Introduction to compressed sensing

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ABSTRACT In this passage we want to introduce the compressed sensing roughly in from the aspects of its defination,the early papers on compressive sampling, and the application.

Conventional wisdom and common practice in acquisition and reconstruction of images from frequency data follow the basic principle of the Nyquist density sampling theory. This principle states that to reconstruct an signal, the number of Fourier samples we need to acquire must match the desired resolution of the signal, for example the number of pixels in a image. But as our modern technology-driven civilization acquires and exploits ever-increasing amounts of data, It is acknowledged that most of the data we acquire can be thrown away with almost no perceptual losswitness the broad success of lossy compression formats for sounds, images, and specialized technical data. The phenomenon of ubiquitous compressibility raises very natural questions: why go to so much effort to acquire all the data when most of what we get will be thrown away? Can we not just directly measure the part that will not end up being thrown away? Now emerging the theory which goes by the name of compressive sampling, and which says that, perhaps surprisingly, it is possible to reconstruct images or signals of scientific interest accurately and sometimes even exactly from a number of samples which is far smaller than the desired resolution of the image/signal, e.g. the number of pixels in the image. The field has existed for at least four decades, but recently the field has exploded, in part due to several important results by David Donoho, Emmanuel Candes, Justin Romberg and Terence Tao.[A]

Compressive sampling, also known as compressive sensing, compressed sensing and sparse sampling, is a technique for acquiring and reconstructing a signal that is sparse or compressible. Suppose there exists a coding matrix it can compress a signal with length by the equation to get a (proximately) recovery of maximum items of where the length of is only . This kind of method coding x into very short y, but capable of recovering most of xs information at the same time is very useful under specific circumstances. So how to get that matrix is the prioror task. The main idea behind compressed sensing is to exploit that there is some structure and redundancy in most interesting signalsthey are not pure noise. In particular, most signals are sparse, that is, they contain many coefficients close to or equal to zero, when represented in some domain. (This is the same insight used in many forms of lossy compression.) Compressed sensing typically starts with taking a limited (possibly randomized) amount of samples in a different basis from the basis the signal is known to be sparse in. Since the amounts of samples are limited, the task of converting the image back into the intended domain would involve solving an underdetermined matrix equationthat is, there is a huge amount of different candidate images that could all result in the given samples, since the number of coefficients in the full image are fewer than the number of samples taken. Thus, one must introduce some additional constraint to select the best candidate. The classical solution to such problems would be minimizing the L2 normthat is, minimizing the amount of energy in the system. This is usually simple mathematically (involving only a matrix multiplication by the pseudo-inverse of the basis sampled in). However, this leads to poor results for most practical applications, as the unknown (not sampled) coefficients seldom have zero energy. A more attractive solution would be minimizing the L0 norm, or equivalently maximize the number of zero coefficients in the new basis. However, this is NP-hard (it contains the subset-sum problem), and so is computationally infeasible for all but the tiniest data sets. Thus, following Tao et al, the L1 norm, or the sum of the absolute values, is usually what is minimized. Finding the candidate with the smallest L1 norm can be expressed relatively easily as a linear program, for which efficient solution methods already exist. This leads to comparable results as using the L0 norm, often yielding results with many coefficients being zero.(wiki) Suppose is an unknown vector in (a digital image or signal); and it has a sparse representation in some orthonormal basis (e.g., wavelet, Fourier) or tight frame (e.g., curvelet, Gabor)so the coefficients belong to an ball for 0 1. The most important coefficients in that expansion allow reconstruction with 2 error (1 2 1). It is possible to design = (log( )) nonadaptive measurements allowing reconstruction with accuracy comparable to that attainable with direct knowledge of the most important coefficients. Moreover, a good approximation to those important coefficients is extracted from the measurements by solving a linear programBasis Pursuit in signal processing. The nonadaptive measurements have the character of random linear combinations of basis/frame elements. Our results use the notions of optimal recovery, of -widths, and information-based complexity. We estimate the Gelfand -widths of balls in high-dimensional Euclidean space in the case 0 1, and give a criterion identifying near-optimal subspaces for Gelfand -widths. We show that most subspaces are near-optimal, and show that convex optimization (Basis Pursuit) is a near-optimal way to extract information derived from these near-optimal subspaces.[C]

The early papers on compressive sampling have spurred a large and fascinating literature in which other approaches and ideas have been proposed. Rudelson and Vershynin have used tools from modern Banach space theory to derive powerful results for Gaussian ensembles . In this area, Pajor and his colleagues have established the existence of abstract reconstruction procedures from subgaussian measurements (including random binary sensing matrices) with powerful reconstruction properties. In a different direction, Donoho and Tanner have leveraged results from polytope geometry to obtain very precise estimates about the minimal number of Gaussian measurements needed to reconstruct S-sparse signals . Tropp and Gilbert reported results about the performance of greedy methods for compressive sampling . Haupt and Nowak have quantified the performance of combinatorial optimization procedures for estimating a signal from undersampled random projections in noisy environments . Finally, Rauhut has worked out variations on the Fourier sampling theorem in which a sparse continuoustime trigonometric polynomials is randomly sampled in time . Because of space limitations, we are unfortunately unable to do complete justice to this rapidly growing literature.[wiki]

We would like to emphasize that there are many aspects of compressive sampling that we have not touched. For example, we have not discussed the practical performance of this new theory. In fact, numerical experiments have shown that compressive sampling behaves extremely well in practice. For example, it has been shown that from 3S?4S nonadaptive measurements, one can reconstruct an approximation of an image in a fixed basis which is more precise than that one would get by measuring all the coefficients of the object in that basis and selecting the S largest . Further, numerical simulations with noisy data show that compressive sampling is very stable and performs well in noisy environments.

There could be massive innovation and countless application in compressive sensing. For example, when todays high resolution digital camera becomes a trend, it would be shameful to have one with resolution of 5 million pixels. But it is noticeable that most pictures do not contain that amount of information. Pictures with 5 million pixels would only occupy a disk space of 500k. So why not reduce the data of that amount of pixels when taking pictures instead of compressing the pictures afterwards? With new data acquisition protocols and careful design of pixel-acquire locations, One-million-pixel cameras would have the same effect as five-million-pixel ones. We have another case on the data flow processing. If we want to know the top ten IP address for accessing one website most frequently(or 50 most hot search items for a search), we could create an accounting array with space of , the corresponding counter adds up by one every time a New visit occurs. But its too wasteful in terms of memory occupation. With compressed sensing, we could create a sensing matrix A. Everytime a visit takes place, where is the ith row of . Since contains the necessary information according to compressive sensing, the total disk occupation is decreased

It is believed that compressive sampling has far reaching implications. it suggests the possibility of new data acquisition protocols that translate analog information into digital form with fewer sensors than what was considered necessary. This new sampling theory may come to underlie procedures for sampling and compressing data simultaneously.

Reference

[A]Robust Uncertainty Principles: Exact Signal Reconstruction from Highly Incomplete Frequency Information Emmanuel Candesy, Justin Rombergy, and Terence Tao] y Applied and Computational Mathematics, Caltech, Pasadena, CA 91125] Department of Mathematics, University of California, Los Angeles, CA 90095 June 2004; Revised August 2005

[B] Compressive sampling Emamnuel J. Candes.

[C] Compressed Sensing David L. Donoho, Member, IEEE IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 52, NO. 4, APRIL 2006 1289