Project Report II
Capacity, Coverage and Connectivity of Wireless Networks, multi-hoc, multi-rate

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I. Outline

The research method we used, as others did, is to firstly establish a simple network model, then some other conditions added step by step, such as the motion state of the nodes, the distribution of the nodes, the setting of the antenna, the power of the emission, the resistance of the band width, etc. This is a vital and common way to seize the core of the problem, but it is sometimes inevitable to reach the conclusions with limitations. For instance, some conclusions only can be utilized in static state, some dynamic state, some UWB technology, and something like that.

This paper will be focus on how to improve the utilization rate under the fixed capacity of multi-hoc networks. Firstly, we will introduce ad hoc network, the capacity, the utilization rate and the leaf node in our work. Subsequently, we describe the basic theory that the capacity of a system is limitary and a bound on the degree of the nodes. Secondly, we assume the capacity of the ad hoc network is changeless, we try to find the routes that have the best transmission performance through the ad hoc network model from the novel point of view and could achieve the maximum utilization rate when the network resources (including area, bandwidth, power, load and so on) are used up minimally on the successful transmission condition. Finally, we draw to the conclusion that the leaf nodes should have higher priority (more rapid velocity, less waiting time, higher transmission probability, etc.)

II. Definition

A. Ad Hoc Network

Ad hoc network is a collection of communications devices that wish to communicate, but have no fixed infrastructure available, and have no pre-determined organization of available links. Ad hoc network consists of a number of nodes that communicate with each other over a wireless channel without any centralized control, whose main technology is the theory of self-organized network.[1] Nodes could cooperate in routing each others data packets. Lack of any centralized control and possible node mobility give rise to many issues at the network.

B. the Capacity

The capacity of wireless ad hoc networks is measured by the data transmitted successfully per second on average.

In this paper, we just consider a system, which has large amounts of fixed nodes. In practice, we calculate the capacity in this way as follows. In a period time T, all the nodes in the system monitor the package sizes they receive, and we claim the sum of the package size CT. Then we get the capacity of the network, C, the unit of which is bit/s.

C. the Utilization Rate

According to flooding, the way of the information spread in ad hoc networks, some transmissions are necessary because the packages are successfully transferred from the source to the destination effectively, while others are not. They take up the capacity of the network but have no use. We call the packages that transmitted in the effective route per second as the effective capacity.

We definite *the utilization rate* as the ratio of the effective capacity and the capacity.

D. the Leaf Node

In many practical ad hoc networks, there always exist some nodes that just can connect only a little nodes because of some objective conditions such as the position. In other words, according to the graph theory, these kind of nodes have low degree, which we definite as *the leaf node*.

III. Basic Theory

A. the Capacity of a System is Limitary

We can consider two classic networks, according to Gupta, Arbitrary Networks and Random Networks. *Arbitrary Networks*, where the node locations, destinations of sources, and traffic demands, are all arbitrary, and *Random Networks*, where the nodes and their destinations are randomly chosen. Our project model to be discussed later is constructed on the basis of the upper limit theory proved by Gupta and Kumar[2], which we want to describe again in more detail so as to cement the model.

1) Arbitrary Networks Physical: Assumption: Let X_i denote the location of a node; we will also use X_i to refer to the node itself. Let $X_k, k \in \tau$ be the subset of nodes simultaneously transmitting at some time instant over a certain channel. Let P_k be the power level chosen by node X_k, for $k \in \tau$. Then the transmission from a node $X_i, i \in \tau$, is successfully received by a node X_j if

$$\frac{\frac{P_i}{|X_i - X_j|^\alpha}}{N + \sum_{k\in\tau, k\neq i} \frac{P_k}{|X_k - X_j|^\alpha}} \geq \beta \tag{1}$$

This models a situation where a minimum signal-to interference ratio (SIR) of β is necessary for successful receptions, the ambient noise power level is N,and signal power decays with distance r as $\frac{1}{r^\alpha}$.

2) Discursion for the upper limit: For the convenience of discursion, we put forward some assumptions,

1) There are n nodes arbitrarily located in a disk of unit area on the plane.
2) The network transports $\lambda n T$ bits over T seconds.
3) The average distance between the source and destination of a bit is $\overline{L}$
4) Each node can transmit over any subset of M subchannels with capacities r_m bits per second, $1 \leq m \leq M, \sum_{m=1}^{M} r_m = r$

Theorem: In the Physical Assumption, the transport capacity $\lambda n \overline{L}$ is bounded as follows:

$$\lambda n \overline{L} \leq (\frac{2\beta + 2}{\beta})^{1/\alpha} \frac{1}{\sqrt{\pi}} r n^{\alpha - 1/\alpha} \tag{2}$$

Proof: Consider bit b , where $1 \leq b \leq \lambda n T$. Let us suppose that it moves from its origin to its destination in a sequence of $h(b)$ hops, where the hth hop traverses a distance of d_b^h, so we get

$$\sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} d_b^h \geq \lambda n T \overline{L} \tag{3}$$

Note now that in any slot at most $n/2$ nodes can transmit. Hence for any subchannel m and any slot s we have $\sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} 1 \leq \frac{r_m \tau_0 n}{2}$, where τ_0 means the duration of the slot s, and there are total T/τ_0 slots in T seconds. Thus we get

$$H \triangleq \sum_{b=1}^{\lambda n T} h(b) \leq \frac{rTn}{2} \tag{4}$$

Suppose that X_j is receiving a transmission from X_i over the mth subchannel at the same time that X_e is receiving a transmission from X_k over the same subchannel. First we accept the condition for successful reception by node X_j when every other node X_k simultaneously transmitting over the same subchannel, with $\Delta > 0$ where a guard zone is specified to prevent a neighboring node from transmitting on the same subchannel at the same time,

$$|X_k - X_j| \geq (1 + \Delta)|X_i - X_j|. \tag{5}$$

Then from the triangle inequality we know

$$\begin{aligned} |X_j - X_e| &\geq |X_j - X_k| - |X_e - X_k| \\ &\geq (1 + \Delta)|X_i - X_j| - |X_e - X_k|. \end{aligned} \tag{6}$$

Similarly,

$$|X_j - X_e| \geq (1 + \Delta)|X_k - X_e| - |X_j - X_i. \tag{7}$$

Adding the two inequalities, we obtain

$$|X_j - X_e| \geq \frac{\Delta}{2}(|X_k - X_e| + |X_i - X_j|) \tag{8}$$

Since at most $r_m \tau$ bits can be carried in slot s from a receiver to a transmitter over the mth subchannel, we have

$$\sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} \frac{\pi \Delta^2}{16} (d_b^h)^2 \leq r_m \tau_0. \tag{9}$$

Summing over the subchannels and the slots gives

$$\sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} \frac{\pi \Delta^2}{16} (d_b^h)^2 \leq rT \tag{10}$$

This can be rewritten as

$$\sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} \frac{1}{H} (d_b^h)^2 \leq \frac{16rT}{\pi \Delta^2 H}. \tag{11}$$

Note now that the quadratic fnction is convex. Hence

$$(\sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} \frac{1}{H} d_b^h)^2 \leq \sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} \frac{1}{H} (d_b^h)^2. \tag{12}$$

Combining the two equations above, we get

$$\sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} d_b^h \leq \sqrt{\frac{16rTH}{\pi \Delta^2}}. \tag{13}$$

Then we substitute the $\lambda n T \overline{L}$ condition to the above, it gives us

$$\lambda n T \overline{L} \leq \sqrt{\frac{16rTH}{\pi \Delta^2}}. \tag{14}$$

So we get the protocol transport capacity theorem as follows with the unit, bit-meters per second:

$$\lambda n \overline{L} \leq \frac{\sqrt{8}}{\pi} \frac{1}{\Delta} r \sqrt{n}. \tag{15}$$

In the physical model, supposing X_i is transmitting to $X_{j(i)}$ over mth subchannel at power level P_i at some time, and let K denote the set of all siultaneous transmitters over the mth subchannel at that time, the signal-to-interference requirement can be written as

$$\frac{\frac{P_j}{|X_i - X_{j(i)}|^\alpha}}{N + \sum_{k\in K} \frac{P_k}{|X_k - X_{j(i)}|^\alpha}} \geq \frac{\beta}{\beta+1}. \tag{16}$$

Hence

$$\begin{aligned} |X_i - X_{j(i)}|^\alpha &\leq \frac{\beta+1}{\beta} \frac{P_i}{N + \sum_{k\in K} \frac{P_k}{|X_k - X_{j(i)}|^\alpha}} \\ &\leq \frac{\beta+1}{\beta} \frac{P_i}{N + (\frac{\pi}{4})^{\alpha/2} \sum_{k\in K} P_k}. \end{aligned} \tag{17}$$

Since $|X_k - X_{j(i)}| \leq \frac{2}{\sqrt{\pi}}$, we sum over all transmitter-receiver pairs

$$\begin{aligned} \sum_{k\in K} |X_i - X_{j(i)}|^\alpha &\leq \frac{\beta+1}{\beta} \frac{\sum_{k\in K} P_i}{N + (\frac{\pi}{4})^{\alpha/2} \sum_{k\in K} P_k} \\ &\leq 2^\alpha \pi^{-(\alpha/2)} \frac{\beta+1}{\beta} \end{aligned} \tag{18}$$

Summing over all slots and subchannels gives

$$\sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} d^\alpha(h,b) \leq 2^\alpha \pi^{\frac{-\alpha}{2}} \frac{\beta+1}{\beta} rT \tag{19}$$

The rest of the proof proceeds long lines similar to the Protocol one, and after all of proof we know that every ad-hoc system, due to its physical condition, it always has the transmission upper limit and the capacity is just not infinite but limitary. This theorem will support our later construction in a useful way.

B. A Bound on the Degree of the Nodes

In the following theorem, we will prove that if $\gamma > 0$, the number of neighbors of each node is bounded from above.

Theorem : Each node can have at most $1 + 1/\gamma\beta$ neighbors.[3]

Proof: Pick any node (called hereafter Node 0), and let N be the number of its neighbors (i.e. the number of nodes to which Node 0 is connected). If N $\leq$1, the claim is trivially proven. Suppose next that N $>$ 1, and denote by 1 the node whose signal power received by Node 0 is the smallest but is non zero, namely is such that

$$P_1 L(x_1 - x_0) \leq P_i L(x_i - x_0), \text{i=2...N}. \tag{20}$$

Since it is connected to Node 0, (1) imposes that

$$\frac{P_1 L(x_1 - x_0)}{N_0 + \gamma \sum_{i=2}^{\infty} P_i L(x_i - x_0)} \geq \beta \tag{21}$$

Taking (20) into account, (21) implies that

$$\begin{aligned} P_1 L(x_1 - x_0) &\geq \beta N_0 + \beta\gamma \sum_{i=2}^{\infty} P_i L(x_i - x_0) \\ &\geq \beta N_0 + \beta\gamma(N-1) P_i L(x_1 - x_0) \\ &+ \beta\gamma \sum_{i=N+1}^{\infty} P_i L(x_i - x_0) \\ &\geq \beta\gamma(N-1) P_1 L(x_1 - x_0) \end{aligned} \tag{22}$$

from which we deduce that

$$N \leq 1 + \frac{1}{\beta\gamma} \tag{23}$$

In CDMA cellular networks, this kind of bound is known under the name of pole capacity. [4]

As a consequence of Theorem, we see that if $\gamma > 1/\beta$, each node has at most one neighbor. This is a very general and may be happen in the nature. [5] As mentioned above we call these kind of nodes *the leaf nodes*.

IV. Ad Hoc Network Model

The basic idea of our model is to find the effective routes that have the best transmission performance (low bit error, high successful transmission condition, low time delay) under the network resources (including area, bandwidth, power, load and so on). Then we could achieve the maximum utilization rate.

Model: The ad hoc network we considered is defined as $N = (V, W)$, V denotes the set of nodes in the network, W is the wastage of nodes transmission in the network, where nodes are either in the plane or on the sphere and are classified as four types: source node V_s, destination node V_d, relay node V_r, and isolation node is V_i, which satisfy the equation: $V = V_s \bigcup V_d \bigcup V_r \bigcup V_i$.

The minimal resources consumed between two nodes i, j are defined as

$$c_{ij} = \min_{P_1 \in P_{ij}} \{ \sum_{ij\in P_1} \omega_{ij} \}, i,j \in V \tag{24}$$

where P_{ij} represents the set of all possible transmission paths from V_i and V_j and P_1 is one of the paths belonging to P_{ij} . Let f_{ij} be data streams that need not relay between node i and j. S_{ij} is the transmission resource consumed at per unit data. $\omega_{ij} = S_{ij} f_{ij}$ denotes the wastage of direct transmission from V_i and V_j.

Target:

$$c_{ij} = \min_{P_1 \in P_{ij}} \{ \sum_{ij\in P_1} \omega_{ij} \}, i,j \in V \tag{25}$$

Restriction:

$$\begin{aligned} &\sum_j f_{ij} \leq a_i, i \in V_s \\ &\sum_j f_{ij} - \sum_j f_{ji} = 0, i \in V_r \\ &\sum_j f_{ji} \geq b_i, i \in V_d \\ &0 \leq f_{ij} \leq u_{ij} \end{aligned} \tag{26}$$

$\sum a_i$ is the maximum capacity in the network. $\sum b_i$ is the minimal capacity guaranteeing successful transmission in the network. u_{ij} is the maximum capacity between nodes not needing relay. If the number of nodes is large, the complexity of the arithmetic according to the linear equation is too great. We consider the characteristics of ad hoc network as additional conditions in order to reduce the complexity.

The traffic pattern of nodes transmission in ad hoc network could be classified as three types. The first type is no traffic. The second type is the traffic with which is communicated directly. The third type is the relaying traffic. Three conditions below are satisfied simultaneously when the network capacity attains the maximum:

1) The number of the first type node is as small as possible. We can improve the power of these nodes and increase the distance of each hoc to realize.
2) The second traffic satisfies: $c_{sd} = s_{sd} f_{sd}$.
3) The third traffic satisfies:

$$c_{sd} = \min \sum_{sd \in P_1} s_{ij} f_{ij}, i \in (V_s \bigcup V_r), j \in (V_r \bigcup V_d)$$

Then we can use the shortest path problem between arbitrary two nodes in the network in the graph theory, the study of graphs in mathematics and computer science, which has been used in lots of fields of economics, architectonics and so on. Next we will study about the shortest path basing on the ad hoc network model which makes c_{sd} between source node and destination node attain the minimum.

Let $c_{sd}^{\star}$ be the minimal resources consumed between V_s and V_d, ω_{ij} as wastage of the resources consumed between V_i and V_j with which communicates directly. We consider V_i has no choice to communicate directly with V_j if ω_{ij} goes to the infinity. V_i could reach V_j in theory when the condition $s_{ij} b_{ij} \leq \omega_{ij} \leq s_{ij} b_{ij}$ ($s_{ij} b_{ij}$ is the critical wastage under successful transmission) is satisfied. Now we calculate the resources consumed c_{ij} between V_i and V_j. The resources consumed are $\overline{c_{ij}} = c_{ik} + c_{kj}$ after the node gets across the arbitrary relay node V_r. The value of c_{ij} will not be changed if $\overline{c_{ij}} < c_{ij}$, otherwise c_{ij} is updated as $\overline{c_{ij}}$. The algorithm needs to check every relay node between V_s and V_d in the network and find some V_r through which makes c_{ij} decrease, and then we obtain the new value of c_{ij}. $c_{sd}^{\star}$ will be achieved after repeating the above process many times and then all relay nodes between V_s and V_d in the network will satisfy the condition $c_{ik} + c_{kj} \geq c_{ij}, i \neq j \neq k$.

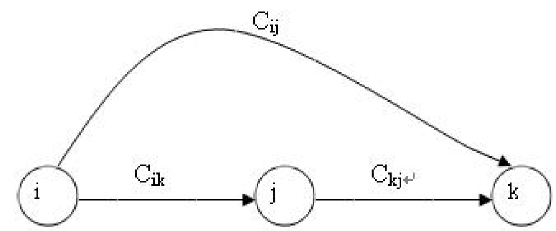

Figure1 The minimal resources consumed between V_i and V_j

As shown in Figure 1, $c_{ik} + c_{kj}$ substitutes for c_{ij} if $c_{ij} > c_{ik} + c_{kj}$. c_{ij} denotes the current value of the resources consumed between V_i and V_j. Let c_{ij}^r be the least value of resources consumed after the r^{th} iterative algorithm and use calculation equation is:

$$\begin{aligned} & c_{ij}^{r+1} = \min\{c_{ij}^r, c_{ik}^r + c_{kj}^r\}, i \neq j \neq k, \\ & i \in (V_s \bigcup V_r), k \in V_k, j \in (V_r \bigcup V_d) \end{aligned} \tag{27}$$

We construct two matrixes during the iterative algorithm. The first matrix is the minimal resources consumed matrix. Every element in the matrix represents the current value of the least resources consumed. Let the first matrix be $C^r = [c_{ij}^r]$ after the r^{th} iterative algorithm. The initialization of the matrix is $C^0 = [c_{ij}^0]$ and $c_{ij}^0 = \omega_{ij}$. C^r is derived from C^{r-1}. The algorithm will be terminated until nodes in the network satisfy the relation of the equation (27).

The second matrix is the relay node matrix, which points out relay nodes when the resources consumed attain the minimum. Let the second matrix be $R^r = [R_{ij}^r]$ after the r^{th} iterative algorithm. R_{ij}^r is the first relay node between V_i and V_j on the least resources consumed condition. The initialization of the matrix is $R^0 = [R_{ij}^0]$ and $R_{ij}^0 = j$. Elements in R^r are achieved according to the below equation:

$$R_{ij}^r = \begin{cases} r & c_{ij}^{r-1} > c_{ir}^{r-1} + c_{rj}^{r-1} \\ R_{ij}^{r-1} & \text{otherwise} \end{cases} \tag{28}$$

We calculate the above two matrixes after they are initiated and repeat the below steps:

Step 1: lay off all elements in the r^{th} row and r^{th} line which are named after the axes of row and line;

Step 2: choose the elements not belonging to the axes of row and line and define them as $c_{ij}^{r-1}, j \neq r$.

We compare $c_{ir}^{r-1} + c_{rj}^{r-1}$ with c_{ij}^{r-1}. The values of i and j choose new values and continue the process to next c_{ij}^{r-1} when $c_{ij}^{r-1} > c_{ir}^{r-1} + c_{rj}^{r-1}$, and then we repeat step 2. $c_{ij}^r = c_{ir}^{r-1} + c_{rj}^{r-1}$ substitutes for c_{ij}^{r-1} and r substitutes for the corresponding element in relay node matrix if $c_{ir^{r-1}} + c_{rj}^{r-1} < c_{ij}^{r-1}$. The algorithm will return step 1 and r is added one after all elements are checked. The process of algorithm is terminated when r is equal to the value p, while we achieve the optimal minimal resources consumed matrix and the corresponding relay node matrix.

V. Discussions and Conclusions

From the basic theories related, we have got some results. The equation (2) gives some ideas that the capacity of an ad hoc network is limitary. In addition, the equation (23) reflects that the leaf nodes exist generally in the network.

In order to get to the conclusion, we could consider in a simple way first. Assume that there is no wastage in the ad hoc network, the transmission between two nodes will happen each time lice, and the monitor distance each time is the same, Υ. A source node wants to send the package, then it monitors the nodes within the Υ and spread the data package. If the available nodes is six, as Figure 2. In the next times lice, these six nodes will transfer the information to its available nodes, and so on and so forth. The process won't cease until the package reach the destination node. Obviously, each time transmission take place, it will cost the capacity, but there is only one route that transfers the package from the source to the destination successfully. We intitule it the efficient route. The ad hoc model we established helps us to find such efficient routes. For the first time lice, the utilization rate is 1/6, one of the six routes is efficient one. If we just consider the utilization rate in a time slice, the utilization rate of the leaf nodes, we mentioned above, will be larger than others. In order to gain the utilization rate in a period of time as great as possible, we should try to make sure the transmission passed by the leaf nodes successful and stable. That's better to finish these kind of transmission as soon as possible. From this, we conclude that the leaf nodes should have higher priority in the system (more rapid velocity, less waiting time, higher transmission probability, etc.)

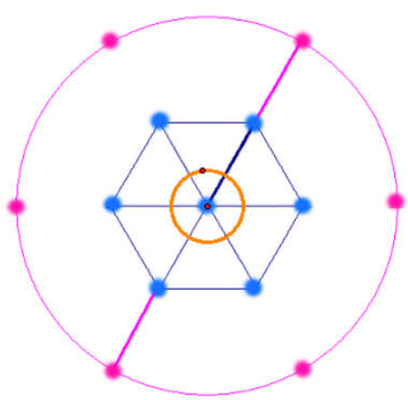

Figure2 The ideal model for ad hoc network

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