Joint Computing Offloading and Resource Allocation for Propagation in Blockchain

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***Abstract*—The mining process in blockchain requires solving a proof-of-work puzzle, which is resource expensive to implement in mobile devices due to the high computing power and energy needed. Meanwhile, Mobile edge computing (MEC) providing information technology and cloud-computing capabilities within the radio access network is an emerging technique in fifth-generation networks. In this paper, we consider Mobile edge computing as an enabler for mobile blockchain. In order to minimize the energy consumption on miners, we jointly optimize the offloading selection, radio resource allocation, and computational resource allocation coordinately. We formulate the energy consumption minimization problem as a mixed interger nonlinear programming problem, which is subject to specific application latency constraints. We show by simulation that the joint offloading task execution is more energy-efficient than local execution and remote execution for miners.**

***Index Terms*—mobile blockchain, proof-of-work puzzle, Mobile edge computing, computation offloading, energy minimization, mining, game theory.**

I. Introduction

Electronic trading with digital transactions is becoming popular than ever in e-commerce society, where the consensus is reached through trusted centralized authorities. The introduction of centralized authorities incurs additional cost, i.e., nominal fees which become more excessive when the number of digital transactions becomes large. In 2008, a new peer-to-peer electronic payment system called Bitcoin was introduced that avoids this additional cost caused by digital transactions. As one popular digital cryptocurrency, Bitcoin can record and store all digital transactions in a decentralized append-only public ledger called blockchain. The Bitcoin is the first application of blockchain technologies. Subsequently, the blockchain technologies have generated remarkable public interests via a distributed network with independence from central authorities. With blockchain, a transaction can take place in a decentralized fashion, which greatly save the cost and improve the efficiency. Since its launch in 2009, Bitcoin economy has experienced an exponential growth, and its capital market now has reached over 70 billion dollars.After the success of Bitcoin, blockchain has been applied in many applications, such as access control systems, smart contracts, content delivery networks, cognitive radio networks, and smart grid powered systems.

The core issue of the blockchain is a computational process called mining, where the transaction records are added into the blockchain via the solution of computational difficult problem, i.e., the proof-of-work puzzle. Confirming and securing the integrity and validity of transactions are processed by a set of participants called miners. However, blockchain has not been adopted widely in mobile applications. This is because blockchain mining needs to solve a proof-of-work puzzle, which is expensive to implement in mobile devices due to the high computing power needed. Thus, deploying blockchain in a mobile environment is truly challenging. So, numerous efforts have been made by researchers from academia and industry to design the fifth generation (5G) mobile communication with the advance of network technologies and the innovation of mobile services to solve the energy consumption problem. One of the prominent characteristics of 5G is strong data processing capability in order to cope with increasing content requests. People make continuous efforts on mobile network operators and network equipment vendors to enhance the wireless link bandwidth. In addition, large number of IoT terminal equipments are applied to various vertical industries. However, this migration not only increase the network load but also causes the delay fluctuation which influences the latency-sensitive application.In order to increase the bandwidth and decrease the latency, energy consumption and network load for computation offloading, European Telecommunications Standards Institute (ETSI) has proposed a promising approach, Mobile Edge Computing (MEC). In this paper, we consider MEC as a network enabler for the mobile blockchain.

In the MEC framework, cloud computing capabilities are provided within the Radio Access Network (RAN) in close proximity to miners need to solve the mining problem. In the computation offloading of MEC, a mining problem can be executed on the mobile application (local execution), or on the MEC server (edge execution). Due to the short distance between the MEC server and SMDs, the MEC paradigm can provide low latency, high bandwidth and computing agility in computation offloading. However, both radio and computational resources are limited in MEC. Motivated by the differences between MEC and traditional MCC, we dedicate to design a computation offloading mechanism for MEC. In this paper, we investigate an energy minimization problem, which is subject to specified delay constraints, in order to optimizes offloading selection, radio resource allocation and computation resource allocation jointly. We learn an algorithm to solve the problem with adjustable solving accuracies. The main contributions of this paper are as follows:

1) : We adopt offloading to MEC server to solve the mining problem in blockchain. we consider Mobile edge computing as an enabler for mobile blockchain. In order to minimize the energy consumption on miners, we jointly optimize the offloading selection, radio resource allocation, and computational resource allocation coordinately. We formulate the energy consumption minimization problem as a mixed interger nonlinear programming problem, which is subject to specific application latency constraints.

2) : To effectively save energy consumption on miners, we jointly optimize the offloading selection, radio resource allocation and computational resource allocation coordinately in the energy minimization problem. To the best of our knowledge, there are few works optimizing these three aspects jointly to minimize the energy consumption in a multi-users system.

3) : We learn a Reformulation-Linearization-Technique based Branch-and-Bound method (RLTBB) with adjustable solving accuracy to solve the energy minimization problem.We show by simulation that the joint offloading task execution is more energy-efficient than local execution and remote execution for miners.

The rest of the paper is organized as follows.In Section II, we review related work. In Section III, we present the system model of mobile blockchain with MEC computation offloading and formulate the energy minimization problem as an MINLP problem. In Section IV, we analyze the optimal distribute of task for miners in detail. In Section V,we present the simulation results. Finally, the conclusion is drawn in Section VI,.

II. Related Work

Recently,there have been several studies on mining schemes management for blockchain network. In [1], the authors designed a noncooperative game among the miners, i.e., the players. The miners strategy is to choose the number of transactions to be included in a block. In the model, solving the proof-of-work puzzle for mining is modeled as a Poisson process. The solution of the game is the Nash equilibrium which was derived only for two miners in [1]. Then, the authors in [2] modeled the mining process as a sequential game where the miners compete for mining reward in sequentially among them. In the game model, the miners are assumed to be rational, and they have to choose whether or not to propagate their solution, i.e., the mined block. It is proved in [2] that there exists a multiplicity of Nash equilibrium. Further, it is found that not propagating is an optimal strategy under certain conditions. Similar to that in [2], the authors in [3] formulated the stochastic game for modeling the mining process, where miners decide on which blocks to extendandwhethertopropagatetheminedblock.Inparticular, two game models in which miners play a complete information stochastic game are studied. In the first model, each miner propagates immediately the mined block that it mines. The strategy of each miner is to select an appropriate block to mine. In the second model, the miner selects which block to mine, but it may not propagate its mined block immediately. For both models, it is proved in [3] that when the number of miners is suf?ciently small, the Nash equilibrium with respect to mining behaviors exists.

To reduce the energy consumption and latency in computation of?oading, ETSI proposed MEC which can provide Information Technology (IT) and cloud-computing capabilities within the RAN in close proximity to mobile subscribers [4]. Recently, there are some works [5] to [7] on computation of?oading in MEC with various objectives.[5] developed an of?oading framework, named Ternary Decision Maker (TDM), which aimed to shorten response time and reduce energy consumption at the same time. A more flexible execution environment for mobile applications was adopted. On account of the comprehensive modeling and the practical simulation environment, [5] gave good contributions on computation offloading. However, [5] considered the single user scenario,and would be better to extend to the multiuser scenario. [6]incorporated dynamic voltage scaling(DVS) into computation of?oading in a single-user scenario. They investigated partial computation of?oading by jointly optimizing the computational speed of SMD, transmit power of SMD, and offloading ratio. An energy-optimal partial computation offloading(EPCO) algorithm was proposed to solve the nonconvex energy consumption minimization problem. Furthermore, a local optimal algorithm was proposed to handle the nonconvex and nonsmooth latency minimization problem.Youetal. [7]studied resource allocation for a multiuser mobile-edge computation of?oading (MECO) system based on time-division multiple access (TDMA) and orthogonal frequency-division multiple access (OFDMA) to minimize the mobile energy consumption. [7] gave comprehensive modeling analyses. Moreover, for the TDMA MECO system with in?nite computation capacity, an optimal policy was designed. For the TDMA MECO system with ?nite computation capacity and the OFDMA MECO system,respectively, two sub-optimal algorithms were designed. But [6], [7] concentrated on the offloading proportion of users mainly, and ignored the joint optimization of radio and computational resources.

In the cloud platform, the data stored in the semi-trusted parties is not safe. To guarantee the confidentiality, integrity and availability of information, it is necessary to encrypt owner's information. Also, attribute-based encryption (ABE) is one of the most popular schemes used in cloud computing [8]. It provided a design and implementation of self-protecting digital information using attribute-based encryption on mobile devices [9]. The system is designed to provide fine-grained encryption and is able to protect individual items within a record, where each encrypted item may have its own access control policy. Li et al. proposed a novel patient-centric framework and a suite of mechanisms for data access control to personal health records (PHRs) stored in semitrusted servers [10]. They leverage ABE techniques to encrypt each patient's PHR file, focus on the multiple data owner scenarios and divide the users in the PHR system into multiple security domains that greatly reduces the key management complexity for owners and users. These schemes improve ABE steadily ceaselessly, protecting people's privacy greatly. However, ABE is not perfect. It has several disadvantages. For example, once a user modifies his access polices, that is, the system needs to execute attribute revocation and encrypt data again based on new attribute sets. This results in extra computational expenditures and slows down the efficiency of the system, especially in papers [11],[12].

III. System Model

We consider a system model like this, where miners can offload their mining computation tasks to the MEC server through a cellular network. The set of miners can be denoted as $N = \{1, 2, ..., N\}$.

Each miner i has a mining computation task $A_i = (D_i, C_i, T_i^{th})$, where D_i denoted the size of mining computation input data including system settings, program codes and input parameters, C_i denotes the number of CPU cycles required to accomplish the mining computation task, and T_i^{th} denotes the corresponding delay constraint.

All the mining tasks can be divided into subtasks. And each computation task can be either executed locally or offloaded to MEC. We define the offloading vector as $\alpha = [\alpha_1, \alpha_2, ..., \alpha_N]$. If task Ai is executed locally, then $\alpha_i = 0$, else $\alpha_i = 1$.

A. Local Execution Model

We define f_i^l as the local computation capability we paid, then the local compute time for mobile i becomes

$$t_i^l = (1-\alpha_i)\frac{C_i}{f_i^l} \quad (1)$$

and corresponding energy consumption of miner i is

$$e_i^l = (1-\alpha_i)\kappa(f_i^l)^2 C_i \quad (2)$$

where κ is the effective switched capacitance depending on the chip architecture. We set $\kappa = 10^{26}$ according to the practical measurement. Considering that the energy consumption grows with the allocated CPU-cycle frequency, we can minimize the energy consumption by controlling α_i.

B. Computation Offloading Model

The computation offloading process can be divided into three steps:

(1) The mobile user i uploads to MEC server through the uplink channel;

(2) MEC server compute the mining task on behalf of the mobile miner i;

(3) MEC server transmits output data back to the mobile miner.

In this case, we assume the output data result is much more smaller than the input data size, so we ignored the step(3), just consider step(1) and step(2).Compared with the local computation, offloading saves the energy of mobile miner, but spends additional time and energy in uplink transmission. And the intracell interference of offloading will increase energy consumption as well.

In order to concentrate more on the algorithm design, we assume tasks are admitted as random Poisson distribution and must be done in slot T_i^{th}.The wireless channel is constituted of L orthogonal frequency subchannels. The achievable uplink rate for miner i in subchannel n can be obtain as

$$r_i^n = W\log(1+\frac{p_i^n h_i^n}{WN_0}), \quad (3)$$

where W is the bandwidth of the uplink channel,p_i^n is the transmit power of miner i in subchannel n,h_i^n is the channel gain of miner i in subchannel n, and N_0 is the noise power spectral density. In order to concentrate more on the algorithm design, we simplify the communication model and make an assumption that the subchannels to be homogeneous for each miner, then equal power is allocated to each assigned subchannel. Then the uplink rate for miner i in each subchannel can be obtain as

$$r_i = W\log(1+\frac{ph_i}{WN_0}) \quad (4)$$

where p is the transmit power of each miner in each assigned subchannel, hi is the channel gain of mining task i in each subchannel. We define F as the maximal CPU-cycle frequency of the MEC server, and define f_i as the assigned CPU-cycle frequency to compute task Ai on the MEC server. When task A_i is executed by edge execution, the required time of A_i is

$$t_i^f = \alpha_i(\frac{D_i}{R_i}+\frac{C_i}{f_i}) \quad (5)$$

and energy consumption of miner i is

$$e_i^f = \alpha_i(P_i^T\frac{D_i}{R_i}+P_i^I\frac{C_i}{f_i}) \quad (6)$$

where P_i^T i is the transmit power of miner i, P_i^I is the power consumption in idle state.

C. Problem Formulate

The objective of the paper is to minimize the total energy consumption of solving the mining problem under speci?ed latency constraints. The total time we need is

$$T_i = t_i^l + t_i^f = (1-\alpha_i)\frac{C_i}{f_i^l} + \alpha_i(\frac{D_i}{R_i}+\frac{C_i}{f_i}) \quad (7)$$

and we must accomplish the task in slot T_i^{th}:

$$T_i \leq T_i^{th} \quad (8)$$

The total energy we cost is

$$E_i = E_i^l + E_i^f = (1-\alpha_i)\kappa(f_i^l)^2 C_i + \alpha_i(P_i^T\frac{D_i}{R_i}+P_i^I\frac{C_i}{f_i}) \quad (9)$$

The objective of the paper is to minimize the total energy consumption of solving the mining problem under specified latency constraints. To guarantee that problem (9) has an optimal solution, we restrict $T_i^{TH} \geq \frac{C_i}{f_i^l}$.

IV. Problem Solution

In this section, we introduce an accuracy-adjustable algorithm, RLTBB. We propose a Reformulation-Linearization-Technique based Branch-and-Bound method (RLTBB) with adjustable solving accuracy to solve the energy minimization problem. We use the ReformulationLinearization-Technique (RLT) relaxation technique to convert the original problem to a Mixed Boolean-convex problem.

To avoid the devide-by-zero error, we introduce two microscales, ε_1 and ε_2, then we has

$$T_i = (1-\alpha_i)\frac{C_i}{F_i^l} + \alpha_i[\frac{D_i}{r_i(\varepsilon_1+\theta_i)} + \frac{C_i}{\theta_2+f_i}] \tag{10}$$

$$E_i = (1-\alpha_i)\kappa(F_i^l)^2C_i + \alpha_i[P_i^T\frac{D_i}{r_i(\varepsilon_1+\theta_i)} + P_i^I\frac{C_i}{\theta_2+f_i}] \tag{11}$$

As we can see, the problem is sensible to ε_1 and ε_2 for obtaining a lower bound of problem before. Now we define two new variables $\beta_i = (\varepsilon_1+\theta_i)^{-1}$ and $\gamma_i = (\varepsilon_2+f_i)^{-1}$, then the new problem can be define as

$$\begin{aligned}
\min_{\alpha,\beta,\gamma} \sum_{i=1}^{N} & [(1-\alpha_i)\kappa(F_i^l)^2C_i + \alpha_i(\frac{P_i^T D_i}{r_i}\beta_i + P_i^I C_i\gamma_i)] \\
s.t.\quad & C_6: \forall i \in N \\
& C_7: (1-\alpha_i)\tfrac{C_i}{f_i^l} + \alpha_i(\tfrac{D_i}{r_i}\beta_i + C_i\gamma_i) \le T_i^{th}, \\
& C_8: \tfrac{1}{\alpha_i L+\varepsilon_1} \le \beta \le \tfrac{1}{\varepsilon_1}, \\
& C_9: \tfrac{1}{\alpha_i F+\varepsilon_2} \le \gamma \le \tfrac{1}{\varepsilon_2}, \\
& C_{10}: \textstyle\sum_{i=1}^{N} \tfrac{1}{\beta_i} \le L + N\varepsilon_1, \\
& C_{11}: \textstyle\sum_{i=1}^{N} \tfrac{1}{\gamma_i} \le F + N\varepsilon_2,
\end{aligned}$$

It is a nonconvex problem because of the discrete variables and the second order terms in the form of $x \cdot y$. RLT can linearize the second order terms in the form of $x \cdot y$ [13],[14].Therefore, we can get a convex relaxation problem based on RLT and the relaxation $0 \le \alpha_i \le 1$. Particularly, we adopt the RLT to linearize the objective function and constraint T_i in Problem before. For the second order term $\alpha_i \cdot \beta_i$, we fefine $\mu_i = \alpha_i \cdot \beta_i$. And for the secong order term $\alpha_i \cdot \gamma_i$, we define $\omega_i = \alpha_i \cdot \gamma_i$.

After substituting μ_i and ω_i into the objective, we obtain another term of the convex optimization problem as follow,

$$\begin{aligned}
\min_{\alpha,\beta,\gamma,\mu,\omega} \sum_{i=1}^{N} & [(1-\alpha_i)\kappa(F_i^l)^2C_i + \alpha_i(\frac{P_i^T D_i}{r_i}\mu_i + P_i^I C_i\omega_i)] \\
s.t.\quad & C_6: \forall i \in N \\
& C_7: (1-\alpha_i)\tfrac{C_i}{f_i^l} + \alpha_i(\tfrac{D_i}{r_i}\mu_i + C_i\omega_i) \le T_i^{th}, \\
& C_8: \tfrac{1}{\alpha_i L+\varepsilon_1} \le \beta \le \tfrac{1}{\varepsilon_1}, \\
& C_9: \tfrac{1}{\alpha_i F+\varepsilon_2} \le \gamma \le \tfrac{1}{\varepsilon_2}, \\
& C_{10}: \textstyle\sum_{i=1}^{N} \tfrac{1}{\beta_i} \le L + N\varepsilon_1, \\
& C_{11}: \textstyle\sum_{i=1}^{N} \tfrac{1}{\gamma_i} \le F + N\varepsilon_2, \\
& C_{12}: 0 \le \alpha_i \le 1,
\end{aligned}$$

The optimal value of this problem is a lower bound of $\min E_i$ in problem(11).

We define $N_1 = \{i \mid i \in N, \alpha_i = 1\}$ and $N_0 = \{i \mid i \in N, \alpha_i = 0\}$. Obviously, when α is determined, problem(11) can be converted as

$$\min_{\theta,f} \sum_{j\in N_0} \kappa(F_i^l)^2C_i + \sum_{i\in N_1}(\frac{P_i^T D_i}{r_i\theta_i} + \frac{P_i^I C_i}{f_i}) \tag{12}$$

The optimal value of this problem is a upper bound of $\min E_i$ in problem(11). Therefore, we adopt the BB^4 method based on (15), (16), which can be solved by the state-of-the-art convex optimization algorithms, to solve the problem (11).

In order to implement the BB method, we build a search tree, which is generated based on the depth-?rst strategy. The root node of the tree represents problem(11). RLTBB can converge and its computation complexity is exponential.

V. Simulation Results

There is an orthohexagonal region, which is covered by an eNB located at the center, with 500m in diameter. miners are randomly scattered over the region. There is an MEC server located in the miners, whose computation capability is $F = 5GHz/sec$. The transmission power of the miner in idle state are set to be $P_i^I = 10mWatts and P_i^T = 100mWatts$. And the computation capability of each miner is $F_i^l = 0.5GHz$. The data $size^8$ of the computation of?oading and total number of CPU $cycles^9$ are Gaussian distributions, $D_i \sim N(400, 100)$ and $C_i \sim N(1000, 100)$.

The proposed algorithms are compared with three methods. The optimal results are obtained by brute-force search. In this case, when the offloading selection is given, we use the convex optimal method to calculate the radio and computational resource allocations. "All-Local" stands for that all miners execute their applications locally. "$RLTBB_{-\varepsilon} = 0.6$" stands for the RLTBB with solving accuracy $\varepsilon = 0.6$. "$RLTBB_{-\varepsilon} = 0.2$" stands for the RLTBB with solving accuracy $\varepsilon = 0.2$ and "$RLTBB_{-\varepsilon} = 0$" stands for the RLTBB with solving accuracy $\varepsilon = 0$.

CDFs of the energy consumption under different algorithms are shown in fig.1. From the curves, $RLTBB_{-\varepsilon} = 0$ can obtain the optimal result. $RLTBB_{-\varepsilon} = 0.6$ and

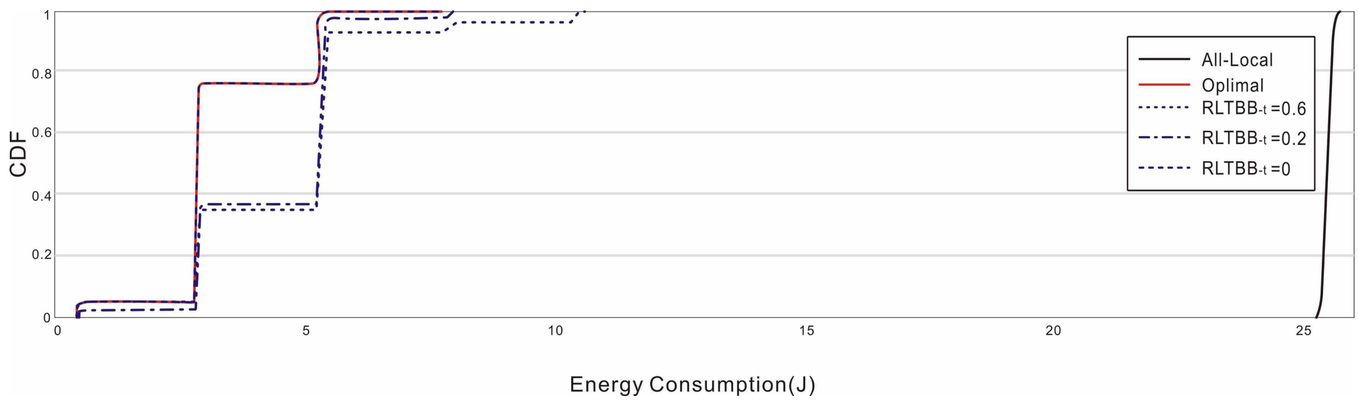

Fig. 1. formulation.jpg

$RLTBB_{-\varepsilon} = 0.2$ can not achieve the optimal result, but their results save much energy compared with ALL-Lacal. The energy savings of $RLTBB_{-\varepsilon} = 0.2$ and $RLTBB_{-\varepsilon} = 0.6$ achieve 97.97%, 94.69% of the optimal energy saving. The result of $RLTBB_{-\varepsilon} = 0.2$ is superior to the result of $RLTBB_{-\varepsilon} = 0.6$ on account of that the solving accuracy of $RLTBB_{-\varepsilon} = 0.2$ is smaller. In addition, we see that $RLTBB_{-\varepsilon} = 0.2$ and $RLTBB_{-\varepsilon} = 0.6$ achieve the same result usually. The reason is that the gap between the upper bound and lower bound in RLTBB decreases leapingly.

VI. Conclusion

In this paper, we consider Mobile edge computing as an enabler for mobile blockchain. In order to minimize the energy consumption on miners, we jointly optimize the offloading selection, radio resource allocation, and computational resource allocation coordinately. We formulate the energy consumption minimization problem as a mixed interger nonlinear programming problem, which is subject to specific application latency constraints. We show by simulation that the joint offloading task execution is more energy-efficient than local execution and remote execution for miners. In this paper, we investigated the MEC computation offloading in a multi-users system. In order to minimize the energy consumption on miners, we jointly optimized the offloading selection, radio resource and computational resource allocations. We formulated an energy consumption minimization problem under specific application latencies. To solve the MINLP problem, we proposed the RLTBB method which can not only obtain the optimal result but also calculate a specific suboptimal result with the adjustable solving accuracy. We also conducted numerous simulations, which validate the energy saving enhancement in our proposed RLTBB.

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