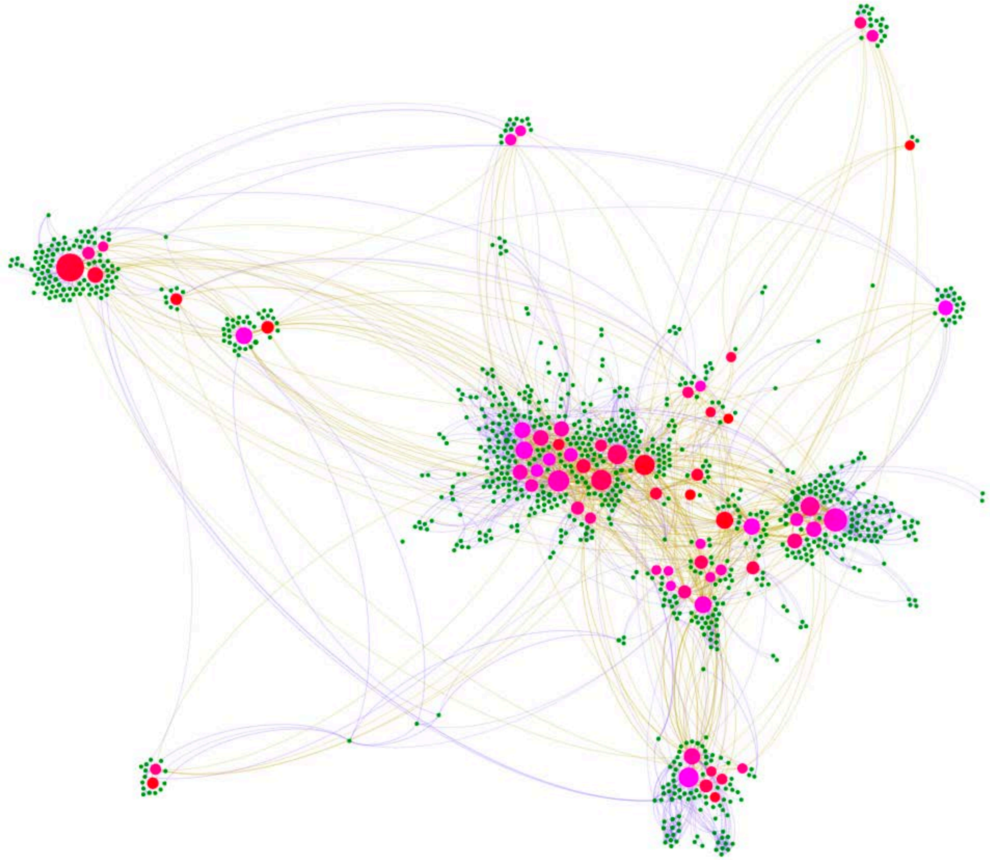
Generalized Model for Core Percolation in Interdependent Network

Mobile Internet Course

Nan Wang

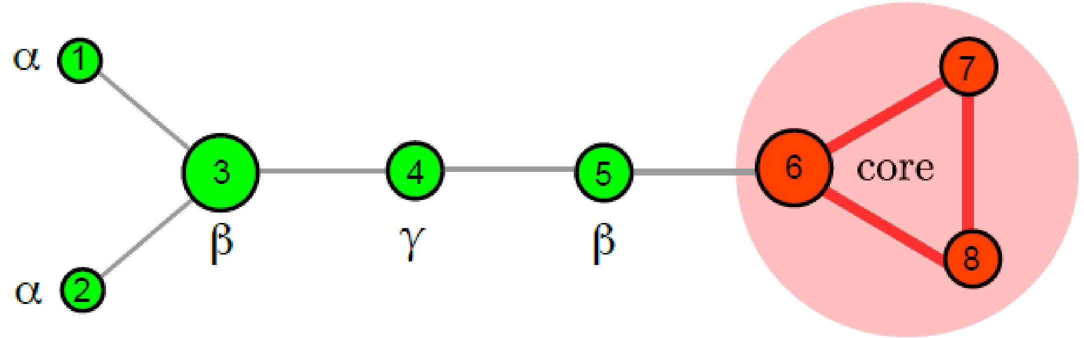
Introduction

- Core percolation, as a fundamental structural transition resulted from preserving codes nodes in the network, is crucial in network controllability and robustness.
- We can consider core nodes as stable nodes in a network.



GLR Procedure.

- Prior works are mainly based on single, non-interacting network where core nodes are obtained by a classic Greedy Leaf Removal (GLR) procedure that takes off leaf nodes along with their neighbors iteratively.



- α-removable: can become isolated without directly removing themselves; β-removable: which can become a neighbor of a leaf; γ-removable: which can be removable but neither α-removable nor β-removable.

Generate Networks

- **Static Scale-free Model** （SSF）

Start from N disconnected vertices, each one of them indexed by an integer number i, i =1,...N. To each vertex, a normalized probability pi is assigned as,

$$p_i = \frac{i^{-\alpha}}{\sum_{j=1}^{N} j^{-\alpha}} \approx i^{-\alpha} \frac{1-\alpha}{N^{1-\alpha}}, \text{for large N and } \alpha \in [0,1].$$

Two different vertices i and j are randomly selected from the set of N vertices, with probability pi and pj, respectively to generate edges. Repeat this process by E=m*N times.

Degree Distribution of SSF Model

- For a static scale-free network model,

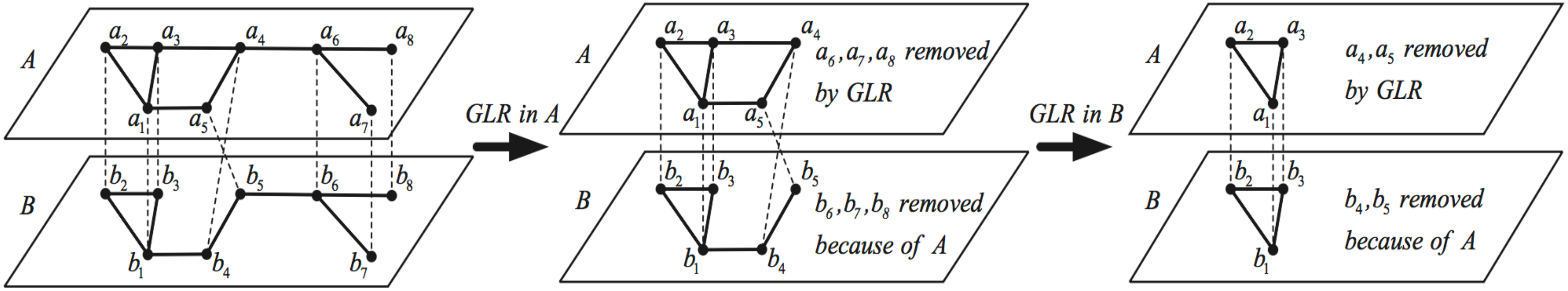
$$P(k) = \frac{[2m(1-\alpha)]^{\frac{1}{\alpha}}}{\alpha} \frac{\Gamma\left(k-\frac{1}{\alpha}, 2m(1-\alpha)\right)}{\Gamma(k+1)}.$$

For large k, $$P(k) \cong \frac{[2m(1-\alpha)]^{\frac{1}{\alpha}}}{\alpha} \frac{\Gamma(k-\frac{1}{\alpha})}{\Gamma(k+1)} \sim k^{-(1+\frac{1}{\alpha})} = k^{-\gamma}$$

Where k is the degree of one node and P(k) denotes the probability of one node to have degree k.

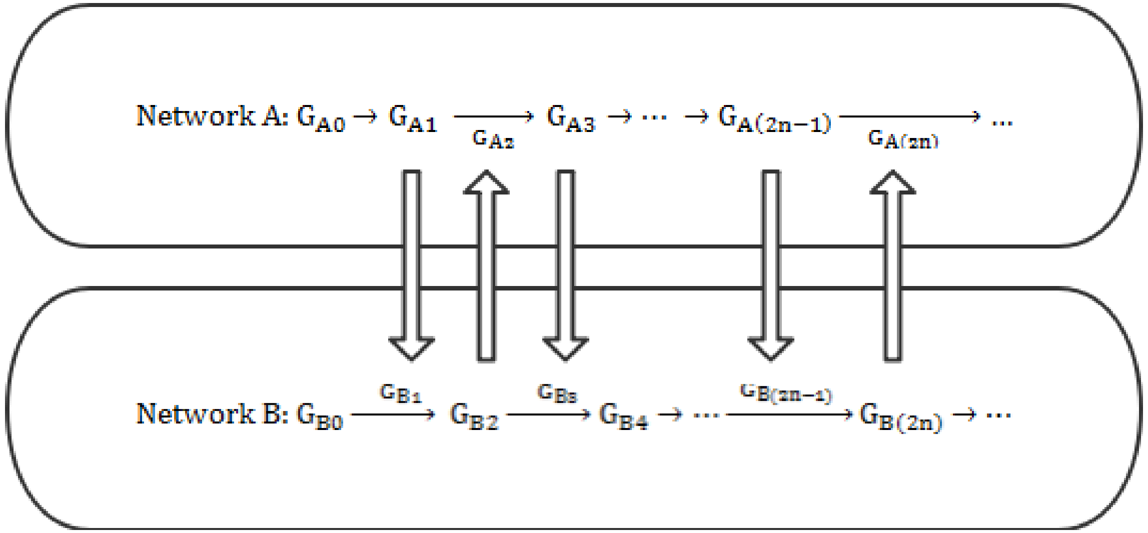
Alternating GLR Procedure

- Consider Two networks A and B. Assume that each node in A depends on a node in B with a probability q(0<q⩽1) and vice versa. We use Alternating GLR Procedure to get the core of the interdependent networks.



Fully-Interdependent Networks

- Firstly consider the condition of q=1, which means fully interdependency.



GLR procedure in two interdependent networks

Partially-Interdependent Networks

- A more general condition: Nodes between networks A(with M nodes) and B(with N nodes) are partially dependent with probability q_A and q_B respectively

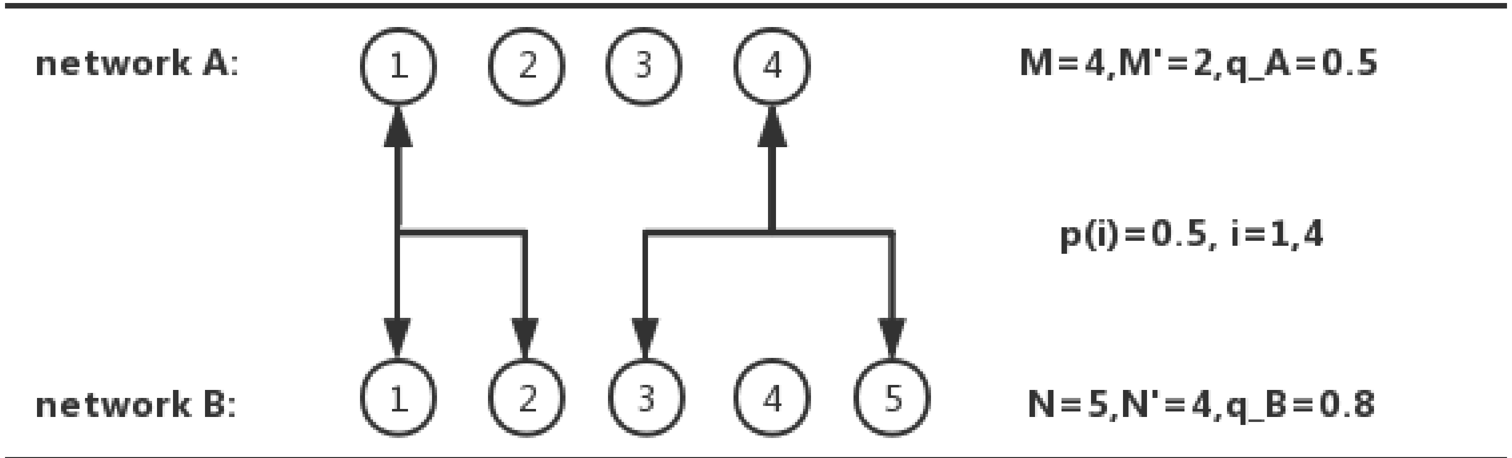
where $0<q_A,q_B\leqslant 1$, and $M * q_A = N * q_B \stackrel{\text{def}}{=} R$

Revised G function:

$$P_{i\,n} = G\left(P_{i(n-1)}, u\right) = \sum_{k'=k}^{\infty} P_{i(n-1)}(k') \binom{k'}{k} (1 - q_i(1-u))^k (q_i(1-u))^{k'-k}$$

One-to-Multiple Interdependent Networks

- Assume N'=N*q_B nodes in network B have a partner in network A, and assume they are partners of node i in network A with probability p_i(i=1,2,⋯,M'). M is the number of nodes in A that may have partners, and denote q_A=M'/M. Denote X_i as the number of nodes being partners of node A_i. Then we have
- $P(X_1 = x_1, X_2 = x_2, \ldots, X_{M'} = x_{M'}) = M'! \prod_{i=1}^{M'} \frac{p_i^{x_i}}{x_i!} \quad (x_i \geq$



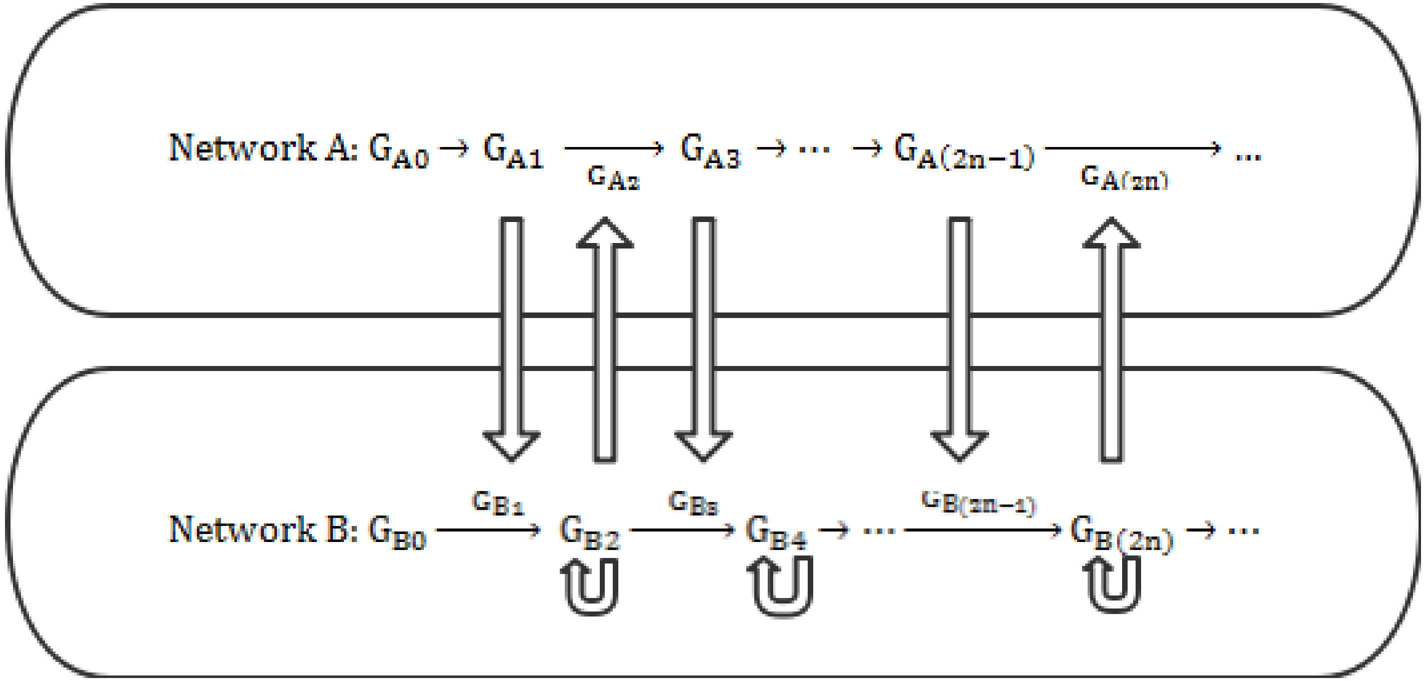
Remaining Nodes

- Further assume there are t_i nodes being partners of A_i(i=1,2,⋯,N', and denote $t=\sum_i t_i$) in the removing nodes in network B. We will remove A_i and all its partners if t_i does not equal to 0, which makes the remaining nodes to be $R=\sum_{i=1}^{M'} x_i 1_{[t_i=0]}$.
- We can get the expectation of remaining nodes
- $E(R)=$

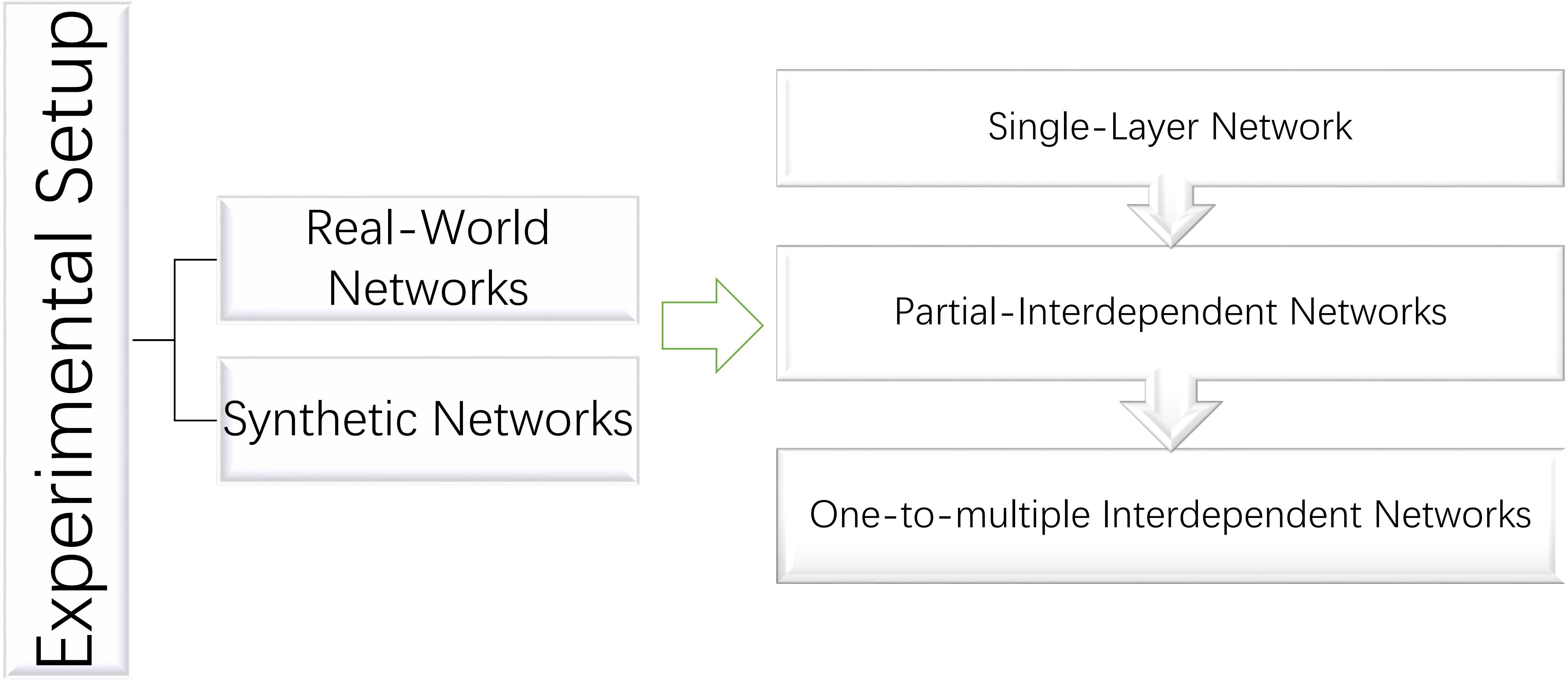
$$\sum_{x_1+x_2+\cdots+x_{M'}=N'} P(x_1,x_2,\ldots,x_{M'})\cdot\left[\sum_{t_1+t_2+\cdots+t_{M'}=t}\frac{C_{x_1}^{t_1}C_{x_2}^{t_2}\ldots C_{x_{M'}}^{t_{M'}}}{C_{N'}^{t}}\cdot R\right]=$$

$$\sum_{x_1+x_2+\cdots+x_{M'}=N'} M'!\prod_{i=1}^{M'}\frac{p_i^{x_i}}{x_i!}\cdot\left[\sum_{t_1+t_2+\cdots+t_{M'}=t}\frac{C_{x_1}^{t_1}C_{x_2}^{t_2}\ldots C_{x_{M'}}^{t_{M'}}}{C_{N'}^{t}}\cdot\sum_{i=1}^{M'} x_i 1_{[t_i=0]}\right]$$

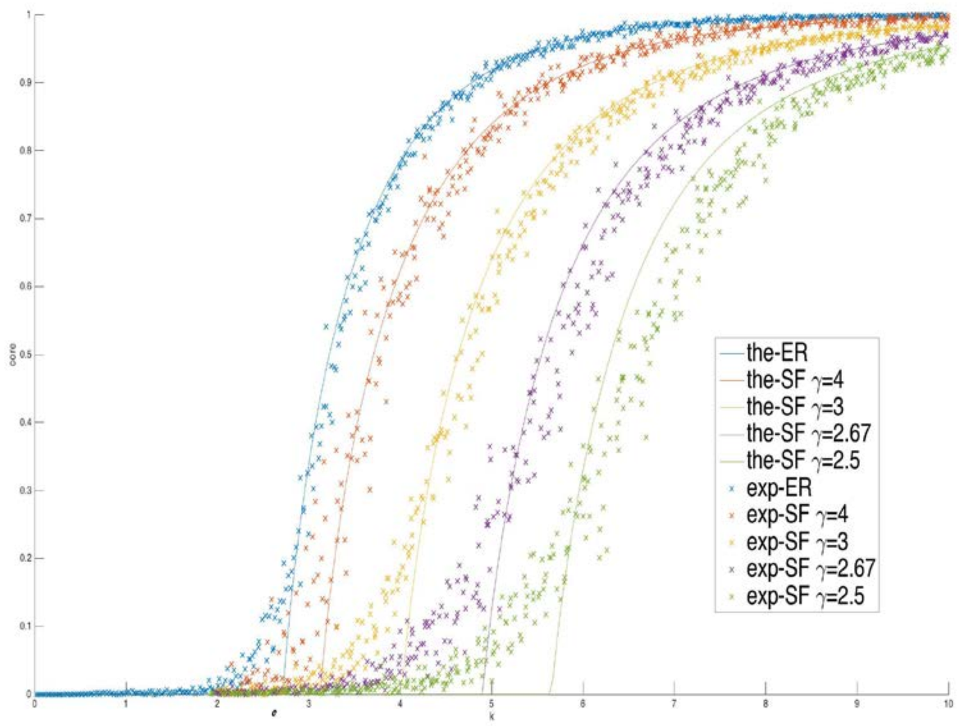
Revised AGLR Procedure



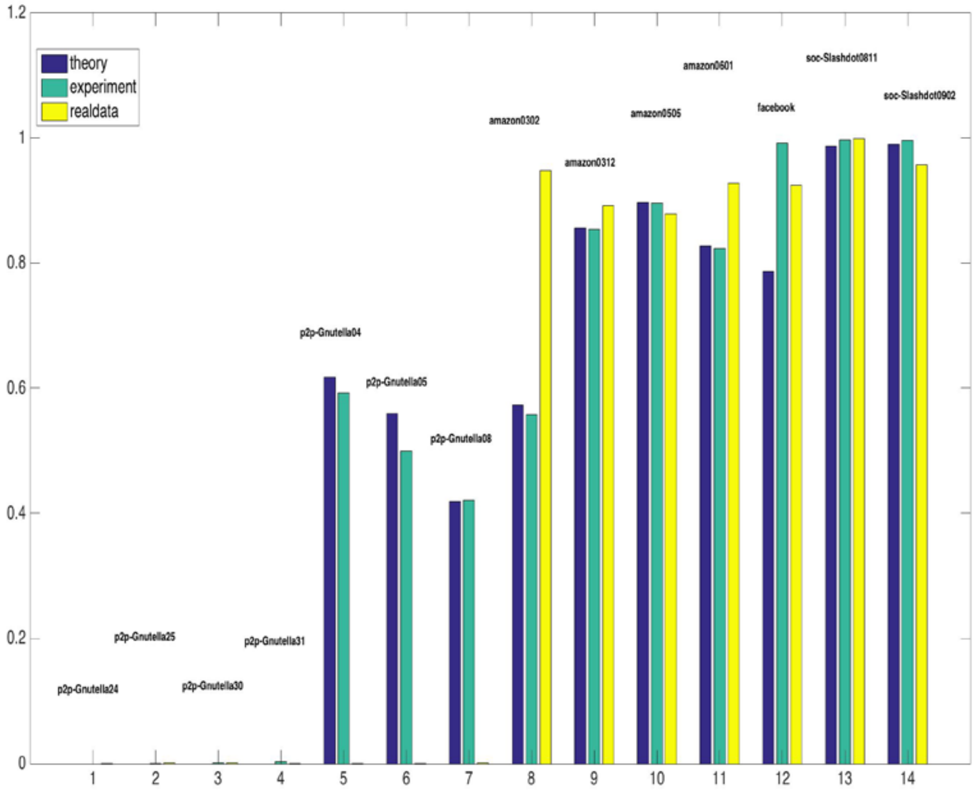
Experimental Study



Single-Layer Network

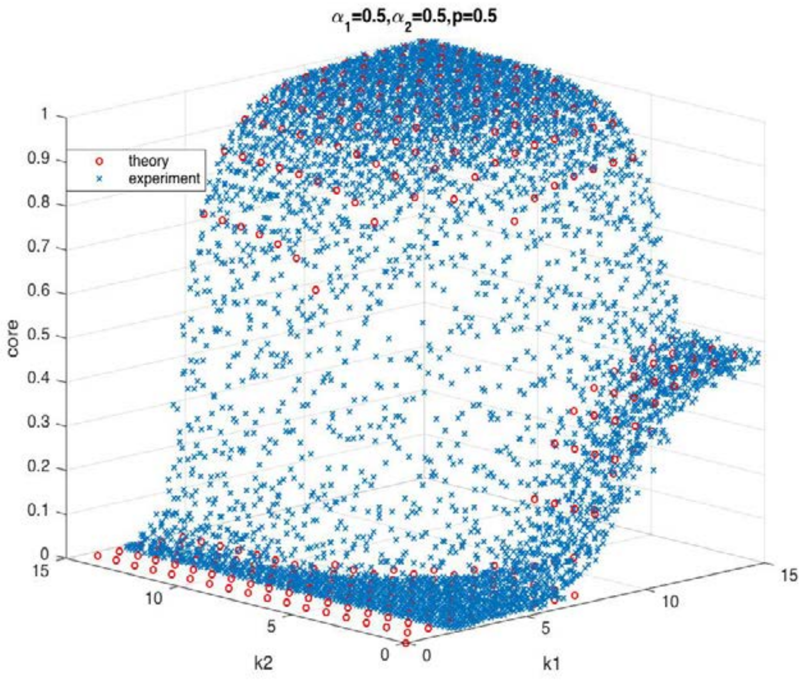


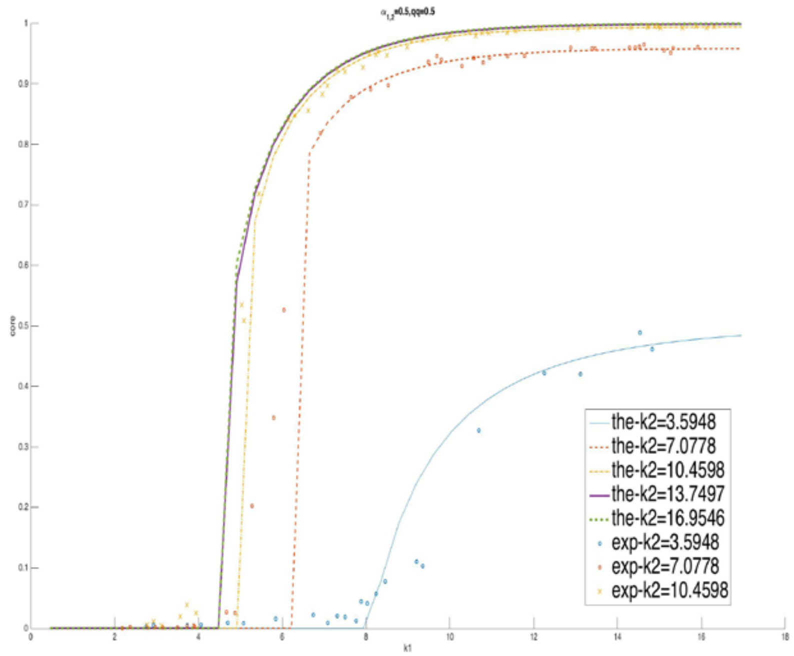
Synthetic Networks



Real-world Networks

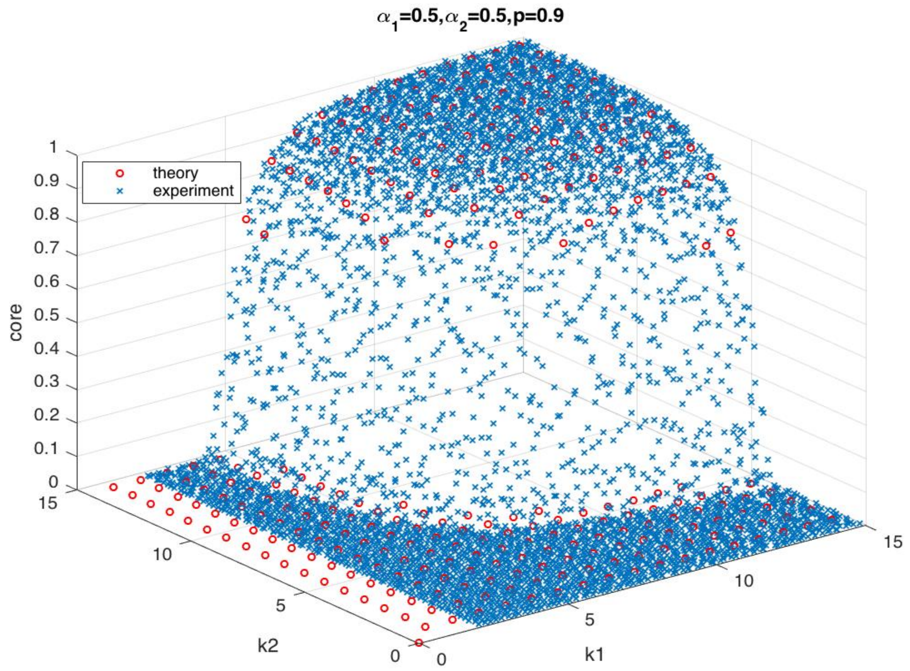
Partial-Interdependent Networks

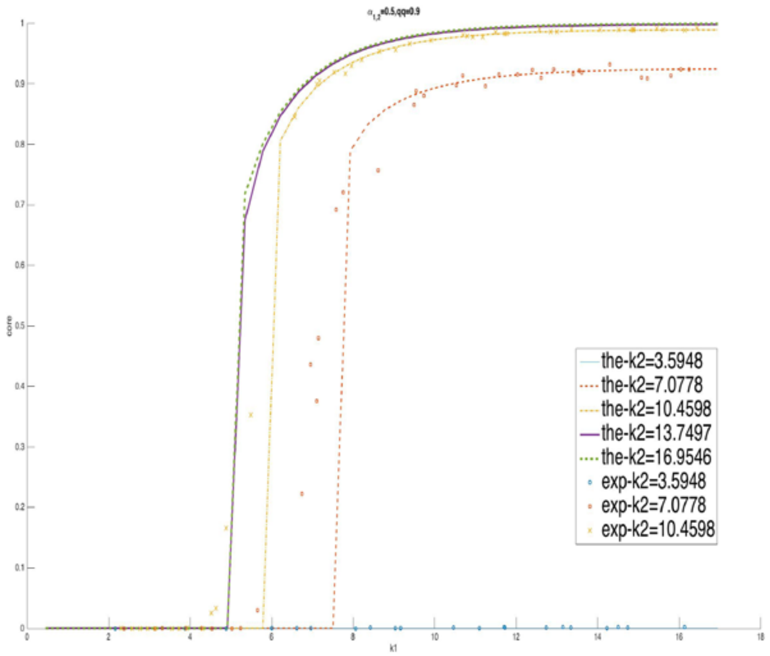




The results of the first layer in double-layer networks where figure (b) are sections of figure (a)

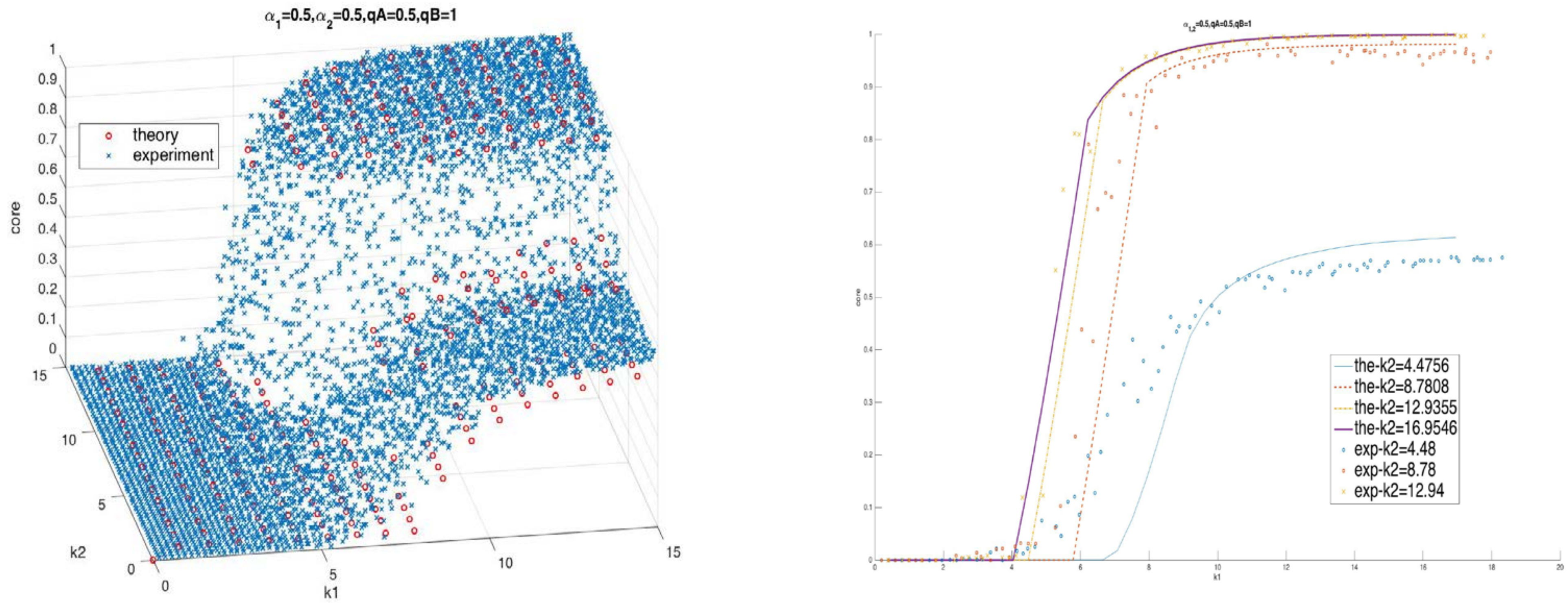
Partial-Interdependent Networks





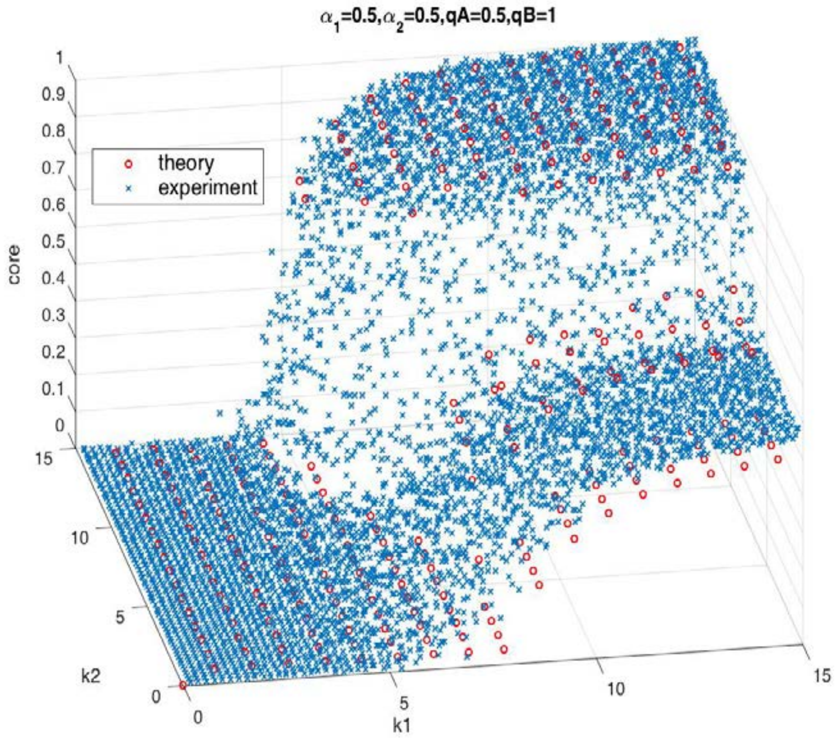
The results of the first layer in double-layer networks where figure (b) are sections of figure (a)

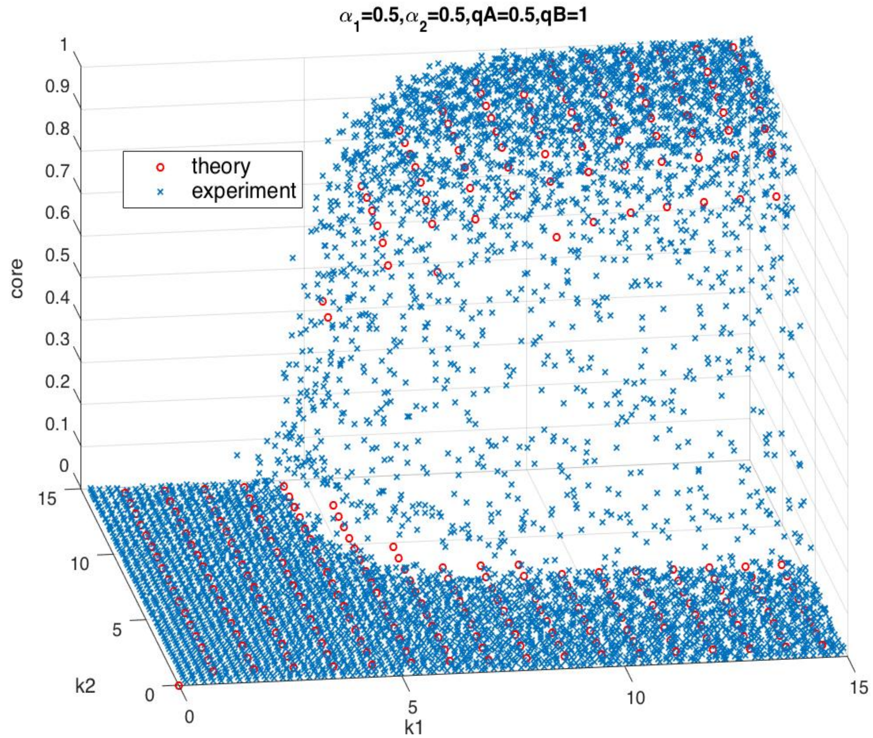
One-to-multiple Interdependent Networks



The results of theoretical derivation and experiments on synthetic networks, where figure (b) is the sections of (a).

One-to-multiple Interdependent Networks





The results of in the case of one-to-multiple interdependent networks

Thanks!