Throughput-Outage Tradeoff of Wireless One-Hop Caching Networks with Affiliation

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- Background and Motivation
- Network Model
- Outer Bound
  - Constraints and Analysis
  - Results
- Inner Bound
  - Constraints and Analysis
  - Results
- Sum up
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Background and Motivation

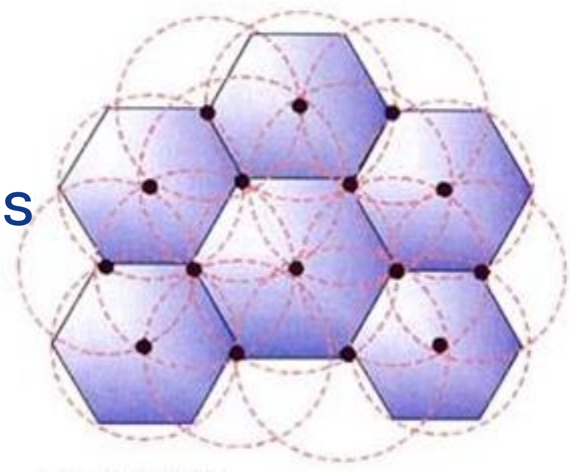
- Increased number of users

-> Increase the bandwidth

-> build more base stations

    -Ineffective

    -too expensive



- Caching network: Storage -> Bandwidth
- Realistic network users' relationships are evolving over time

Background and Motivation

- Current research:
  - Based on static network :
  - Disadvantages :

    Can not reflect evolutionary properties
  - Difference:

    Evolution improve cache performance

My Work

- For the first time to study the evolution of network outage and throughput tradeoffs

- To observe the impact to the tradeoff under the affiliation

Definition

- **Outage:** $p_o = \frac{1}{n}\mathbb{E}[\mathsf{N}_o] = \frac{1}{n}\sum_{u\in\mathcal{U}} \mathbb{P}\left(\mathbb{E}[T_u|\mathsf{f},\mathsf{G}] = 0\right).$
- A user node in a period of time the throughput is expected to be 0, called the point in outage, the ratio of the points in outage for all user.

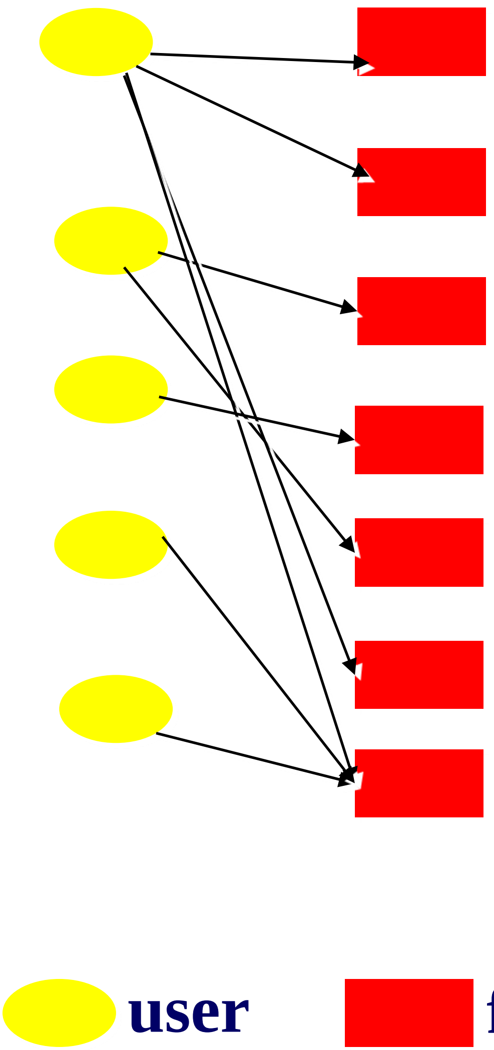
- **Throughput:** $T_u = \sum_{v:(u,v)\in\mathsf{A}} c_{u,v} 1\{\mathsf{f}_u \in \mathsf{G}(v)\}$
- The amount of data transferred between users per slot. (The file is composed of packets)

Definition

- Outer Bound: The upper limit of the throughput associated with the outage.
  (Transmission interference limit)

- Inner Bound: The lower limit of the throughput associated with the outage.
  (File requirements limit)

Affiliation model



$B(Q, U)$

Fix two integers $c_q, c_u > 0$, and let $\beta \in (0, 1)$.

At time 0, the bipartite graph $B_0(Q, U)$ is a simple graph with at least $c_q c_u$ edges, where each node in Q has at least c_q edges and each node in U has at least c_u edges.

At time $t > 0$:

(Evolution of Q) With probability β:

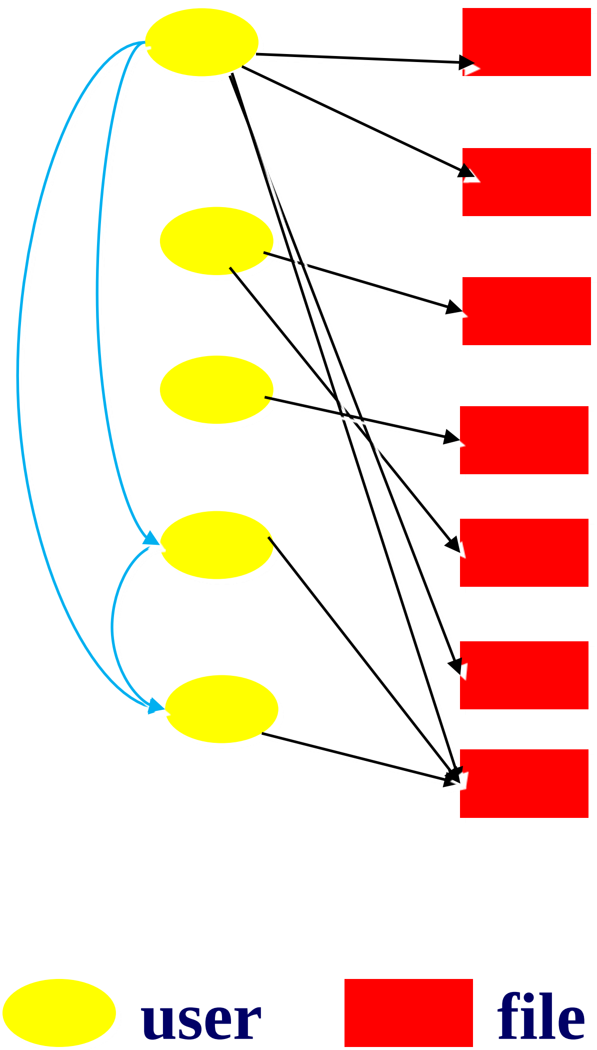
(*Arrival*) A new node q is added to Q.

(*Preferentially chosen Prototype*) A node $q' \in Q$ is chosen as *prototype* for the new node, with probability proportional to its degree.

(*Edge copying*) c_q edges are "copied" from q'; that is, c_q neighbors of q', denoted by $u_1, \ldots, u_{c_q}$, are chosen uniformly at random (without replacement), and the edges $(q, u_1), \cdots, (q, u_{c_q})$ are added to the graph.

(Evolution of U) With probability $1 - \beta$, a new node u is added to U following a symmetrical process, adding c_u edges to u.

Affiliation model



$G(Q, E)$

Fix integers $c_q, c_u, s > 0$, and let $\beta \in (0, 1)$.

At time 0, $G_0(Q, E)$ consists of the subset Q of the vertices of $B_0(Q, U)$, and two vertices have an edge between them for every neighbor in U that they have in common in $B_0(Q, U)$.

At time $t > 0$:

(Evolution of Q) With probability β:

(*Arrival*) A new node q is added to Q.

(*Edges via Prototype*) An edge between q and another node in Q is added for every neighbor that they have in common in $B(Q, U)$ (note that this is done after the edges for q are determined in B).

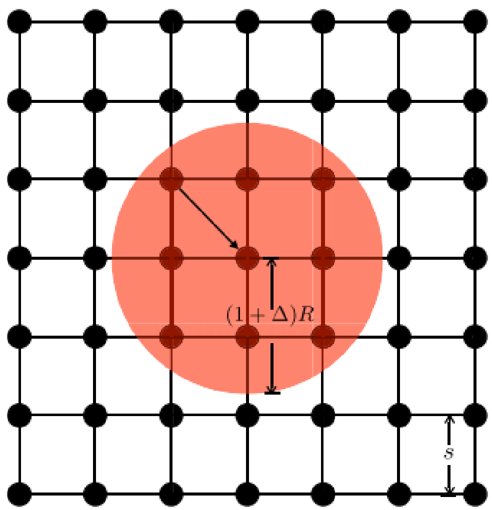
(Edges via evolution of U)
With probability $1 - \beta$:

A new edge is added between two nodes q_1 and q_2 if the new node added to $u \in U$ is a neighbor of both q_1 and q_2 in $B(Q, U)$.

(Preferentially Chosen Edges) A set of s nodes $q_{i_1}, \ldots, q_{i_s}$ is chosen, each node independently of the others (with replacement), by choosing vertices with probability proportional to their degrees, and the edges $(q, q_{i_1}), \ldots, (q, q_{i_s})$ are added to $G(Q, E)$.

Caching network

- The network consists of files and users in a two-dimensional plane.
- File cache in the user, the user does not have their own documents to make a request,
- In the absence of interference a certain range of transmission.



Definition

- There important parameters
- n:Number of users
- M:Number of files
- α （γ）：Ratio of File increase

Theorem 4. *For the bipartite graph $B(Q,U)$ generated after n steps, almost surely, when $n \to \infty$, the degree sequence of nodes in Q (resp. U) follows a power law distribution with exponent* $\alpha = -2 - \frac{c_q\beta}{c_u(1-\beta)} \left(\alpha = -2 - \frac{c_u(1-\beta)}{c_q\beta}\right)$, *for every degree smaller than* n^γ, *with* $\gamma < \frac{1}{4+\frac{c_q\beta}{c_u(1-\beta)}} \left(\gamma < \frac{1}{4+\frac{c_u(1-\beta)}{c_q\beta}}\right)$.

Outer bound result

$$T^{*}_{sum}(g) \leq T^{ub}_{sum}(g) = \frac{16}{\Delta^2} \cdot k \cdot c \cdot H(\gamma - 1, 1, n)\left[\left(1 - p^{lb}(g)^{(1+\frac{2}{3}\Delta^2)g}\right)\frac{n}{g}\right]$$

$$g = \Theta(m^{\alpha}) = \rho m^{\alpha}$$

Or

$$T = \frac{c}{\alpha} M^{-\gamma} \frac{n^{-\alpha}}{m^{-\alpha}} + o\left(\frac{n^{-\alpha}}{m^{-\alpha}}\right)$$

$$1 - \left(\frac{Mn}{m}\right)^{1-\gamma} \leq p \leq 1.$$

- After adding evolution analysis
- Number of files has resulted in a small increase in the impact of Throughput.
- In some case, Evolution of the file growth ratio is high, the Throughput increase.
- Number of users has little effect.

Inner bound result

$$T = C \cdot \frac{1}{\gamma^3} \frac{1-2g_c(m)^{1-\gamma}}{g_c(m)}$$

$$p_0 = 2^x\left(1 - \frac{x}{2^{\frac{x}{g_c(m)M}}}\right)$$

$$g_c(m) = \frac{x}{log_2 \frac{x}{1-\frac{p_0}{2^x}}} \qquad x = \frac{m\gamma}{E[n_2(t)]}$$

- After adding evolution analysis,
- Number of files has resulted in a small increase in the impact of Throughput.
- Evolution of the file growth ratio is high, the Throughput increase.
- Number of users has little effect.

Outer bound

transfer speed

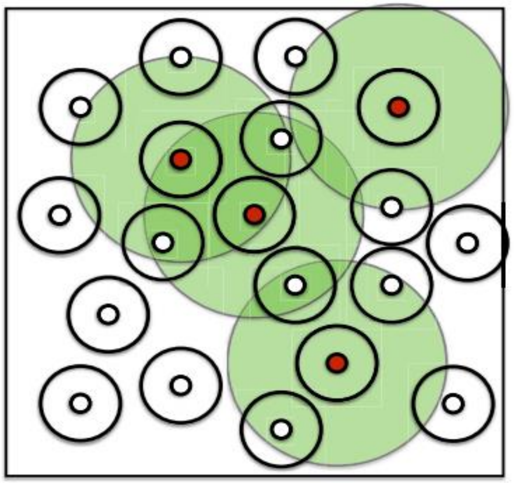
⟷ the user's degree in the B graph

File requirements

⟷ the file's degree in the B graph

- $g_R(m)$:All the points within a transmission radius
- Space constraints :

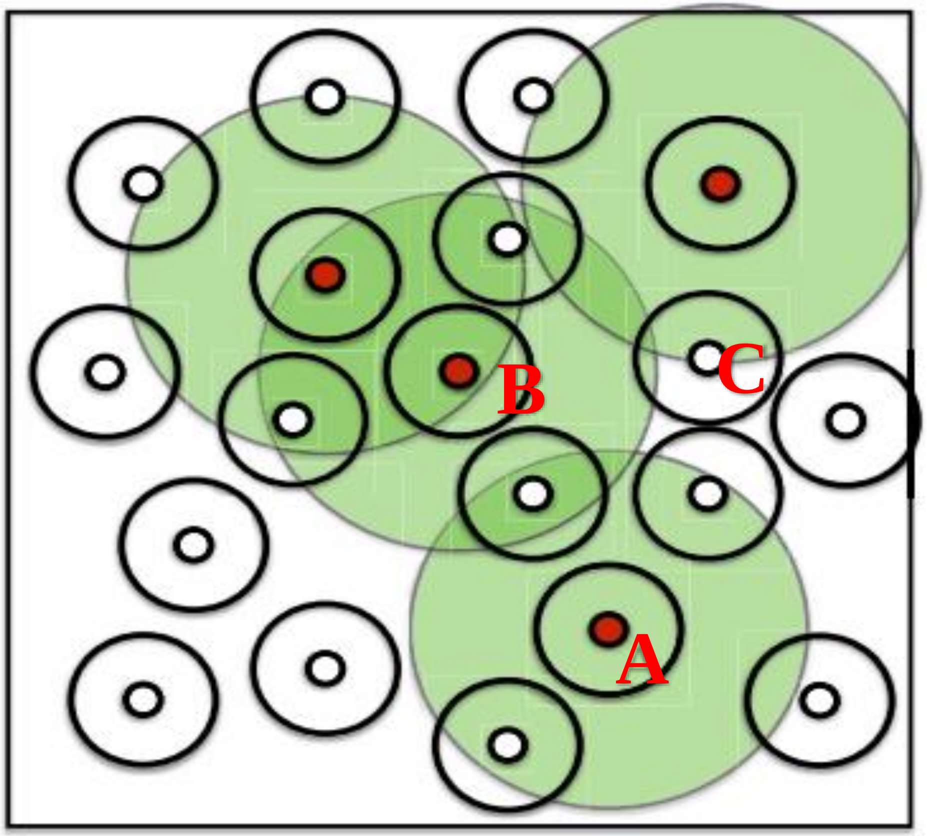
no transfer around $\frac{\Delta}{2}R$ distance from a transmitting point



Outer bound

No transfer around $\frac{\Delta}{2}R$ distance from a transmitting point.

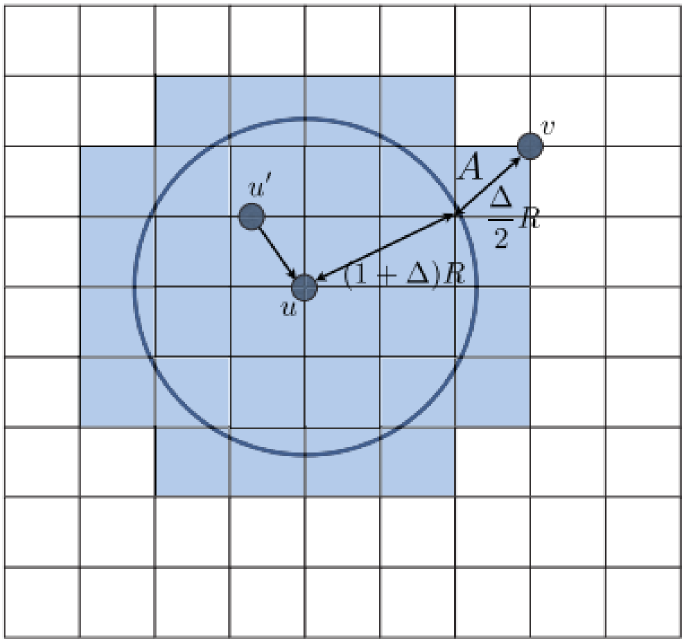
$$
\begin{aligned}
d(j,l) &\geq d(k,j) - d(k,l) \\
&\geq (1+\Delta)R - d(k,l) \\
&\geq (1+\Delta)R - R = \Delta R
\end{aligned}
$$

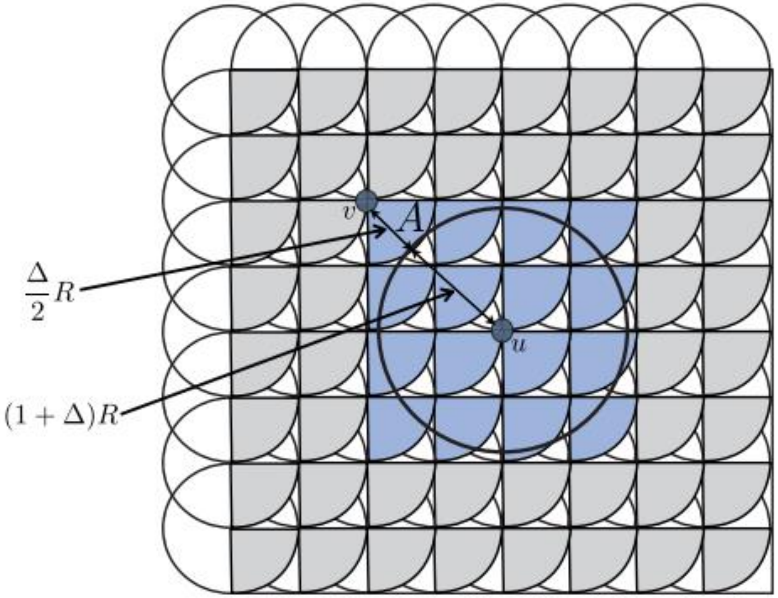


Outer bound

- The distance between transmissions gives the upper limit of the number of transmissions to get the tradeoff between throughput and outage

$$T^{*}_{sum}(g) \le T^{ub}_{sum}(g) = \frac{16}{\Delta^2} \cdot k \cdot c \cdot H(\gamma - 1, 1, n)\left[\left(1 - p^{1b}(g)^{(1+\frac{2}{3}\Delta^2)g}\right)\frac{n}{g}\right]$$





Outer bound

$$[(1-p^{1b}(g)^{(1+\frac{2}{3}\Delta^2)g})\frac{n}{g}]$$

- $g_R(m)$ and m has there relationship
  ① $g = o(m^{\alpha})$ ② $g = \omega(m^{\alpha})$ ③ $g = \Theta(m^{\alpha}) = \rho m^{\alpha}$
- The first and second case has a little effect compare with T
- Third case:(M is caching size)
- ① $\rho > \frac{1}{M}$, γ ↑, T ↑
- ② $\rho > \frac{1}{M}$, γ ↑, T ↓

Outer bound

- When compared with users, the files number too large:

$$T_{\text{sum}} = C \cdot \mathbb{E}[\mathsf{L}]$$

$$\overset{(a)}{\leq} C \sum_{u=1}^{n} \sum_{f=1}^{Mn} P_r(f) = Cn \frac{H(\gamma, 1, Mn)}{H(\gamma, 1, m)}$$

$$1 - \left(\frac{Mn}{m}\right)^{1-\gamma} \leq p \leq 1.$$

- But its P is impossible to decrease

Inner bound

- transfer speed

  ⟷ the user's degree in the B graph

  File requirements

  ⟷ the file's degree in the B graph

  User to user

  ⟷ the user's degree in the G graph

  (users can transfer with each other if they have edge between them)

- Cluster: $g_c(m)$
- The constraint is that only the required files can be found in the cluster.

Inner bound

- Optimal caching:
- the caching distribution Pc that maximizes the probability that any user u ∈ U finds its requested file inside its corresponding cluster

$$
\begin{aligned}
p_u^c &= E[l_u] = P(F^u_{g_c(m)}) \\
&= \sum_{f=1}^{m} P_r(f)((1-(1-P_c(f)))^{ME[n_2(t)\ \frac{m}{g_c(m)}]-M}) \\
&= \sum_{f=1}^{m} \frac{f^{-\gamma}}{H(\gamma,1,n)}((1-(1-P_c(f)))^{ME[n_2(t)\ \frac{m}{g_c(m)}]-M})
\end{aligned}
$$

- Lagrangian function

$$\mathcal{L}(P_c,\xi) = \sum_{f=1}^{m} P_r(f)(1-P_c(f))^{Mg-M} + \xi'\left(\sum_{f=1}^{m} P_c(f)-1\right)$$

$$P_c(f) = \frac{1+\xi-(\xi-1)\cdot 2^{\frac{1}{\xi}}}{\xi} \qquad \xi = \frac{M\cdot E[n_2(t)\frac{g_c(m)}{m}]-M-1}{\gamma}$$

Inner bound

- Found （T, p） with the caching

$$\overline{T}_{sum} = C \cdot E[L] = k \cdot E[n_1(t)] \cdot \frac{E[total\ nuber\ of\ clusters] \cdot P(W>0)}{K}$$
$$= K \cdot cH(\gamma - 1, 1, n) \cdot \frac{E[total\ nuber\ of\ clusters] \cdot P(W>0)}{K}$$

- W is the number of potential links.
- Maximize the probability of P

$$P(W > 0) \geq \frac{E[W]^2}{E[W^2]}$$

- Minimize E[W^2] and Maximize E[W]

$$P(W > 0) = \frac{2\gamma - 2}{(\gamma - 2)^2} \frac{1 - 2(g_c(m))^{1-\gamma}}{1 - g_c(m)^{1-\gamma} + 2\gamma - 2}$$

Inner bound

- Outage:
- Use the simplest probability theory

$$p_0 = P(\text{文件不存在}) + P(\text{连接不存在}) - P(\text{文件不存在且连接也不存在})$$
$$p_0 = A + B - AB$$

- A= $\prod_1^{g_c(m)M}(1 - P_c(f))$ B= $\prod_1^{g_c(m)}(1 - x^{-\gamma})$

  $P_c(f)$ is the optimal caching

- Using Inequality to change,

$$p_0 = \prod_1^{g_c(m)M}[1 - P_c(f)]$$
$$= (1 - \tfrac{1}{\xi})[1 - \tfrac{1+\xi-(\xi-1)\cdot 2^{\frac{1}{\xi}}}{\xi}]^{g_c(m)M-1}$$
$$= \tfrac{\xi-1}{\xi}(\tfrac{(\xi-1)2^{\frac{1}{\xi}}-1}{\xi})^{g_c(m)M-1}$$

$$p_0 = 2^x(1 - \frac{x}{2^{\frac{x}{g_c(m)M}}})$$

$$\frac{m\gamma}{E[n_2(t)]} = x$$

Inner bound

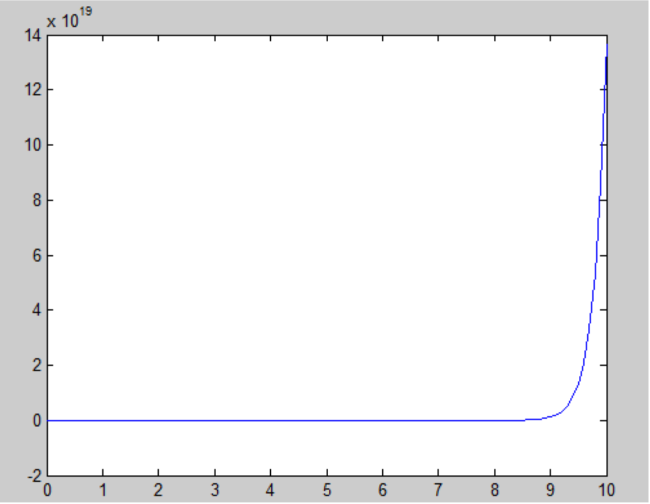
- Plot the $g_c(m)$ and γ

- $$T = C \cdot \frac{1}{\gamma^3} \frac{1-2g_c(m)^{1-\gamma}}{g_c(m)}$$

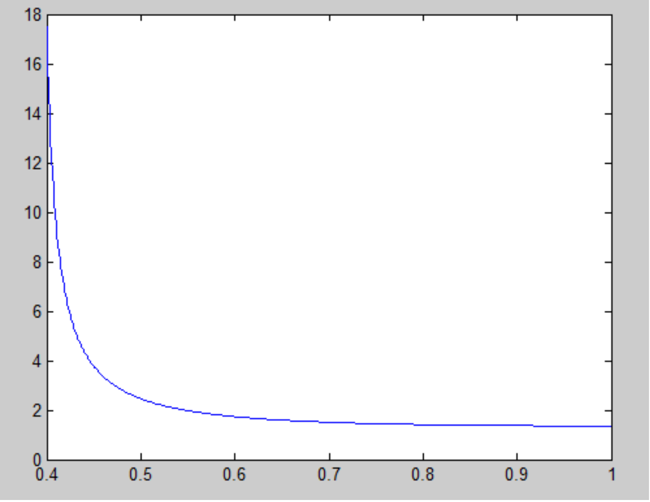
$$g_c(m) = \frac{x}{log_2 \frac{x}{1-\frac{p0}{2^x}}} \qquad x = \frac{m\gamma}{E[n_2(t)]}$$

- $g_c(m)$ ↑, T ↓

- m and $\gamma(\alpha)$ make Throughput increase



T−γ



$g_c(m) - x$

Sum up

- After adding evolution analysis
- In the case that propagation radius large
- Number of files has resulted in a small increase in the impact of Throughput.
- Evolution of the file growth ratio is high, the Throughput increase.
- Number of users has little effect.

Future

- Gap :

    the gap between inner bound and outer bound is not close.

- Complete :

    the cases which is talked are not Include all the circumstances.

- Experiment

Reference

- [1] Silvio Lattanzi,D Sivakumar "Affiliation Networks", in STOC2009 December 13,2010
- [2]Mingyue Ji, Giuseppe Caire, Andreas F.Molisch "Throughput-Outage Tradeoff of Wireless One-Hop Caching Networks " in IEEE TRANSACTIONS ON INFORMATION THEORY Vol 61, NO.12,December 2015

Thank you !