Reconstruct radio map for indoor localization

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The main point of Wi-Fi fingerprinting localization is to collect large amounts of received signal strength(RSS) fingerprints. In my recent work, I am doing the modulation of a new theory where fundamental radio propagation properties are captured based on Huygens' principle to construct the radio map. I first model radio propagation from one location to the neighboring locations using a single-step matrix, and then derive the end-to-end propagation matrix through parallel radio propagation channels modeling. After that I derive the end-to-end propagation matrix using the theory that we investigate, and then compare the derived matrix of these two methods.

Consider a room that is divided by grid as shown figure.1. There are two Wi-Fi access points (Aps) located in the two grey squares. We use $H_{i,j}$ to denote the wireless propagation factor between $cell_i$ and $cell_j$, where the physical meaning of $H_{i,j}$ is how the radio signal can be changed after it arrives to $cell_j$ from $cell_i$ in power level. Consider an extremely small period of time, the signal will only arrive at the neighboring cell of the signal source, as shown in figure.1. We only consider the case where the signal can travel to the four neighboring cells for the simplicity of presentation. In this way, we could use a single-step propagation matrix H to describe the radio propagation, where elements in the matrix is $H_{i,j}$.

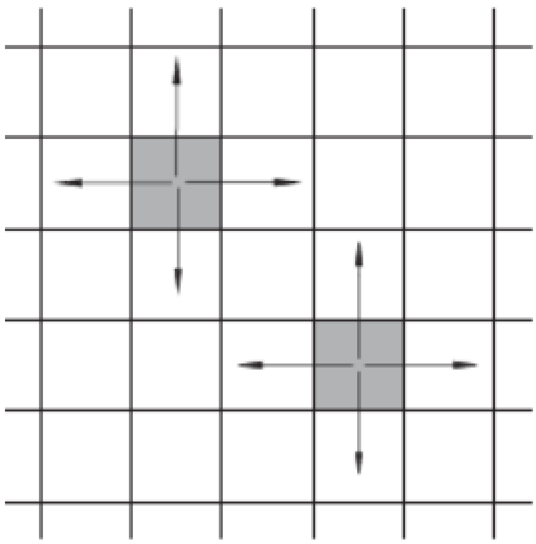

Figure 1: single-step radio propagation model

According to Huygens' principle, every point on a wave-front can be a source of secondary wavelets which propagate in the forward direction, and new wave-front is the tangential surface to all of these secondary wavelets. We can use $H_{i,k}$ and $H_{k,j}$ to denote the channel propagation factor between $cell_i$ and $cell_j$.

We now consider the radio propagation process when the signal hits the walls. We first examine the situation in the corner, as shown in the upper left part of figure.2. The wall will absorb part of the signal and rebound the rest. If we use x to denote the power level of the signal at the center of the square, y the aggregate power level after the walls' effects, and α the transmissivity of the wall ranging from 0 to 1, then the signal strength can be received at each of the neighboring cells is:

$$y = \frac{1}{4}x + \frac{1}{2}x(1\text{-}\alpha)\frac{1}{2} = \frac{1}{2}x - \frac{1}{4}\alpha x,$$

where the rebounded signal can also propagation to the neighboring cells according to Huygens' principle. Similar, if the source of the signal is beside the border of the room as shown in the middle right part of figure.2. The corresponding signal strength is:

$$y = \frac{1}{4}x + \frac{1}{4}x(1\text{-}\alpha)\frac{1}{3} = \frac{1}{3}x - \frac{1}{12}\alpha x.$$

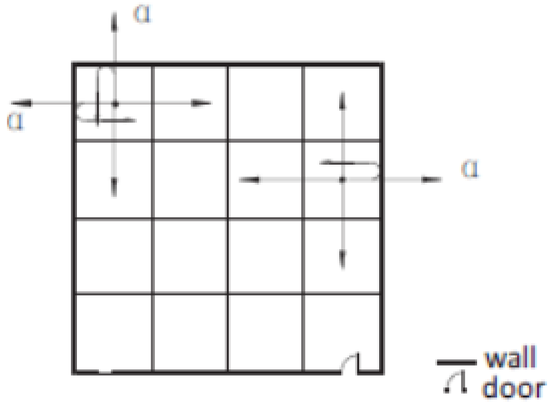

Figure 2: propagation analysis with walls

Recall that H is the single-step propagation matrix. We here specify two assumption: first, in every single cell, signal will not attenuate; second, in extremely short period of time, signal can only propagate to the neighboring cell, thus only certain item in H is non-zero. Here are some characteristics of the single-step matrix H: 1) $H_{i,j} = H_{j,i}$, because propagation is symmetric; 2) for each roll in matrix H, there are at most four non-zero elements. That means the propagation of nonzero elements in H is approximately $4n^2/n^4$. When n is large enough, the propagation of nonzero elements is small, so we can consider H as a sparsity matrix.

We use T to denote the multi-hop radio propagation matrix, where each element:

$$T_{ij} = \sum_{k=1}^{n^2} (H_{ij} + H_{ik}H_{kj} + \cdots + H_{ik}H_{km} \ldots H_{pq}H_{qj}), \quad (1)$$

Where $H_{ik}H_{kj}$ describe the propagation process from $cell_i$ to $cell_j$ obeying Huygens' principle. Each elements of equation (1) represents an independent channel form $cell_i$ to $cell_j$. According to the theory of parallel channels, we can get the final end-to-end propagation results:

$$Tx = Hx + H^2x + H^3x + \cdots, \quad (2)$$

Where Tx is the collection of RSS fingerprints that have been sampled, x is the original signal strength, which can be obtained in the localization system. The physical meaning of the equation is the aggregate influence of the environment on radio propagation.

Removing x from both size of equation (2), we can get:

$$T = H + H^2 + H^3 + \cdots, \quad (3)$$

This is the fundamental relationship between T and H, which can be used in the matrix reconstruction later. By calculating $HT - T$,we finally get the relationship between T and H:

$$T = (E - H)^{-1}H. \quad (4)$$

The difference between T and H is that T_{ij} are all nonzero elements if there is no obstacle in the space, so the uncertainty of T is magnitude of n^4.So the biggest challenge of this theory is how to get the H matrix through collected fingerprints at a limited number of locations.

Recall that the order of uncertainty of T is $\mathrm{O}(\mathrm{n}^4)$ and that of H is $\mathrm{O}(\mathrm{n}^2)$. This means that we need to sample the number of fingerprints in the order of $\mathrm{O}(\mathrm{n}^4)$ to obtain the entire information of T, however if we only sample a subset of locations for fingerprints in practice it is still possible to derive H with those fingerprints available. We now examine the relationship between matrices T and H.

We can get $\mathrm{H} = \mathrm{T} - \mathrm{HT}$ by equation (4). Due to the characteristic of single-step propagation matrix H, for each cell $\mathrm{c_{ij}}$(not at the corner or on the borders), only values in $H_{i-1,j}$, $H_{i+1,j}$, $H_{i-n,j}$, $H_{i+n,j}$ are valid. Thus:

$$H_{i,i-1} = T_{i,i-1} - \sum_{\mathrm{k}} H_{i,k} T_{k,i-1}$$
$$= T_{i,i-1} - H_{i,i-n}T_{i-n,i-1} - H_{i,i-1}T_{i-1,i-1} - H_{i,i+1}T_{i+1,i-1} - H_{i,i+n}T_{i+n,i-1}. \quad (5)$$

Similarly, we can get expressions of $H_{i,i-n}$, $H_{i,i+1}$, $H_{i,i+n}$. Based on these equations above, we can get:

$$\begin{bmatrix} 1+T_{i-n,i-n} & T_{i-1,i-n} & T_{i+1,i-n} & T_{i+n,i-n} \\ T_{i-n,i-1} & 1+T_{i-1,i-1} & T_{i+1,i-1} & T_{i+n,i-1} \\ T_{i-n,i+1} & T_{i-1,i+1} & 1+T_{i+1,i+1} & T_{i+n,i+1} \\ T_{i-n,i+n} & T_{i-1,i+n} & T_{i+1,i+n} & 1+T_{i+n,i+n} \end{bmatrix} \begin{bmatrix} H_{i,i-n} \\ H_{i,i-1} \\ H_{i,i+1} \\ H_{i,i+n} \end{bmatrix} = \begin{bmatrix} T_{i,i-n} \\ T_{i,i-1} \\ T_{i,i+1} \\ T_{i,i+n} \end{bmatrix} \quad (6)$$

Where the first matrix can be denoted by T_c and the term on the right side is denoted by T_r. Note that the algorithm works only if the coefficient matrix T_c Is full rank. Due to the stochastic values we collect in RSS fingerprints in practical environment, there is little possibility that the matrix is not full rank; therefore, this condition holds.

According to the connections between matrix T and H, if we have known some elements in the final propagation matrix T, we can first recover the single-step propagation matrix H, and then recover all elements in the final matrix T using matrix completion. In equation (6), we find that for each $cell_i$, we need T_c, T_r to calculate the propagation factor to its neighboring cells, i.e., $H_{i,i-n}$, $H_{i,i-1}$, $H_{i,i+1}$, $H_{i,i+n}$. To recover the entire matrix H, we should know all the T_c, T_r when $cell_i$ is from $cell_1$ to $cell_{n^2}$. According to the feature of subscript in T_c, T_r, all we need is elements in matrix T that satisfy one of the equations as following: $\mathrm{y = x, y = x + n, y = x - n, y = x + 1, y = x + 1, y = x - 1, y = x + 2, y = x - 2, y = x - 2n, y = x + 2n, y = x + 1 + n, y = x + 1 - n, y = x - 1 + n, y = x - 1 - n}$. Now we have proved that if we have get these elements in T, we can first recover the matrix H and derive the complete T.