. Due to its flexibility, it can be deployed randomly in the geographic regions to obtain a large scale of information and provide network services. Therefore, the capacity and connectivity of such network are of great interest.

Gupta and Kumar[1] first come up with the idea of the capacity of wireless network. The main conclusion is that when n identical randomly located nodes, each capable of transmitting at W bits per second and using a fixed range, form a wireless network, the throughput $\lambda(n)$obtainable by each node for a randomly chosen destination is $\Theta(\frac{W}{\sqrt{nlogn}})$ bits per second under a noninterference protocol.If the nodes are optimally placed in a disk of unit area, traffic patterns are optimally assigned, and each transmissions range is optimally chosen, the bit-distance product that can be transported by the network per second is $\Theta(W\sqrt{n})$ bit-meters per second. Thus even under optimal circumstances, the throughput is only $\Theta(\frac{W}{\sqrt{n}})$.

It shows that for a single-channel single-interface scenario, in a randomly deployed network, per-flow capacity scales as $\Theta(\frac{W}{\sqrt{nlogn}})$bits/s under a Protocol Model of interference, and that if the available bandwidth W is split into c channels, with each node having a dedicated interface per channel, the results remain the same.V Bhandari, NH Vaidya[8] proposed present a lower bound construction that matches the previous upper bound with capacity as$\Theta(W\sqrt{\frac{p_{rnd}}{nlogn}})$ under multi-channel model.

In the Gupta and Kumar's[1] seminal work, they proposed a model for studying the capacity of fixed ad hoc networks, where nodes are randomly located but are immobile. They have shown that for a random ad hoc network of size n(nodes per unit area), the per node throughput capacity is $\Theta(\frac{1}{\sqrt{nlogn}})$. This is a negative result as it implies that ad hoc networks might not be scalable. In order to increase the capacity of wireless network, Grossglauser and Tse[2] proposed a 2-hop relaying algorithm and showed that it can achieve a throughput capacity of $\Theta(1)$per node by adding mobility. However, they did not consider the delay. Delay limited capacity of ad hoc networks has recently been addressed in [3]. The authors have obtained an approximate expression for the achievable throughput capacity under a maximum delay constraint. In order to solve the trade-offs of mobility, delay and capacity, G Sharma, R Mazumdar, NB Shroff[4] have proposed two different classes of mobility models and showed that they both exhibit critical delays which is inversely proportional to the characteristic path length. Bansal and Liu[5] considered a wireless network consisting of static sender-destination(S-D) pairs and mobile relays, and proposed a geographic routing scheme that achieves a near optimal capacity $O(Wm/n)$and studied its delay performance. Q Dai, L Rong, H Hu[6] derived the analytical expression of mathematical expect value on the capacity in hybrid wireless networks with delay and mobility. A Ozgur, O Leveque[7] proposed hierarchical cooperation to achieve better throughput scaling than classical multihop schemes in static wireless networks. Right now, the core problem of capacity associated with delay and mobility is far from finishing.

Besides, recent work by Fragouli and Katabi[9] introduced the concept of network coding, and there are tremendous interest results when employing network coding to wireless network. [9] shows that the upper bound can be increased by a factor of 2 for the multicast pattern, which is rather satisfactory.

The main concept of [9] is to broadcast coded information through intersecting flows and the next hop of each relay is able to decode the information with the received coded information based on all the former broadcasts and local information. In this case, each node in the relay can broadcast the information to its neighbor nodes through only one transmission instead of multiple data flows. The Figure 1 provides an example of this concept. Assume two sources S1 and S2 are transmitting information both taking the way of node R which serves as a router, to the destination D1 and D2 respectively. With the assumption of broadcast pattern, D1 can receive packets from S2, and D2 can hear from S1 in the same case. The node R can combine the packets from both sources with the method of XORing them. The XORed version of packets is useful for both destination nodes. The respectively information can be easily obtained when XORing again the XORed packets and local ones heard from neighbor source. This pattern is the so-called "X" topology. We can also employ this concept to other patterns in which router nodes exit.

The intuition is that network coding can significantly improve the throughput of ad hoc network because it allows the router node to compress the multiple packets into a single XORed version, so that the number of transmissions is reduced.

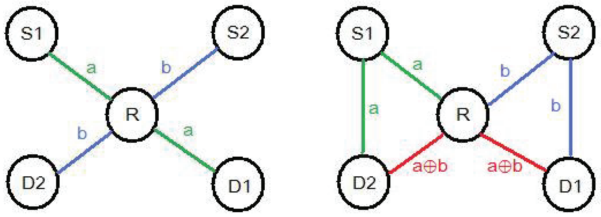

Figure 1: X topology

S. Katti and H. Rahul[10] also proposed the mechanism COPE, a new forwarding architecture that substantially improve the capacity of stationary wireless networks, by inserting a code shim between the IP and MAC layers. The concept of COPE has been put forward by a lot of research work. S. Sengupta and S. Rayanchu[11] employed the COPE-type network coding in traditional unicast network and provide a method for computing source-destination routes and utilizing the best coding opportunities from available ones to maximize the throughput. However, this kind of design often ignore the underlying Phy layer capability and algorithms. Therefore, these schemes are so greedy that it might in fact reduce the throughput of wireless network. P. Chaporkar and A. Proutiere[12] proposed a general method to develop optimal and adaptive joint network coding ana employed it to both COPE-type and XOR-Sym architectures.

2 Model Formulation

In this section, we give some proof and explanations on the

capacity which are mainly about the Gupta and Kumar's work. We list them here just for convenience. In report 1, we only deal with the upper bound of arbitrary networks in both protocol and physical model.

Arbitrary Networks: In the arbitrary network setting, n nodes are located in a certain region, and each node has a destination chosen in advance. That means the location of nodes and traffic patterns can be controlled.

Random Networks: In the random network setting, nodes' location are randomly chosen, i.e. independently and uniformly distributed, in a certain region. Each node also has a randomly chosen destination, which might be the one nearest to the randomly located node.

2.1 Important Assumptions

The conclusions of the report are based on the following assumptions which are very important for the following proof. The assumptions are as follows:

1. There are n nodes arbitrarily located in a disk of unit area on the plane. (The results carry over to any domain of unit area in R^2 which is the closure of its interior.)
2. The network transports λnT bits over T seconds.
3. The average distance between the source and destination of a bit is $\overline{L}$ Note that, together with (2), this implies that a transport capacity of $\lambda n\overline{L}$ bit-meters per second is achieved.
4. Each node can transmit over any subset of M subchannels with capacities W_m bits per second,$1 \leqslant m \leqslant M$, where $\sum_{m=1}^{M} W_m = W$.
5. Transmissions are slotted into synchronized slots of length τ seconds. (This assumption can be eliminated, but makes the exposition easier.)
6. While retaining the restriction (2) for the case of the Physical Model, we can either retain (1) in the Protocol Model or consider an alternate restriction as follows: If a node X_i transmits to another node X_j located at a distance of units on a certain subchannel in a certain slot, then there can be no other receiver within a radius $\triangle r$ of around X_j on the same subchannel in the same slot. This alternate restriction addresses situations where the transmissions are not omnidirectional, but nevertheless there is some dispersion in the neighborhood of the receiver.

2.2 Protocol Model

The Definition of Protocol Model

Suppose node X_i transmits over the m th subchannel to a node X_j. Then this transmission is successfully received by node X_j if

$$|X_k - X_i| \geq (1+ \triangle)|X_i - X_j| \tag{1}$$

for every other node X_k simultaneously transmitting over the same subchannel.The quantity $\triangle > 0$ models situations where a guard zone is specified by the protocol to prevent a neighboring node from transmitting on the same subchannel at the same time. It also allows for imprecision in the achieved range of transmissions.

Main Results and Proof

result(1): In the protocol Model, the transport capacity $\lambda n\overline{L}$ is bounded as follows:

$$\lambda n\overline{L} \leq \frac{\sqrt{8}}{\pi}\frac{1}{\triangle}W\sqrt{n}$$

bits per second.

proof:Comsider bit b ($1 \leq b \leq \lambda nT$).Suppose it that it moves from the origin to its destination in a sequence of $h(b)$ hops, where the hth hop transvers a distance of r_b^h. Then from the assumption(iii), we can get that

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} r_b^h \geq \lambda nT\overline{L}$$

(Because the bit b is counted for at least once.)

Note that in any slot at most $n/2$ nodes can transmit, for if there are more than $n/2$ nodes which transmit, there will be less than $n/2$ nodes that receive.Hence for any subchannel m and any slot s

$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} 1$(The hth hop of bit b is over subchannel m in slot s)$\leq \frac{W_m\tau n}{2}$

Summing over the subchannels and the slots, and noting that there can be no more than $\frac{T}{\tau}$ slots in T seconds, thus

$$H := \sum_{b=1}^{\lambda nT} h(b) \leq \frac{WTn}{2}$$

.

Suppose thatX_j is receiving a transmission from X_i over the m th subchannel at the same time that X_l is receiving a transmission from X_k over the same subchannel. Then from the triangle inequality and the definition of the Protocol Model.

$$|X_j - X_l| \geq |X_j - X_k| - |X_l - X_k| \geq (1+\triangle)|X_i - X_j| - |X_l - X_k|$$

Similarly,

$$|X_l - X_j| \geq (1+\triangle)|X_k - X_l| - |X_j - X_i|$$

Adding together, we can get:

$$|X_l - X_j| \geq \frac{\triangle}{2}(|X_k - X_l| + |X_i - X_j|)$$

Hence disks of radius $\frac{\triangle}{2}$ times the lengths of hops centered at the receivers over the same subchannel in the same slot are essentially disjoint.

Note that at least a quarter of such a disk is within the domain.(for the minimum situation is that the origin is located at one of the four edges of the square).Since at most $W_m\tau$ bits can be carried in slot from a receiver to a transmitter over the m th subchannel, we have

$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)}$ 1(The hth hop of bit b is over subchannel m in slot s)$\frac{\pi\triangle^2}{16}(r_b^h)^2 \leq W_m\tau$

Summing over the subchannels and the slots gives

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)}\frac{\pi\triangle^2}{16}(r_b^h)^2 \leq WT$$

Because

$$H := \sum_{b=1}^{\lambda nT} h(b) \leq \frac{WTn}{2}$$

We can get

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)}\frac{1}{H}(r_b^h)^2 \leq \frac{16WT}{\pi\triangle^2 H}$$

Note that the quadratic function is convex.Hence

$$(\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)}\frac{1}{H}r_b^h)^2 \leq \sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)}\frac{1}{H}(r_b^h)^2$$

Thus

$$\lambda nT\bar{L} \leq \sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} r_b^h \leq \sqrt{\frac{16WTH}{\pi\triangle^2}}$$

And

$$H := \sum_{b=1}^{\lambda nT} h(b)$$

We can get the conclusion:

$$\lambda n\bar{L} \leq \frac{\sqrt{8}}{\pi}\frac{1}{\triangle}W\sqrt{n}$$

2.3 Physical Model

The Definition of Physical Model

In the physical model, the model is mainly constructed by the power level between transmission and receiver nodes compared with the protocol model which is set up on the distance. let $\{X_k; k \in T\}$ be the subset of nodes simultaneously transmitting at some time instant over a certain subchannel. Let P_k be the power level chosen by node $X_k, fork \in T$.Then the transmission from a node $X_i, i \in T$, is successfully received by a node X_j if

$$\frac{\frac{P_i}{|X_i - X_j|^\alpha}}{N + \sum\limits_{k\in T, k\neq i}\frac{P_i}{|X_k - X_j|^\alpha}} \geq \beta$$

This models a situation where a minimum signal-to-interference ratio (SIR) of β is necessary for successful receptions, the ambient noise power level is , and signal power decays with distance γ as $\frac{1}{\gamma^\alpha}$.

Main Results and Proof

result(2): In the physical Model,

$$\lambda n\bar{L} \leq (\frac{2\beta + 2}{\beta})^{1/\alpha}\frac{1}{\sqrt{\pi}}Wn^{\alpha - 1/\alpha}$$

bit-meters per second.

result(3): If the ratio $\frac{P_{max}}{P_{min}}$ between the maximum and minimum powers that transmitters can employ is strictly bounded above by β, then

$$\lambda n\bar{L} \leq \sqrt{\frac{8}{\pi}}\frac{1}{(\frac{\beta P_{min}}{P_{max}})^{1/\alpha} - 1}W\sqrt{n}$$

bit-meters per second.

proof: Including the signal power X_i also in the denominator, the SIR must be a figure less than 1. In order to get the equation of β, let SIR denotes $\frac{\beta}{\beta+1}$. Therefore

$$\frac{\frac{P_i}{|X_i - X_{j(i)}|^\alpha}}{N + \sum\limits_{k\in T}\frac{P_i}{|X_k - X_{j(i)}|^\alpha}} \geq \frac{\beta}{\beta + 1}$$

Because the maximum distance between X_k and $X_{j(i)}$ is the diameter of the radius(ie. $|X_k - X_{j(i)}| \leq \frac{2}{\sqrt{\pi}}$), there is the following inequality

$$|X_i - X_{j(i)}|^\alpha \leq \frac{\beta + 1}{\beta}\frac{P_i}{N + \sum\limits_{k\in T}\frac{P_k}{|X_k - X_{j(i)}|^\alpha}}$$
$$\leq \frac{\beta + 1}{\beta}\frac{P_i}{N + (\frac{\pi}{4})^{\alpha/2}\sum\limits_{k\in T}P_k}$$

Summing over all transmitter-receiver pairs

$$\sum_{i\in T}|X_i - X_{j(i)}|^\alpha \leq \frac{\beta + 1}{\beta}\frac{\sum\limits_{i\in T}P_i}{N + (\frac{\pi}{4})^{\alpha/2}\sum\limits_{k\in T}P_k}$$
$$\leq \frac{\beta + 1}{\beta}\frac{\sum\limits_{i\in T}P_i}{(\frac{\pi}{4})^{\alpha/2}\sum\limits_{k\in T}P_k} \leq 2^\alpha\pi^{-(\alpha/2)}\frac{\beta + 1}{\beta}$$

Noting that $|X_i - X_{j(i)}|$ denotes $\gamma^{\alpha}(h,b)$, summing over all slots and subchannels gives

$$\sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} \gamma^{\alpha}(h,b) \leq 2^{\alpha} \pi^{-(\alpha/2)} \frac{\beta+1}{\beta} WT$$

Similar to the Protocol Model, invoking the convexity of γ^{α} instead of γ^2,

$$(\sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} \frac{1}{H} \gamma(h,b))^{\alpha} \leq \sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} \frac{1}{H} \gamma^{\alpha}(h,b)$$

Because $H \leq \frac{WTn}{2}$, we get the conclusion

$$\lambda n \overline{L} \leq (\frac{2\beta+2}{\beta})^{1/\alpha} \frac{1}{\sqrt{\pi}} W n^{\alpha - 1/\alpha}$$

If X_i is transmitting to X_j at the same time that $X_k$is transmitting to X_l, both over the same subchannel, then $\frac{\frac{P_i}{|X_i - X_j|^{\alpha}}}{\frac{P_k}{|X_k - X_j|^{\alpha}}} \geq \beta$, thus

$$|X_k - X_j| \geq (\frac{\beta P_{min}}{P_{max}})^{\frac{1}{\alpha}} |X_i - X_j| = (1 + \Delta)|X_i - X_j|$$

where $\Delta := (\frac{\beta P_{min}}{P_{max}})^{\frac{1}{\alpha}} - 1$. Thus the same upper bound as for the Protocol Model carries over with Δ defined as above and leads to the main result(3).

3 To Do

1. Take a further insight of the Gupta and Kumar's work[1], mainly in the random network model. Provide necessary proof of the essential concepts.
2. Find out the effective mechanism of network coding and explore the gain of capacity when employing it in the wireless network.

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