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Project Report

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Modeling Information Diffusion in Multi-Sensitive Networks

Abstract

Information diffusion has been widely studied in networks, aiming to model the spread of information among objects when they are connected with each other. Most of the current research assumes the underlying network is working under a diffusion probability which is independent with their network structures (e.g. the number of nodes and edges, the transmission time, etc.). However, I'm trying to propose a model for information diffusion with time-varying and node-varying diffusion rate to address the procedure of information diffusion, and provide an interface between my proposed model and the well-studied SI model with constant diffusion rate. After that, I'm going to find which sensitive type affects the information diffusion procedure most, by distinguishing the power in passing information around for different types of sensitiveness. In addition, I use real diffusion action logs to learn the parameters in these models, which will benefit diffusion prediction in real networks. To distinguishing their diffusion power, machine learning algorithms will be used. Now I've finished the proof part of learning algorithm, and still working on the proof of multi-sensitive model.

1. Introduction

I joined Prof. Wang's lab in this semester. With the help of Dr. Fu, now I'm doing research in the Social Network fields. After several weeks' reading papers, I find myself especially interested in the field of information diffusion in online social networks.

Online Social Networks (OSNs) such as Facebook, Twitter and Weibo have exploded in popularity and drawn much attention from the research community. They offer a unique information sharing mechanism, which allows users to forward information like news articles, public opinions, videos, photos, etc. to their friends, and thus possibly to a wider audience. The convenient interaction and personalized feature of this mechanism makes the form of public information dissemination undergo a significant structural transformation. Under such circumstances, understanding/modeling the dynamics of information diffusion over OSNs has become an important research problem. The applications of modeling this process include locating the most influential users for commercial purpose, finding the source of malicious information and evaluating the social influence of some political and social events, among others.

In terms of modeling information diffusion over OSNs, most of the existing works have relied on Independent Information Cascade and Linear Threshold models. However, almost all of these assume time-invariant and node-invariant spreading speed and mainly concern if the statistical properties obtained from the model, with parameters appropriately adjusted, would match the empirical observation. But in reality, information diffusion on OSNs is usually affected by multiple factors, highly time-sensitive and edge-sensitive.

Considering this phenomenon: two people answer the same question in "Zhihu". One of them has great contribution in his working area, but is almost unknown to the public. Another one also make great contributions, but he's more famous, in other words, he's an academic star. (Someone just like Dr. Li Kaifu!) Now both of them answered this question as best as they can, in other words, the "quality" of their answers are in the same level. Intuitively, because the second person is more popular, public will think his answer is better than the first one's answer, more people will like the answer submitted by the popular one. Although both of the answers are good, the second one receives more likes than the first one. Don't think I'm just joking! It's a common phenomenon in "Zhihu". Sometimes the famous answerers (e.g. Zhang Jiawei and Ma Qianzu) answered a question totally wrong form the professional perspective, but still received lots of likes. Public sees that you have lots of fans, than they think that your answer is right, and vote for it.

So, that's what I'm going to find. I think the rate of information diffusion will change with the number of edges a transmit node have. If a node has more edges than others, it has more "fans", and gets more popular. Then, information provided by the popular node has a stronger ability to propagate than the "unknown" one. Meanwhile, the diffusion rate will attenuate with transmitting time growing. To the best of my knowledge, it's the first work to establish a theoretical framework under which the impact of the shape of infection rate with consideration of the infected node's edges on the information diffusion dynamics is discussed.

There are two major challenges in providing such models. First, how can we model the diffusion process in multi-sensitive networks, with heterogeneous diffusion power for each sensitive type? Second, for a particular network and a concrete diffusion task, how can we automatically determine the best weight for each relation type? In order to solve the first challenge on modeling, I extend SI, the well-known diffusion model in single-relational information networks, into a variational model for multi-sensitive information networks that combines each sensitive type at the sensitiveness level. In other words, the multi-sensitive network is treated as a single-sensitive network by putting different weights on different types of links, and then apply the single-sensitive SI model to determine the activation probability.

In order to determine the weights for the models and therefore make the models applicable for diffusion prediction in the real world, I propose to use diffusion action logs to learn the parameters in these models. The diffusion action logs record the object set that is activated at each timestamp. By maximizing the likelihood of observing the action logs, either obtained from one cascade or multiple cascades, we can find the MLE estimators for the parameters using optimization methods. With the learned parameters, either the weight of each relation-based diffusion model or the weight for relationships of each relation type, we can not only understand the role of each relation type in the diffusion process but also predict the diffusion according to given initial set of activated objects.

2. Related Work

The study of disease propagation in contact networks, which is analogous to the diffusion of news and ideas in formation networks, has long been a base for information diffusion study. In [1], the authors discussed different thresholds in different disease propagation models including SIV, SIS, SIRV and so on. In [2], the paper discussed the co-evolution of content delivery in mobile P2P networks under Linear Threshold Model. [3] And [4] proposed models for more than one kind of information diffused in social networks and their own strength. In [5], author studied the propagation influenced by information overload, which is identified by the length of cascades. In

[6], the authors performed Hawkes Process to analyze the influence of users' relationship and topics' relationship in information diffusion. [7], [8] and [9] proposed the complex multi-layer network model to study information diffusion more realistically. In [10] and [11], authors studied users' privacy in online social networks. In [12], the prediction of the users in next timestamp is studied. [13], [14] and [15] focused on the network structure itself. In [16], an optimization method to find the max degree node was under discussion. Authors of [17] discussed the impact in information diffusion when a single node's infectiveness is enhanced. In [18], authors modeled information diffusion in time-sensitive conditions. In [19], learning algorithm was applied to find the best parameters in multi-relational bibliographic information networks.

3. Diffusion Models for Multi-Sensitive Networks

3.1 Overview

Let $\mathrm{G} = (\mathrm{N}, \mathrm{E})$ be a connected network with a set of finite nodes $\mathrm{N} = \{1,2, \dots, \mathrm{n}\}$ and a set of links E. In this report, I assume that the evolution of the network structure is much slower compared with the speed of information spreading, and thus can be neglected. If node j lists node i as a friend, then i's interface allows node j to access the messages that node I posts or forwards, as well as other activities associate with i, but not vice versa. Then there is a directed link pointing from i to j such that information can flow from i to j, denoted by $(\mathrm{i}, \mathrm{j}) \in \mathrm{E}$. Note that this friend relationship is asymmetric. I assume that G has no self-loops and no multiple links between any two nodes.

For a given topic or a piece of message/information in the network, we say a node i is infected if i either initiates this message or forwards this message from its infected neighbors/friends; otherwise, it is considered as uninfected. I then model the diffusion of this information over G using a process$\mathrm{S(t)} = (S_1(t), S_2(t), \dots, S_n(t)) \in \{0,1\}^n$, where $S_i(t) = 1$ if i has been infected by time t and $S_i(t) = 0$ otherwise. Let $|\mathrm{S(t)}| = \sum_{j \in N} S_j(t)$ be the size of the infected node set (or simply the number of infected nodes) at time t, and $\mathrm{S}(0)$ be the initial set of source nodes. To keep the notation simple, we will also use $\mathrm{S(t)}$ to represent the set of infected notes at time t, i.e., $\{\mathrm{i} \in \mathrm{N} | S_i(\mathrm{t}) = 1\}$ whenever no confusion arises. I allow that the diffusion starts from a single user ($|\mathrm{S}(0)| = 1$) or a connected initial component ($|\mathrm{S}(0)| > 1$). Clearly, all the infected nodes remain connected at any time $\mathrm{t} > 0$. Let $\mathrm{N}\big(\mathrm{S(t)}\big) = \{\mathrm{j} \in \mathrm{N} \backslash \mathrm{S(t)} \mid \exists (\mathrm{i}, \mathrm{j}) \in \mathrm{E}, \mathrm{i} \in \mathrm{S(t)}\}$ be the set of "neighbors" of the infected nodes at time t, and $\partial(\mathrm{S(t)}, \mathrm{j}) = \{(\mathrm{i}, \mathrm{j}) \in \mathrm{E} \mid i \in S(t), j \in N(S(t))\}$ be the set of edges originating from $\mathrm{S(t)}$ to the neighboring node j.

If $\mathrm{Si(t)} = 1$, then all nodes who list i as a friend are exposed to her message, and are willing to forward the message with time-varying rate $\mathrm{t} \geq 0$ because of the influence of node i. Here, t captures people's changing enthusiasm to forward the message depending on how old the message is. For a fresh news/message (small t), a user may be more willing to share it with her friends (followers) on her personal page, while she loses her interest in doing so for not-so-fresh message (e.g., smaller t for large t). In this setting, at time t, a node j will be infected with rate t multiplied by the number of its infected friends. That is,

$$p_i = \beta \times \left|\partial\big(s(t,i)\big)\right| \qquad (1)$$

if $j \in N(S(t))$, and zero otherwise, where $\partial(S(t), j)$ is the number of edges from the set $S(t)$ to the neighboring node j. This model can be considered as the well-known Susceptible-Infected (SI) model on a graph, but with time-dependent and edge-dependent infection rate. Here, I use SI model instead of SIR or SIS since my interest is on the temporal dynamics of $S(t)$ for a given message over time t. Thus, I do not take into account message removal from user's personal page or a user being re-infected by the same message.

My construction above makes $\{S(t)\}_{t\geq 0} \in \Omega$ a time-inhomogeneous discrete-time Markov Chain, where $\Omega = \{S_1, S_2, \ldots, S_{|\Omega|}\} \subset \{0,1\}^n$ consists of 2^n possible states recording whether or not each node is infected. See Figure 1 for illustration.

Specifically, if β is a constant, then my model degenerates to the SI model with constant infection rate on a finite graph. This time-homogeneous diffusion process has been extensively studied in the literature.

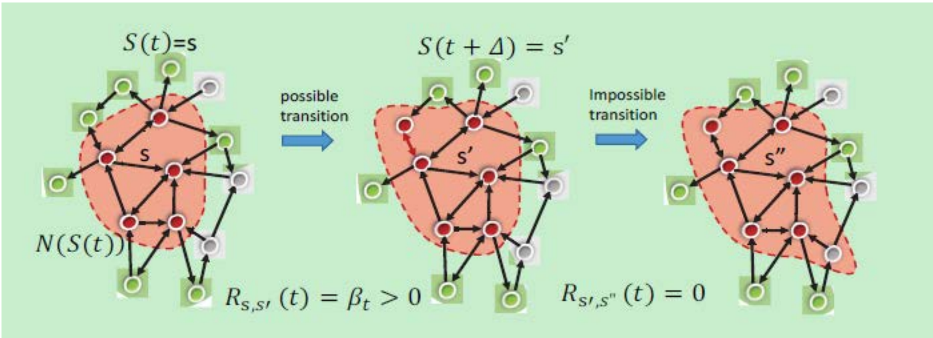

Fig. 1. Information diffusion over G: red nodes are infected; green nodes are in $N(S(t))$; gray nodes are outside of $S(t) \cup N(S(t))$. Here, s' is a possible follow-up state from s, but $s' \to s''$ is not a possible transition.

3.2 Model of Time-sensitive information diffusion

The model of time-sensitive networks has been structured in [18]. According to the paper, when let $m(t) = \int_0^t \beta ds$, Consider the standard process $\tilde{S}(t)$ with $\beta = 1$, (i.e. $m(t) = t$) for all t. In particular, for any increasing sequence $0 < t_1 < t_2 < \cdots < t_r$, $\{S(t_1), S(t_2), \ldots, S(t_r)\}$have the same joint distribution as $\{\check{S}(m(t_1)), \tilde{S}(m(t_2)), \ldots, \tilde{S}(m(t_r))\}$. The shape of t is the key factor in determining when and whether a message will reach a set of prescribed nodes. Here, I use exponential model of β so that the diffusion will end in some timestamp t rather than get pandemic:

$$p_i = \beta \times |\partial(s(t,i))| = \alpha \times t^{\gamma} \times |\partial(s(t,i))| \qquad (2)$$

3.3 Model of Edge-sensitive information diffusion

Firstly, let's consider in the simplest condition: the network G is homogeneous, each node has m edges points to other nodes. Let the type of β be $\beta = \alpha|\partial(s(t))|^{\gamma}$, it means the more "friends" (edges) a node has, and the more infectious it will be.

Notice that,

$$\left|\partial(\mathrm{S(t)})\right| = \sum\nolimits_{j\in N(S(t))} |\,\partial(\mathrm{S(t,j)})|$$

On the discrete time condition, we can get the mathematical expectation of V_t, which is the set of infected nodes at timestamp t:

$$\mathrm{E}(\,V_t) = \sum_{i\in N(t)} p_i = \alpha|\partial(s(t-1))|^{\gamma} \sum_{i\epsilon N(t)} |\partial\big(s(t-1,i)\big)| = \alpha|\partial\big(s(t-1)\big)|^{1+\gamma}$$

In the homogeneous network,

$$\mathrm{E}(\left|\partial\big(\mathrm{s(t-1)}\big)\right|) = \mathrm{m} \times \frac{N-|s(t-1)|}{N-1} \times |s(t-1)|$$

Where N is the total number of nodes in network G. After applying the Recursive formula,

$$\mathrm{s(t) = s(t-1) +}\ V_t$$

When given the initial state of $\mathrm{S}(0)$, we can obtain the mean number of infected nodes at timestamp t step by step:

$$\mathrm{E}(\,V_t) = \alpha(\mathrm{m} \times \frac{N-|s(t-1)|}{N-1} \times |s(t-1)|)^{1+\gamma}$$

Notice that $\mathrm{E}(|\,\partial(\mathrm{s(t)})|)$ is a quadratic function, and its quadratic coefficient is below zero. So we can predict that, V_t's rate of change will firstly become bigger and then reduced. When $\mathrm{E}(\,V_t)$ is reduced under a small threshold, we can regard it as the transmission procedure is stopped.

When the network become more realistic, in other words, an inhomogeneous network, the mathematical expectation of V_t can also be inferred by checking every node's status,

$$p_i = \beta \times \left|\partial\big(s(t,i)\big)\right| = \alpha \times \left[\sum\nolimits_{i\epsilon R_t} \sum\nolimits_{j\in\tau_t(i,k)} \delta(j,t)\right]^{\gamma} \times \sum\nolimits_{j\in\tau_t(i,k)} \delta(j,t) \qquad (3)$$

Where R_t means the set of uninfected nodes at timestamp $t-1$, $\delta(j,t)$ represents the state of node j at time t. If j is infected, $\delta(j,t)=1$, otherwise $\delta(j,t)=0$ and $\tau_t(i,k)$ represents the neighbor set of object i at timestamp t.

Obviously, whether the nodes are infected or not follow the Bernoulli distribution. Then,

$$\mathrm{E}(V_t) = \sum\nolimits_{i\epsilon R_t} p_i = \alpha \times \left[\sum\nolimits_{i\epsilon R_t} \sum\nolimits_{j\in\tau_t(i,k)} \delta(j,t)\right]^{\gamma+1}$$

3.4 Model Information Diffusion in Multi-sensitive networks

The information is propagated on the mixed set of links from any types of relationships, with the weights of different relation types being treated differently, as shown in Figure 2.

This assumption stipulates that the multi-relational network can be converted to a single-relational network by aggregating all the edges together, where links from different types carry different hyper-level weights. Under this assumption, the activation probability of object i at timestamp $\mathrm{t}+1$ can be defined as:

$$p_i = \sum\nolimits_k \mu_k \times (\beta_i^{(k)} \times \sum\nolimits_{j\in\tau_t(i,k)} \delta(j,t)) \qquad (4)$$

Where μ_k denotes the hyper-level weight for relation type k.

Then we can easily get the property of this multi-sensitive model:

PROPERTY 1: Let $p_i^{(-k)}(t+1)$ be the infection probability of i at timestamp $\mathrm{t}+1$ without relation type k, under the multi-sensitive model, we have

- $\min(p_i^{(k)}(t+1), p_i^{(-k)}(t+1)) \le p_i(t+1) \le \max(p_i^{(k)}(t+1), p_i^{(-k)}(t+1))$

- when $\mu_k \to 0,\ p_i(t+1) \to p_i^{(-k)}(t+1)$

- when $\mu_k \to \infty,\ p_i(t+1) \to p_i^{(k)}(t+1)$

From this property, we can see that under this model when a new relation type is added, it might cause a decrease of the activation probability, if the activation probability for this single relation type is smaller than the current overall activation probability.

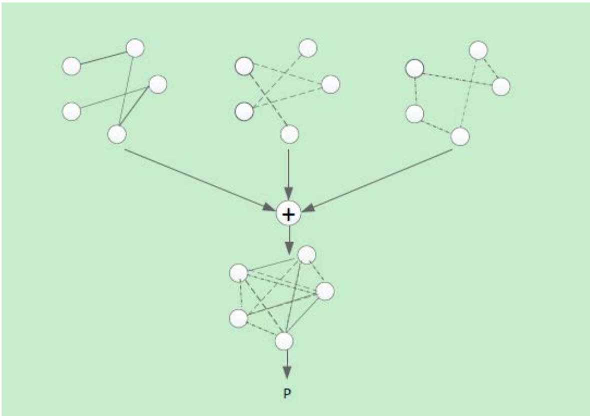

Fig.2. Illustration of the Relation Interdependent Diffusion

4. Diffusion Model Learning From Action Log

4.1 The Learning Framework

For each diffusion process, or cascade, an action log is a sequence of object set recording when an object is activated: $A = \{V_t\}_{t=1}^{T}$. In my setting, t is collected from discrete timestamps. The general learning framework is then to find the best parameters, i.e., the weight μ_k for each relation type E , in the diffusion models that can maximize the likelihood of observing these actions recorded by the action logs. At a timestamp t, the activation probability $p_u(t)$ of every uninfected object u, $u \in R_t = V \setminus \cup_{t'=1}^{t-1} V_t$, follows the Bernoulli distribution.

In other words, at timestamp t, for any object $u \in R_t$, it would be infected with probability $p_u(t)$, and stay uninfected with probability $1 - p_u(t)$. Therefore, the probability of observing the set of objects V_t infected at time t, and the set of objects $R_t \setminus V_t$ not infected at time t, can be calculated as follows:

$$P(V_t, R_t \setminus V_t) = \prod_{u \in V_t} p_u(t) \prod_{u \in R_t \setminus V_t} (1 - p_u(t)) \qquad (5)$$

The probability of observing the action log of a cascade is then:

$$L = \prod_{t=1}^{T} P(V_t, R_t \setminus V_t)$$

If multiple cascades are available, the likelihood is then the product of probabilities of each of these cascades:

$$\mathrm{L} = \prod_k \prod_{t=1}^{T^{(k)}} P({V_t}^{(k)}, R_t \backslash {V_t}^{(k)}) \qquad (6)$$

Now I introduce the learning algorithms for the model. The goal is to find the best μ_k that can maximize the likelihood, i.e., the MLE estimators, when plugging $p_u(t)$ into the likelihood function.

4.2 Learning Algorithm

By plugging Eq. (4) into the likelihood function Eq. (6), we can get the log-likelihood function as:

$$\log(\mathrm{L}) = \sum_t \left\{ \sum_{u \in V_t} log p_u(t) + \sum_{u \in V_t \backslash R_t} \log\big(1 - p_u(t)\big) \right\}$$

$$= \sum_t \left\{ \sum_{u \in V_t} log \sum_k \mu_k \beta_u^{(k)} \sum\nolimits_{j \in \tau_t(u,k)} \delta(j,t) + \sum_{u \in V_t \backslash R_t} log \left[1 - \sum_k \mu_k \beta_u^{(k)} \sum\nolimits_{j \in \tau_t(u,k)} \delta(j,t) \right] \right\}$$

I apply coordinate descent method to find the best non-negative k's that maximizes that log-likelihood, by setting the learning step η smartly.

According to gradient descent method, a local maximum of μ_k can be derived by iteratively updating the following formula:

$$\mu_k^{new} = \mu_k^{old} + \eta \frac{\partial logL}{\partial \mu_k} \qquad (7)$$

Where $\frac{\partial logL}{\partial \mu_k}$ is the first derivative of function $\log(L)$:

$$\frac{\partial logL}{\partial \mu_k} = \sum_t \sum_{u \in V_t} \frac{\beta_u^{(k)} \sum_{j \in \tau_t(u,k)} \delta(j,t)}{\sum_{k'} \mu_{k'} \beta_u^{(k')} \sum_{j \in \tau_t(u,k')} \delta(j,t)} - \sum_t \sum_{u \in V_t \backslash R_t} \frac{\beta_u^{(k)} \sum_{j \in \tau_t(u,k)} \delta(j,t)}{1 - \sum_{k'} \mu_{k'} \beta_u^{(k')} \sum_{j \in \tau_t(u,k')} \delta(j,t)} \qquad (8)$$

In order to get non-negative updates for μ_k's, let

$$\eta = \frac{\mu_k^{old}}{\sum_t \sum_{u \in V_t \backslash R_t} \frac{\beta_u^{(k)} \sum_{j \in \tau_t(u,k)} \delta(j,t)}{1 - \sum_{k'} \mu_{k'} \beta_u^{(k')} \sum_{j \in \tau_t(u,k')} \delta(j,t)}}$$

Then we have updating formula:

$$\mu_k^{new} = \mu_k^{old} + \eta \frac{\partial logL}{\partial \mu_k} = \mu_k^{old} \left\{ \frac{\sum_t \sum_{u \in V_t} \frac{\beta_u^{(k)} \sum_{j \in \tau_t(u,k)} \delta(j,t)}{\sum_{k'} \mu_{k'}^{old} \beta_u^{(k')} \sum_{j \in \tau_t(u,k')} \delta(j,t)}}{\sum_t \sum_{u \in V_t \backslash R_t} \frac{\beta_u^{(k)} \sum_{j \in \tau_t(u,k)} \delta(j,t)}{1 - \sum_{k'} \mu_{k'}^{old} \beta_u^{(k')} \sum_{j \in \tau_t(u,k')} \delta(j,t)}} \right\} \qquad (9)$$

For the coordinate descent method, we update μ on one dimension μ_k each time, and then update μ_{k+1} using the updated μ_k, repeat this process cyclically until $\log(L)$ convergences.

4.3 Action Prediction

Once we have learned the parameters $\check{\mu}$ in the models, we can utilize these parameters to predict the future action of objects given an initial set of active objects.

These probabilities can be used to do (1) ranking: who are most likely to be activated at the next timestamp, and (2) prediction: predict the total number of activated objects at a future timestamp. Note that the total expected number of activated objects at timestamp t can be calculated as $E(\sum_{u \in R_t} Y_u(t))$, which is equal to $\sum_{u \in R_t} E(Y_u(t))$, assuming the independence of the activation behavior among inactive objects. For Bernoulli distribution, $E(Y_u(t)) = p_u(t)$, therefore $E(\sum_{u \in R_t} Y_u(t)) = \sum_{u \in R_t} p_u(t)$

5 Future Work

In next step, I will try my best to find the theoretical proof of edge-sensitive model. After that, I will do experiments using real data from Weibo to find the parameters that fits the time-edge-sensitive model best. Then, I'll use learning algorithms to study the weight of these two conditions, and trying to find which one plays a greater part in information diffusion.

6 Conclusion

I begin my research with the help of Dr. Fu in the beginning of this semester. In these weeks, I explored a whole new area and accumulated lots of useful knowledge. In conclusion, I read some papers (which are listed in the References part), I found an area which attracts me best, proposed an idea which is never proposed in other authors' work and trying to prove it. To prove the feasibility of a model is a nontrivial task, and I'm trying my best to do it. It's an unforgettable memory to study in Prof. Wang's lab!

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