Compressive Sensing Report 1

Group 11

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***Abstract*—In this report, we give a brief introduction to the recently very popular topic–compressive sensing(CS for short), we explain the advantages of CS over traditional sampling, and discuss the sample methods and reconstruction algorithms, also we suggest several applications in wireless communication which we might concentrate later in work.**

***Index Terms*—sparsity, incoherence, convex optimization, UWB.**

I. Introduction

As Nyquist sampling theorem suggests, to avoid missing information about a signal, one should sample at least twice the highest frequency of the signal bandwidth. But in many situations, such as medical imaging and video capturing, the Nyquist rate is so high that it is very expensive to implement.However, since signals are sparse in many occasions, they can be represented sparsely on certain basis, such as spikes, sinusoids, wavelets, Gabor functions, curvelets and so on. In this case, if we sample the signal randomly much fewer times than the origin signal length, we can reconstruct the signal exactly with a high probability. Consequently, we sample the signal at a average much lower rate than the Nyquist rate. In wireless communication systems, there are many cases of sampling and coding, by applying this cs technique, we can improve system capability dynamically.

II. Review of Compressive Sensing

A. sampling

consider a real-valued, finite-length, one-dimensional, discrete-time signal $\mathbf{x}$ with length N, which can be viewed as N column vectors in R^N, then any signal can be represented through a $N \times N$ basis matrix Ψ, whose raw vectors are ψ_i, i=1,2,3...N. Then the signal can be expressed as:

$$\mathbf{x} = \sum_{i=1}^{N} \psi_i * s_i, s_i iselementof \mathbf{s}$$

Here, the signal $mathbfx$ is *K-sparse* if only K elements in $mathbfs$ are none-zero, where $K \ll$ N.For such kinds of signals, we can sample them with a measurement matrix to get M linear projections of $\mathbf{s}$, by carefully design the measurement matrix Φ, we can restore complete information about the signal. Consider the example in figure1, the CS measurement proceed with a random Gaussian measurement matrix Φ and discrete cosine transform (DCT) matrix Ψ.Here the signal $mathbfx$ is 4-sparse, as we can see, by dimension reduction, the sample become more concentrated than the origin signal. Since Φ is gaussian random and Ψ is a fixed DCT pattern, the result Θ will also be gaussian random.

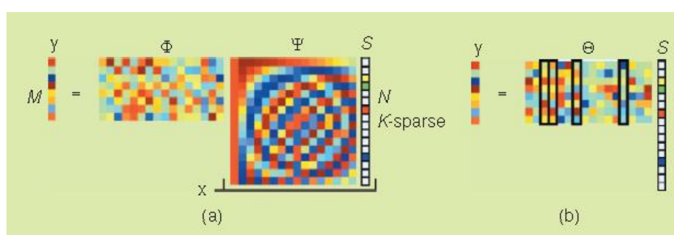

Fig. 1.

We will discuss the constraints on the size of sample matrix in the next section.

B. reconstruction

Another key term is how to testify the efficiency of the matrix and guarantee the exact recovery of the sparse or compressive signal. Terance Tao has proved that if we construct the sample by uniformly selecting M DFT samples at random in frequency domain,then for a certain parameter μ, if K satisfiy

$$|K| < C_\mu \cdot (logN)^{-1} \cdot |M|$$

then with probability 1-$O(N^\mu)$, the solution to certain optimization problem is unique and equal to the origin signal $mathbfs$, which we will discuss later. More generally, a sufficient condition for a stable solution is refered to as RIP:
For any arbitrary 3K-sparse ν and some ε¿0, if

$$1 - \varepsilon \leq \frac{||\Theta\nu||_2}{||\nu||_2} \leq 1 + \varepsilon$$

then the K-sparse signal is unique and stable. A related condition is that ΦandΨmatrix should be incoherent. We will proceed using these conclusions without prove them in details.

Suppose we've constructed some deterministic nonadaptive measurement matrix, to recover the signal, we would solve the equation:

$$\Theta s = y$$

since Θ is of size $M \times N$, the solution to this equation is not unique. What we expect is s' = argmin$||s||_0$ also called p0 problem.However this is a combinational optimization problem and also a NP-complete, which indicates complexity and unstability of the solver. Other alternatives are the p1 and p2 problem. p2 type is the minimum energy solution of this equation and has many convenient methods to solve, however, as we will see later, p2 solution has a very low probability of matching the p0 solution. For p1 type, we can see it match the p0 type with a much higher probability although the algorithm is more limited than the that of the p2 type. In figure 2, a K=3-sparse signal is used. The plane is the solution of the equation, in (b)the l2 ball has very low probability to be sparse while in (c)the l1 ball has high probability to match the sparse condition.

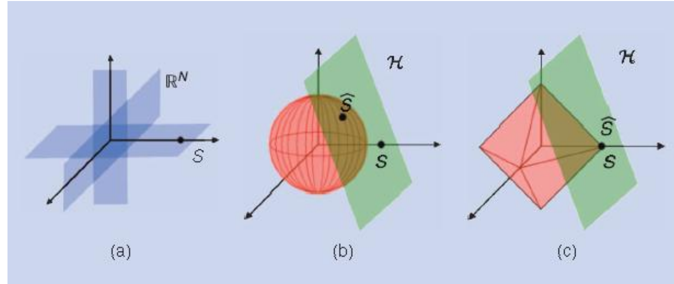

Fig. 2.

For the p1 optimization problem, we can apply several traditional linear programming such as Basic Pursuit or OMP which we will use in the next section.

C. *experiment*

For simplicity, all the examples are 1D signals in this report.In this section, we try to get familiar with the construction and reconstruction process in CS by transferring directly a sparse impulse train signal into sample vectors. We tried two methods to construct the measurement matrix, iid gaussian matrix and PN-shift matrix and compare the performance of them.
First, we create a realvalued discrete signal with length N=100 and have K=10 spikes, then we create measurement matrix Θ with a size of 40×100 following the steps below:
1.iid-Gaussian matrix:$\Theta = randn(40, 100)$,to improve performance, we make the operation $\Theta = orth(\Theta')'$ to guarantee the raws of the matrix are orthogonal. The reconstruction algorithm is primal-dual in Basic Pursuit, we will not discuss this method in this report.As we can see in figure 3 is the gaussian measurement matrix, figure 4 is the original signal, figure 5 is the p2 solution of the equation, which badly-matches the sparse condition, figure 6 is the p1 solution, with $MSE = 2.8869^{-4}$.

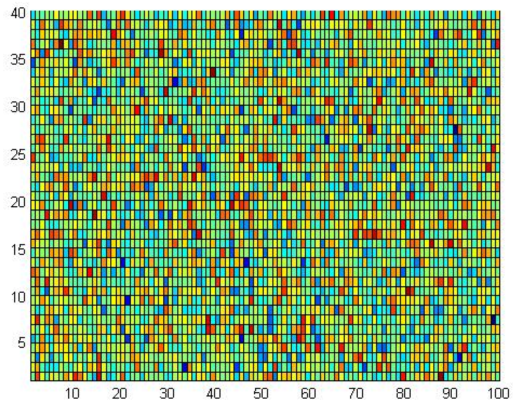

Fig. 3.

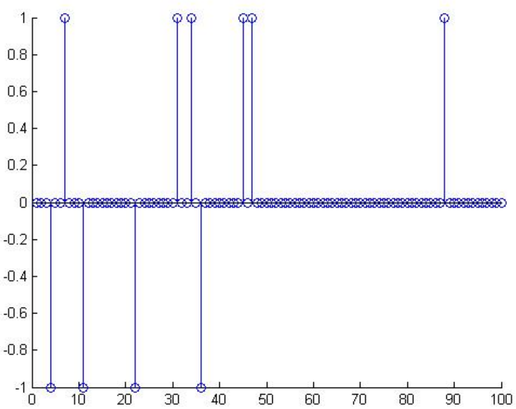

Fig. 4.

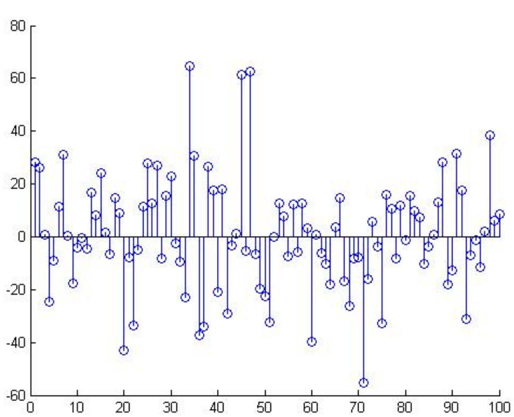

Fig. 5.

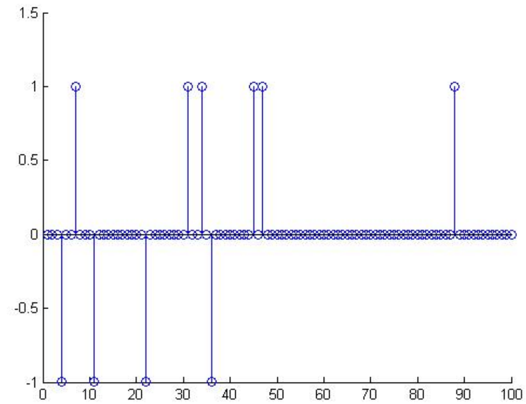

Fig. 6.

2.shifted PNsequence matrix:$\Theta_{m,n} = pn([N/M]m - n)$,where pn is the PN sequence, whose we assume to be N,the matrix generated are in figure 7, the reconstruction algorithm is the same as above. As we can see in figure 10, the signal recovered has missed some information due to the somewhat ill-designed matrix, the $MSE = 1.9752$.

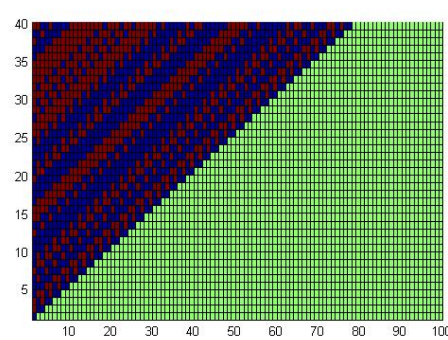

Fig. 7.

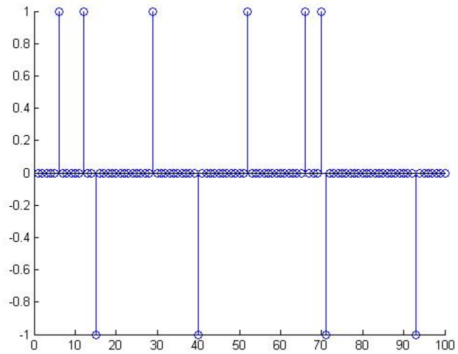

Fig. 8.

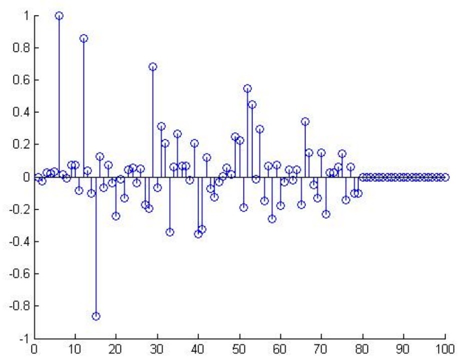

Fig. 9.

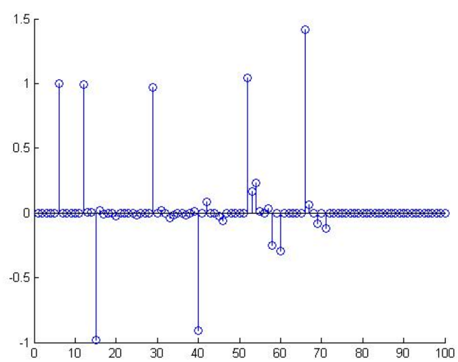

Fig. 10.

How to design a efficient deterministic matrix is of key importance, noticing Iwen has given some kind of powerful matrix in his writings, we will study more on this topic.

III. What to do next

A. learn more about CS

After several days' learning,we've got the basic idea of CS,however,we still have to spend much more time on studying the following facets of CS :

1:We haven't quite understood the details of the so many Theorems and Lammas,which are important for us to deeply absorb the ideas of CS,we will concentrate more on two classical papers[2]and[3],in which the authors have done excellent theoretical work.

2:We will read carefully the paper of Iwen *Simple Deterministically Constructible RIP Matrices with Sublinear Fourier Sampling Requirements* to learn a new method to construct the deterministic RIP matrix,which is significant for signal sampling.Then we can design some high-performance measurement rather than the limited two kinds of matrix used in the experiment.

3:We should do more experiments and simulations with the help of l1magic matlab package that Mr. liang offers us,and get more information about convex optimization.

B. Applying CS in wireless communication system

Generally,the CS technique is widely used in almost every area requiring sampling or(and) coding,in wireless communication systems,CS can improve the system capacity in many occasions,as we know,in OFDM system,CS can be used to reduce carrier and thus improve spectrum effciency.Also in MIMO system,it can be proved that the MIMO channels are sparse thus CS can further improve system throughput.

What we may continue to study is the UWB system,since in certain thus system,signals are transmitted by extreme short modulated impulses,which are obviously sparse in time,however,we should at first learn more about the UWB system and explore ways to apply CS in thus systems.

IV. Conclusion:

We've worked together for long to learn related techniques and do this report,however,we have to say,we've indeed choose a difficult topic since there are a lot of mathmetical deductions in related papers and materials,which seems forbidden for us ,however,the topic itself is really exciting in that it overthrows some traditional understandings of sampling and we hope to apply this magic technique in certain wireless communication system, maybe UWB.

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