Revised SEIR model for COVID-19 in China

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ABSTRACT

The coronavirus disease has influenced the life of many people around the world since the beginning of 2020. In this paper,we modify the traditional SEIR model, and propose two new models named **SEIR Under Quarantine** and **SEIR Under Migration** to help analyze and predict the spread of COVID-19 in China. We provide several possible ways to derive analytical solution, and then use Euler method to simulate. The parameters are set based on the statistics of Wuhan, and our model fits the real statistics well. We further do experiments on one or more cities to show how the change of parameters will influence the spread. Our new models and analysis on them can serve as a basis for future disease analysis and decision making.

KEYWORDS

Epidemiology, COVID-19, SEIR, SEIR Under Quarantine, SEIR Under Migration, Euler Method

1 INTRODUCTION

1.1 Background

Coronavirus disease 2019 (COVID-19) is an infectious disease caused by severe acute respiratory syndrome coronavirus (SARS-CoV-2). The first wave of outbreak happened in China, especially Wuhan, Hubei Province in the first quarter of 2020. By June 2020, it is still widely-spread in many places around the world, although it has been controlled largely in China.

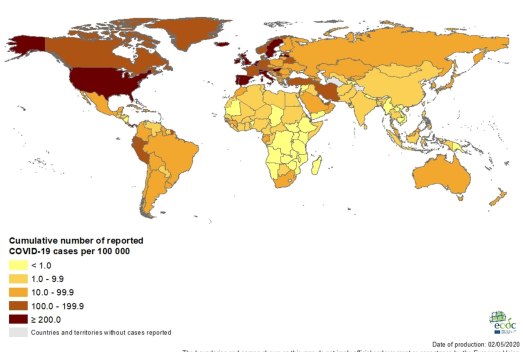

Figure 1: Spread of COVID-19 around the world

1.2 Previous Work

To model the spread of the diseases, many problems arising in epidemiology may be described, in a first formulation, by means of differential equations. This means that the models are constructed by averaging some population and keeping only the time variable.

To the best of our knowledge, the first mathematical model of epidemiology was formulated and solved by Daniel Bernoulli in 1760. Since the time of Kermack and McKendrick[1], the study of mathematical epidemiology has grown rapidly, with a large variety of models having been formulated and applied to infectious diseases[2–4]. Among them, the most popular one is SEIR model.

Consider a population which remains constant and which is divide into four classes: the susceptibles, denoted by S, who can catch the disease; the exposed, denoted by E, who are infected but still under the incubation period (no typical symptoms) and can transmit the disease to the suspectibles; the infectives, denoted by I, who are infected, have typical symptoms and can transmit the disease to the susceptibles, and the recovered, denoted by R, who had the disease and recovered and have developed immunity. Since from the modeling perspective only the overall state of a person with respect to the disease is relevant, the progress of individuals is schematically described by

$S \longrightarrow E \longrightarrow I \longrightarrow R$

1.3 Our Contribution

Our main contribution can be mainly summarized as follows:

- We modified the original SEIR model and proposed a new model named. **SEIR under quarantine**. It takes the quarantine measures that the government into consideration.
- We further consider the influence of inter-city mobility, and proposed another model named **SEIR under migration**.
- We propose several possible ways to find analytical solution, and use Euler Method to do simulation in our experiments.
- The simulation result of our models is with the real statistics in Wuhan, which displays the usability of our model. We also test on several toy examples to show the need to take measures early and decisively.

2 SEIR UNDER QUARANTINE

2.1 Model Description

Classical SEIR model divide people into several categories as: S (susceptible), E (exposed), I (infected), R (recovered). This model assume that people who carry virus (E and I) have the power to infect S. Thus S (susceptible) will transfer to incubation period and become E. After the incubation period, E (exposed) get sick and become I. When I(infected) people recover, they will have the antibodies, i.e., R will not be infected again.

Considering the effective measure of quarantine by Chinese government, we newly introduce three states: SQ (susceptible under quarantine), EQ (exposed under quarantine) and IQ (infected under quarantine). Note that the infected under quarantine will be transferred to hospital immediately, so we replace IQ with H (hospitalized patients) to denote these people.

Thus our revised SEIR model, namely **SEIR Under Quarantine** are proposed. Before illustrating the transition relationship, we first introduce some related variables.

- β represents the infection probability. It measures the ability of disease to spread.[1]
- σ represents the probability that E transmit to I.
- μ_I represents the death rate of I, μ_H represents the death rate of H.
- γ_I represents the recover rate of I, γ_H represents the recover the rate of H.
- q_S represents the portion of quarantine people from S, q_E represents the portion of quarantine people from E.
- λ represents the rate of release from quarantine after 14 days' watching. So, usually we take $\lambda = \frac{1}{14}$.
- δ_E represents the rate that EQ get sick thus sent to hospital , δ_I represents the portion that I (infected) go to hospital.

Thus their transition relationship are presented in Fig 2

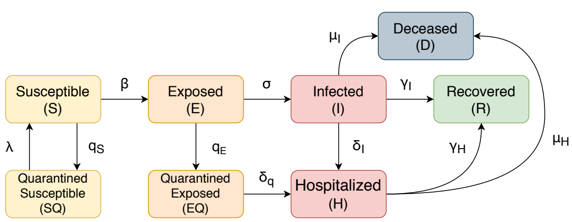

Figure 2: SEIR Under Quarantine

2.2 Model Analysis

Based on the flow in transition graph, we can construct the differential equations to analyze the relationship between these states.

Definition 1 (Contact Rate). *It is defined as the total number of contacts per unit time of one person. We denote it as c.*

According on the flow in Fig 2, the changing rate of S, namely $\frac{dS}{dt}$ should be minus infection rate plus release rate from quarantine minus quarantine rate.

A person who carries the virus meets a healthy people with the probability $\frac{S}{N}$. And he infects the healthy person with the probability β. If he meets c persons a day, then the infection rate will be $\frac{c \cdot S}{N}(\beta_E E + \beta_I I)$.

$$\frac{dS}{dt} = -\frac{c \cdot S}{N}(\beta_E E + \beta_I I) + \lambda SQ - q_S S \tag{1}$$

According on the flow in Fig 2, the changing rate of E, namely $\frac{dE}{dt}$ should be infection rate minus quarantine rate minus morbidity rate.

$$\frac{dE}{dt} = \frac{c \cdot S}{N}(\beta_E E + \beta_I I) - (\sigma + q_E)E \tag{2}$$

[1]Specifically, β_E and β_I represents the infection probability of E (exposed) and I (infected) respectively.

According on the flow in Fig 2, the changing rate of SQ, namely $\frac{dSQ}{dt}$ should be quarantine rate from S minus release rate from quarantine.

$$\frac{dSQ}{dt} = q_S S - \lambda SQ \tag{3}$$

According on the flow in Fig 2, the changing rate of EQ, namely $\frac{dEQ}{dt}$ should be quarantine rate from E minus hospitalized rate.

$$\frac{dEQ}{dt} = q_E E - \delta_q EQ \tag{4}$$

According on the flow in Fig 2, the changing rate of I, namely $\frac{dI}{dt}$ should be morbidity rate of E minus death rate minus recovered rate minus hospitalized rate.

$$\frac{dI}{dt} = \sigma E - (\delta_I + \mu_I + \gamma_I)I \tag{5}$$

According on the flow in Fig 2, the changing rate of H, namely $\frac{dH}{dt}$ should be hospitalized rate of E and I minus death rate minus recovered rate.

$$\frac{dH}{dt} = \delta_q EQ + \delta_I I - (\gamma_H + \mu_H)H \tag{6}$$

According on the flow in Fig 2, the changing rate of R, namely $\frac{dR}{dt}$ should be recovered rate from H and I.

$$\frac{dR}{dt} = \gamma_I I + \gamma_H H \tag{7}$$

3 SEIR UNDER MIGRATION

3.1 Model Description

In the above analysis, the city is regarded as an isolated island, and we simulate and analyze our model based on the assumption. Nonetheless, things are not that simple. There are actually migration between different cities, and the migration can also influence the spreading of COVID-19.

When considering the migration between different regions, **SEIR Under Quarantine** may not be able to deal with this situation. Thus we need a little improvement to enhance the robustness of our model.

Under the restriction of Chinese government, people move from regions to regions need to take temperature tests. If someone's temperature is rather high, he would be sent to hospital. With this regulation, there are several cases should be considered (when a person moves from A to B):

- S in A → S in B.
- E in A → E in B.
- I in A → H in B.
- R in A → R in B.

Thus we propose a new model named **SEIR Under Migration**. In this model, there are $\mathcal{M}$ regions, denoted as 1, 2, ..., M. In each region j, we have these seven states: $S_j, E_j, SQ_j, EQ_j, I_j, H_j, R_j$. And we use some variables to describe the migration between regions.

- We use H_{ij} to denote the migration from region i to region j.
- S_{ij} represents the number of susceptible people from region i to region j.

- E_{ij}, I_{ij} and R_{ij} are defined similarly.

Note that there are some relations between them, i.e., $H_{ij} = S_{ij} + E_{ij} + I_{ij} + R_{ij}$. And we assume that that $S_{ij} = H_{ij} \times \frac{S_i}{N_i}$, $E_{ij} = H_{ij} \times \frac{E_i}{N_i}$, $I_{ij} = H_{ij} \times \frac{I_i}{N_i}$ and $R_{ij} = H_{ij} \times \frac{R_i}{N_i}$.

3.2 Model Analysis

With the migration analysis, formula are derived as follows: we only need to add the migration based on the above cases.

$$\frac{dS_j}{dt} = -\frac{c \cdot S_j}{N_j}(\beta_E E_j + \beta_I I_j) + \lambda SQ_j - q_S S_j + \sum_{m \neq j} S_{mj} - \sum_{m \neq j} S_{jm} \tag{8}$$

$$\frac{dE_j}{dt} = \frac{c \cdot S_j}{N_j}(\beta_E E_j + \beta_I I_j) - (\sigma + q_E)E_j + \sum_{m \neq j} E_{mj} - \sum_{m \neq j} E_{jm} \tag{9}$$

$$\frac{dSQ_j}{dt} = q_S S_j - \lambda SQ_j \tag{10}$$

$$\frac{dEQ_j}{dt} = q_E E - \delta_q EQ_j \tag{11}$$

$$\frac{dI_j}{dt} = \sigma E_j - (\delta_I + \mu_I + \gamma_I)I_j - \sum_{m \neq j} I_{jm} \tag{12}$$

$$\frac{dH_j}{dt} = \delta_q EQ_j + \delta_I I_j - (\gamma_H + \mu_H)H_j + \sum_{m \neq j} I_{mj} \tag{13}$$

$$\frac{dR_j}{dt} = \gamma_I I_j + \gamma_H H_j + \sum_{m \neq j} R_{mj} - \sum_{m \neq j} R_{jm} \tag{14}$$

4 SOLUTION APPROACH

In this section, we propose some possible approaches to solve the above differential equations.

4.1 Homotopy Analysis Method

A powerful, easy-to-use analytic tool for nonlinear problems in general, namely the *homotopy analysis method*, is further improved and systematically described through a typical nonlinear problem in [5]. Two rules, the rule of solution expression and the rule of coefficient ergodicity, are proposed, which play important roles in the frame of the homotopy analysis method and simplify its applications in science and engineering. An explicit analytic solution is given for the first time, with recursive formulas for coefficients. This analytic solution agrees well with numerical results and can be regarded as a definition of the solution of the considered nonlinear problem. And the authors apply this method in SIS and SIR model in [6] which could also be generalized in our model.

4.2 Markov Chain

A Markov chain is a mathematical system that experiences transitions from one state to another according to certain probabilistic rules. The defining characteristic of a Markov chain is that no matter how the process arrived at its present state, the possible future states are fixed. In other words, the probability of transitioning to any particular state is dependent solely on the current state and time elapsed. The state space, or set of all possible states, can be anything: letters, numbers, weather conditions, baseball scores, or stock performances.

In our model, transitions between different states are displayed in Fig 2, we can apply the idea of Markov Chain to explore the relations of these states.

4.3 Euler Method

In mathematics and computational science, the Euler method (also called forward Euler method) is a first-order numerical procedure for solving ordinary differential equations (ODEs) with a given initial value. It is the most basic explicit method for numerical integration of ordinary differential equations and is the simplest Runge–Kutta method. The Euler method is a first-order method, which means that the local error (error per step) is proportional to the square of the step size, and the global error (error at a given time) is proportional to the step size. The Euler method often serves as the basis to construct more complex methods, e.g., predictor–corrector method.

For example, if we discretize the time and use $S^{(n)}$ to denote the susceptibles in time slot n, then we have:

$$S^{(n+1)} = S^{(n)} - \frac{c \cdot S^{(n)}}{N^{(n)}}(\beta_E E^{(n)} + \beta_I I^{(n)}) + \lambda SQ^{(n)} - q_S S^{(n)} \tag{15}$$

It is used to simulate the results.

5 EXPERIMENT

We use **Euler Method** to do simulation in our experiments, because other methods are rather difficult to be applied.

5.1 SEIR Baseline Model for WUHAN

In this subsection, we try to apply the original SEIR baseline model to Wuhan. Information in both demography and epidemiology is used to help determine our parameters. Specifically, The initial value for equations are set as follows.

- S(susceptible): set to 11800000. According to statistics about population in Wuhan
- E(exposed): set to 2000. Approximately the number of infected people between January 23 and January 29
- I(infected): set to 1000. Estimated value
- R(recovered): set to 23. According to official statistics on Jan 23

The parameters of our model are chosen mainly based on previous studies, while the exact values are also adjusted to fit our model better.

- β_E: set to $2.41 * 10^{-8}$.
- β_I: set to $2.41 * 10^{-8}$. Here we assume that $\beta_E = \beta_I$
- σ: set to 1/7. We expect an exposed person to take 7 days to get infected
- μ_H: set to $2.7 * 10^{-4}$. Estimated according to official statistics
- γ_H: set to 0.11. We expect a patient to take 9 days to recover

With the above settings, we try to use the original SEIR model to fit the epidemic statistics from January 23 in Wuhan. The result are shown in figure 3.

We use our model to approximate the transmission of COVID-19 in Wuhan since January 23, when the central government of China imposed a lockdown there. From figure 3, we can see that

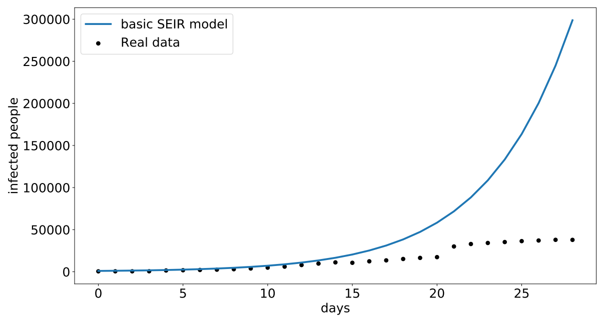

Figure 3: Simulated SEIR model in Wuhan

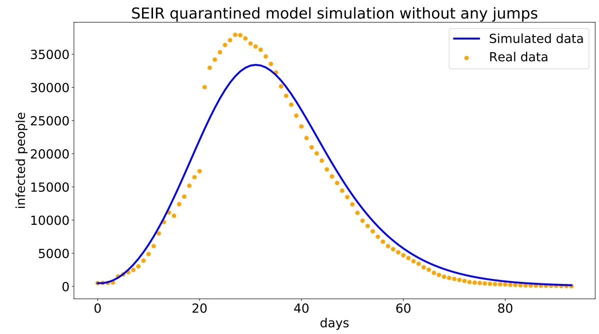

Figure 4: SEIR quarantined model simulation without any jumps from Jan 23

the original SEIR model fit the situation in Wuhan well in the first 15 days. However, the simulated number of infected people is much more than that in reality as time goes by.

We are not able to find any faults in the SEIR model despite scrutinization. The inference we make is that the model is not so suitable for Wuhan, and we propose our revised SEIR model for Wuhan later.

5.2 SEIR Under Quarantine for WUHAN

The impact of quarantine is considered in our revised model. Our new model inherits the original one, and it has several more parameters related to quarantined people. The new parameters are set as described below.

- λ: set to $\frac{1}{14}$. We expect a healthy people to be lifted from quarantine after 14 days.
- q_S: set to $1.8 * 10^{-7}$. It is the portion of susceptible people who are exposed
- q_E: set to 0.35 It is the portion of exposed people who are quarantines
- H: set to 485. According to official statistics on Jan 23
- δ_q: set to 0.13. Similar to σ (0.14)
- δ_I: 0.33. We expect it to take 3 days for an infected people to get hospitalized
- γ_H: 0.11. We expect a patient to recover after 9 days
- γ_I: 0.11. Assume $\gamma_I = \gamma_H$
- μ_H: $2.7 * 10^{-4}$. Assume $\mu_H = \mu_I$

We try to fit the real statistics with the above parameters, but we find that the simulated value of infected people is still too much.

We further adjust our model to deal with the problem. We assume that they both increase as time goes by, benefiting from the strict measures that the government takes. To be more exact, a shifted version of sigmoid function is used to approximate the change of δ_I and q_E.

Note that the start date is set as January 23 in this experiment, because it is the time when the whole city, Wuhan, was blockaded. Consequently, the city can be viewed as an isolated node from then on.

As shown in figure 4, our model fits well with the real statistics most of the times. However, a huge gap appears on approximately the 23rd day. Tracking back the statistics, we get to know that the jump of confirmed cases in Wuhan is caused by a change in statistical caliber. To better simulate the real case, we add a jump to the simulated curve on the exact date, and arrive at Figure 5.

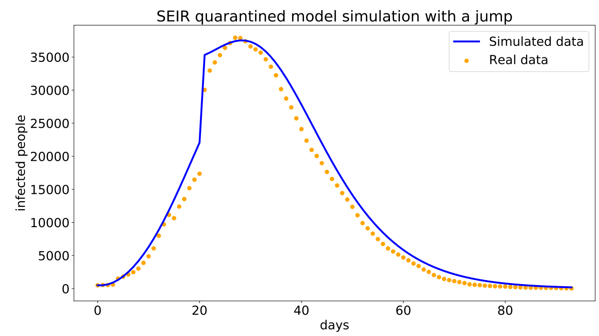

Figure 5: SEIR quarantined model simulation with a jump from Jan 23

Apart from testing more people and making more patients hospitalized, The Wuhan government also encourages people to stay at home most of the time by propaganda, which can lead to lower contact rate. In the next experiment, we try to analyze how the change of contact rate can make an impact on the numebr of infected people. The original contact rate in our model is 2. Besides the original setting, We set the parameter to 1.8 and 2.2 as well, and the curves are shown in figure 6

5.3 SEIR Under Migration

In this subsection, we conduct several experiments to illustrate the influence, and we further investigate into how the reaction of government will makes a difference.

The best way to examine our model of migration is to use real statistics to test. However, it is not so easy to finish the task in reality. The first problem we face is that it is difficult to design good parameters in our model for each city. Even if we do set the parameters well, doing simulation on them will probably lead to too much time to solve all the equations.

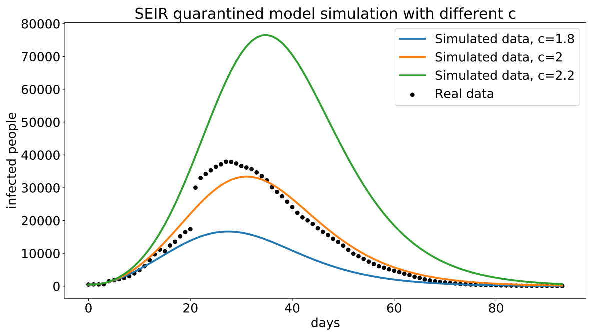

Figure 6: SEIR quarantined model simulation with different contact rates

Considering these facts, we decide to test our model only on several toy examples. As we have learned proper parameters for the city Wuhan, we do our simulation on several cities, and each one has the same parameters as Wuhan. We test on these cities by simulation.

The first topic is the influence of reducing inter-city mobility. Suppose there are four cities A, B, C, D, and each one has the same property as Wuhan. There are different percent of people moving from A to B, C, D every day, and the Figure 7 shows the result.

The peak appears almost simultaneously in the three cities, and the peak value is approximately proportional to the transmission rate. It enlightens us that reducing inter-city mobility can greatly flatten the curve, but have little influence on when the summit appears.

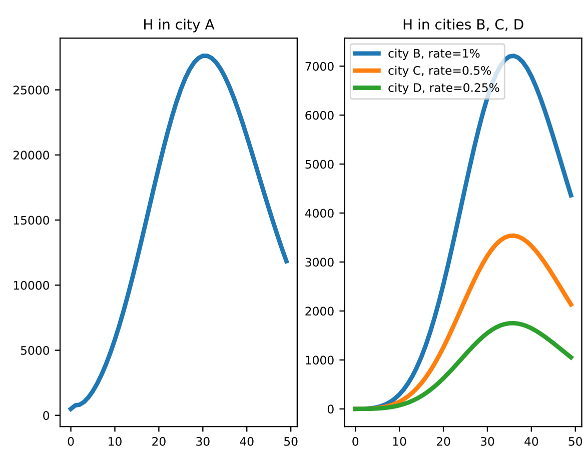

Figure 7: SEIR quarantined model simulation with different transmission rate

Next, we want to find out how the reaction time of the government will influence the curve. As we mentioned before, we use a shifted version of sigmoid function to approximate the change of δ_I and q_E. In the problem setting, the shift can be thought to be linear to the reaction time.

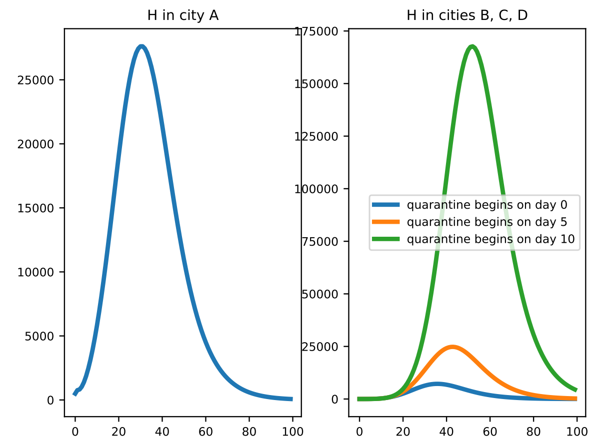

Figure 8: SEIR quarantined model simulation with different reaction time

The reaction time will influence both where the peak appears and what the peak value is. The shorter the reaction time, the earlier the peak and the less the peak value. It illustrates that taking actions early and decisively is important to the control of spreading.

6 CONCLUSION

Coronavirus disease 2019 (COVID-19) is an infectious disease which is now widely spread around the world. Traditional epidemiology models such as SEIR can not simulate the spread of the disease in China well ,and we introduce two new models, **SEIR Under Quarantine** and **SEIR Under Migration**, to solve the problem. The two considers the influence of quarantine and migration separately. After providing several possible ways to solve them theoretically, we do numerical simulation with Euler method. With proper parameters, the simulation result of our model is very similar to the real statistics. We also test them on some toy examples to show the need to take measures early and decisively. Our new models provides us with a new prospective to the spread of COVID-19, and they can be modified to analyze the spread of other diseases.

REFERENCES

[1] W. O. Kermack and A. G. McKendrick, "A contribution to the mathematical theory of epidemics," *Proceedings of the royal society of london. Series A, Containing papers of a mathematical and physical character*, vol. 115, no. 772, pp. 700–721, 1927.

[2] O. Diekmann, J. A. P. Heesterbeek, and J. A. Metz, "On the definition and the computation of the basic reproduction ratio r 0 in models for infectious diseases in heterogeneous populations," *Journal of mathematical biology*, vol. 28, no. 4, pp. 365–382, 1990.

[3] H. W. Hethcote, "The mathematics of infectious diseases," *SIAM review*, vol. 42, no. 4, pp. 599–653, 2000.

[4] R. W. West and J. R. Thompson, "Models for the simple epidemic," *Mathematical biosciences*, vol. 141, no. 1, pp. 29–39, 1997.

[5] S. Liao, "On the homotopy analysis method for nonlinear problems," *Applied Mathematics and Computation*, vol. 147, no. 2, pp. 499–513, 2004.

[6] H. Khan, R. N. Mohapatra, K. Vajravelu, and S. Liao, "The explicit series solution of sir and sis epidemic models," *Applied Mathematics and Computation*, vol. 215, no. 2, pp. 653–669, 2009.

WORK DIVISION

- Wenze Ma: propose **SEIR Under Quarantine** and **SEIR Under Migration** model; further derive a series of differential equations and analyze some possible way to solve them; devise several experiments to test our hypothesis.
- Xiaoyi Bao:write codes to do simulation on model SEIR, **SEIR Under Quarantine**, **SEIR Under Migration** with numerical method;use a web crawler to grab statistics from the internet; devise several experiments to test our hypothesis.
- Wenze Ma: Subsection 2, 3, 4
- Xiaoyi Bao: Subsection 1, 5, 6