Fundamental limits of channel state information

RSSI and LDPL(Log-distance path loss) model

RSSI->distance & triangulation location

multipath effect -> **rough**!!!!!

Fingerprint localization

location is related to the characteristic of rssi

take the multipath into account

CSI(Channel State Information)

amplitude and phase to get submeter-level accuracy

In reality, the measurement of RSSI and phase are both affected by the noise as well as the multipath effect.

This causes the measured result has deviation from the theoretical value.

The deviation affects the localization precision.

Can we evaluate the measurement error?

Can we give an error bound of the measurement result?

then the Fisher information takes the form of an $N \times N$ matrix, the Fisher Information Matrix (FIM), with typical element:

$$(\mathcal{I}(\theta))_{i,j} = -\operatorname{E}\left[\frac{\partial^2}{\partial\theta_i\,\partial\theta_j}\log f(X;\theta)\middle|\theta\right].$$

Where f(X; θ) is the probability density function for X under the parameters θ

The FIM for a N-variate multivariate normal distribution, X ~ N (μ(θ),Σ(θ)), has a special form.

Let the K-dimensional vector of parameters be $\theta = [\theta_1 , . . . , \theta_k]T$, and the vector of random normal variables be $X =[X_1 , . . . , X_N]^T$, with mean values $\mu(\theta) = [\mu_1 (\theta),...,\mu_N (\theta)]^T$, and let Σ(θ) be the covariance matrix.

$$\mathcal{I}_{m,n} = \frac{\partial \mu^{\mathrm{T}}}{\partial \theta_m} \Sigma^{-1} \frac{\partial \mu}{\partial \theta_n}$$

The received signal can be written as

$$r(t) = \sum_{k=1}^{I} \alpha_k s(t - \tau_k) + z(t)$$

α_k and τ_k are the amplitude and delay, respectively, of the k_{th} path. And I is the number of multipath components, $z(t)$ represents the observation noise modeled as additive white Gaussian processes with two-side power spectral density $\frac{N_0}{2}$.

Here is the parameter vector:

$$\theta = [\tau_1, \alpha_1, \tau_2, \alpha_2, . . . , \tau_I, \alpha_I]$$

As mentioned before, the fisher equality need the pdf(probability distribution function) of received signal. Therefore, we sample the received signal with the sampling period T to transform the continuous signal into discrete variables.

$$X_1 = r(T) = \sum_{k=1}^{I} \alpha_k s(T - \tau_k) + z(\mathrm{T})$$

$$X_2 = r(2T) = \sum_{k=1}^{I} \alpha_k s(2T - \tau_k) + z(2T)$$

…..

$$X_n = r(nT) = \sum_{k=1}^{I} \alpha_k s(nT - \tau_k) + z(nT)$$

Then we can obtain a series of Gaussian random variables and the covariance between each two variables is equal to zero. It means the covariance matrix of these random variables is just in proportion to the identity matrix. This can greatly simplify the further calculation.

Covariance matrix:

$$\operatorname{cov}(X_i, X_j) = E(z(iT) * z(jT)) = 0(i \neq j)$$

$$D(X_i) = D(X_j) = \frac{N_0}{2}$$

$$\Sigma = \frac{N_0}{2} E$$

where E is the identity matrix.

Now, we combine the random variables into a joint probability and we can get the actual probability distribution function and all estimations of the parameters in the parameter vector are based on the function

Then, we can calculate the fisher information matrix.

The elements of the information matrix:

$$I_{2i,2j} = \frac{2}{N_0} \sum_{k=1}^{I} s(kT - \tau_j) s(kT - \tau_i)$$

$$I_{2j-1,2i} = I_{2i,2j-1} = \frac{2}{N_0} \sum_{k=1}^{I} -\alpha_j s'(kT - \tau_j) s(kT - \tau_i)$$

$$I_{2i-1,2j-1} = \frac{2\alpha_i \alpha_j}{N_0} \sum_{k=1}^{I} s'(kT - \tau_j) s'(kT - \tau_i)$$

Taking into the multipath effect and the noise existing in the surrounding area, the fisher matrix seems somewhat complex due to its high dimensions. To circumvent direction matrix inversion and gain insights into the localization problem, we first introduce the notions of EFI——Equivalent Fisher Information Matrix. We divide the parameter vector into two parts, only the first part includes the parameters of our interests. Due to the symmetry, the fisher information matrix can also be divided into the four parts as shown.

Given

$$\theta = [\theta_1^T, \ \theta_2^T]$$

The FIM

$$J_\theta = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$$

Where

$$\theta \in R^N, \theta_1 \in R^n, A \in R^{n\times n}, B \in R^{n\times(N-n)}, C \in R^{(N-n)\times(N-n)}$$

$$J_e(\theta_1) \triangleq A - BC^{-1}B^T$$

Then, the equivalent Fisher information matrix for θ_1 is given by this. Because EFIM retains all the necessary information

to derive the information inequality for the parameter vector θ_1.

$$\left[J_\theta^{-1}\right]_{n\times n} = J_e^{-1}(\theta_1)$$

since this equation is correct. So we can reduce the dimension of the original FIM.

To display our results more clearly, we applied the above steps to calculation the phase error bound of a simple model. Regardless of the multipath effect, we only consider the noise.

$$\theta = [\tau_1, \alpha_1]$$

Then, we can calculate the fisher information matrix and here is the result. We applied it to determine the phase error bound, the final expression are shown like this.

The FIM:

$$I_{1,1} = \frac{2\alpha_1^2}{N_0} \sum_{k=1}^{J} s'^2(kT - \tau_1)$$

$$I_{1,2}=I_{2,1} = \frac{2}{N_0}\sum_{k=1}^{I}[-\alpha_1 s'(kT-\tau_1)s(kT-\tau_1)]$$

$$I_{2,2} = \frac{2}{N_0}\sum_{k=1}^{I} s^2(kT-\tau_1)$$

The time delay error bound :

$$bound(\tau_1) = \frac{1}{SNR}\frac{\sum_{k=1}^{I} s^2(kT-\tau_1)}{\sum_{k}^{I} s'^2(kT-\tau_1)\sum_{k}^{I} s^2(kT-\tau_1) - (\sum_{k}^{I} s'(kT-\tau_1)\sum_{k}^{I} s(kT-\tau_1))^2}, SNR = \frac{\alpha_1^{\;2}}{2N_0}$$

SNR ↑ error bound ↓

Supposing

$$s(t) = A\cos(wt), bound(\varphi) = \omega * bound(\tau) \propto \frac{1}{\omega}$$

Higher Frequency signal suffers less from the effect of noise.

Future work

1. The relationship between T/N and bound
2. AP deployment
3. Experiments to confirm results