Localized Topology Control Algorithms for Heterogeneous Wireless Networks

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Most existing algorithms on topology control assume homogeneous wireless nodes with uniform maximum transmission ranges, and cannot be directly applied to heterogeneous wireless multi-hop networks in which the maximum transmission range of each node may be different. In this paper, we present two localized topology control algorithms for heterogeneous networks: Directed Relative Neighborhood Graph (DRNG) and Directed Local Spanning Subgraph (DLSS). In both algorithms, each node selects a set of neighbors based on the locally collected information. We prove that (1) the topologies derived under DRNG and DLSS preserve the network connectivity; (2) the out-degree of any node in the resulting topology by DLSS and DRNG is bounded by a constant; and (3) the topologies generated by DRNG and DLSS preserve the network bi-directionality. Simulation results indicate that DRNG and DLSS outperforms the other known topology control algorithms that can be applied to heterogeneous networks in several aspects.

Categories and Subject Descriptors: C.2.1 [**Network Architecture and Design**]: distributed networks, network topology, wireless communication; C.2.4 [**Distributed Systems**]: distributed applications

General Terms: Algorithms, Performance

Additional Key Words and Phrases: Connectivity, energy management, heterogeneous networks, topology control

1. INTRODUCTION

The development of wireless networks has posed many new challenges in system design and analysis. Energy efficiency [Jones et al. 2001] and network capacity [Gupta and Kumar 2000] are among the most important issues in wireless ad hoc networks and wireless sensor networks. Topology control algorithms have been proposed to maintain network connectivity while improving energy efficiency and network capacity. Instead of transmitting using the maximal power, nodes in a wireless multi-hop network collaboratively determine their transmission power and define the network topology by forming the proper neighbor relation under various topology control algorithms.

This work was supported in part by National Science Foundation (NSF) under a grant with No. ANI-0221357, and by MURI/AFOSR under a contract with No. F49620-00-1-0330.
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A preliminary version of this manuscript appeared in IEEE INFOCOM 2004, Hong Kong, March 2004 [Li and Hou 2004].
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By enabling wireless nodes to use adequate transmission power (which is usually much smaller than the maximal transmission power), topology control can not only save energy and prolong network lifetime, but also improve spatial reuse (and hence the network capacity) and mitigate the MAC-level medium contention [Narayanaswamy et al. 2002]. Several topology control algorithms [Rodoplu and Meng 1999; Ramanathan and Rosales-Hain 2000; Li et al. 2001; Narayanaswamy et al. 2002; Kawadia and Kumar 2003; Borbash and Jennings 2002; Li et al. 2002; Li et al. 2003] have been proposed to create power-efficient network topology in wireless multi-hop networks with limited mobility. (A summary will be given in Section 3). However, most of them assume homogeneous wireless nodes with uniform maximum transmission ranges (except [Rodoplu and Meng 1999]).

The assumption of homogeneous nodes does not always hold in practice, since even devices of the same type may have slightly different maximal transmission power, let alone devices of dramatically different capabilities. As will be exemplified in Section 3, most existing algorithms cannot be directly applied to heterogeneous wireless multi-hop networks in which the transmission range of each node may be different.

In this paper, we propose two localized topology control algorithms for heterogeneous wireless multi-hop networks with non-uniform transmission ranges: Directed Relative Neighborhood Graph (DRNG) and Directed Local Spanning Subgraph (DLSS). In both algorithms, the topology is constructed by having each node build its neighbor set and adjust its transmission power based on the locally collected information. We prove that (1) the topology derived under DRNG or DLSS preserves network connectivity, i.e., if the original topology generated by having each node use its maximal transmission power is strongly connected, then the topology generated by either DRNG or DLSS is also strongly connected; (2) the out-degree of any node in the topology by DLSS or DRNG is bounded by a constant; and (3) the topology generated by DRNG or DLSS preserves network bi-directionality, i.e., if the original topology by having every node use its maximal transmission power is bi-directional, then the topology generated by either DRNG or DLSS is also bi-directional after some simple operations.

Simulation results indicate that, compared with other known topology control algorithms that can be applied to heterogeneous networks, DRNG and DLSS derive topologies with smaller average node degrees (both logical and physical) and smaller average link lengths. The former reduces the MAC-level contention, while the latter implies that smaller transmission power is needed to maintain the network connectivity. To the best of our knowledge, this is the first effort to formally address the connectivity and bi-directionality issues in the context of topology control in heterogeneous wireless networks.

The rest of the paper is organized as follows. The network model is first given in Section 2. In Section 3, we summarize previous work on topology control, and give examples to show why existing algorithms cannot be directly applied to heterogeneous networks. Then we present both DRNG and DLSS algorithms in Section 4, and prove several of their useful properties in Section 5. Finally, we evaluate the performance of the proposed algorithms in Section 6, and conclude the paper in Section 7.

2. NETWORK MODEL

Let the network topology be denoted as a directed simple graph $G = (V(G), E(G))$ in the plane, where $V(G) = \{v_1, v_2, \ldots, v_n\}$ is the set of randomly distributed nodes (vertices) in the network and $E(G)$ is the set of links (edges). Assume that the transmission area of each node is a disk centered at the node. (We will relax this assumption in Section 5.5.) We define the (maximum transmission) range of a node v_i as the radius of the disk that v_i can cover using its maximal transmission power, denoted r_{v_i}. In a heterogeneous network, the transmission ranges of nodes may not be the same. Let $r_{min} = \min_{v \in V}\{r_v\}$ and $r_{max} = \max_{v \in V}\{r_v\}$.

We assume G is geometric, i.e., $E(G) = \{(u, v) : d(u, v) \leq r_u, u, v \in V(G)\}$, where $d(u, v)$ is the Euclidean distance between node u and node v. Note that (u, v) is an ordered pair representing an edge from node u to node v, i.e., (u, v) and (v, u) are two different edges. A unique id (such as an IP/MAC address) is assigned to each node. Here we let $id(v_i) = i$ for simplicity.

The information needed by the proposed algorithms is the knowledge of all existing edges in G. An edge that was not formed in the network, whether because the two end-nodes of the edge are too far away to communicate, or because there exists an obstacle in between, does not have any impact on the results. As long as the original topology (which has taken into account of the obstacles in the network) is strongly connected, our algorithms can be applied to preserve the connectivity. This implies whether or not obstacles exist in the deployment area does not affect the correctness of the proposed algorithms.

Before delving into the technical discussion and algorithm description, we give the definition of several terms that will be used throughout the paper.

Definition 2.1 *Weight Function.* Given two edges $(u_1, v_1), (u_2, v_2) \in E$ and the Euclidean distance function $d(\cdot, \cdot)$, the weight function $w : E \mapsto R$ satisfies:

$$\begin{aligned}
& w(u_1, v_1) > w(u_2, v_2) \\
\Leftrightarrow\ & d(u_1, v_1) > d(u_2, v_2) \\
\text{or}\ & (d(u_1, v_1) = d(u_2, v_2) \\
& \quad \&\&\ \max\{id(u_1), id(v_1)\} > \max\{id(u_2), id(v_2)\}) \\
\text{or}\ & (d(u_1, v_1) = d(u_2, v_2) \\
& \quad \&\&\ \max\{id(u_1), id(v_1)\} = \max\{id(u_2), id(v_2)\} \\
& \quad \&\&\ \min\{id(u_1), id(v_1)\} > \min\{id(u_2), id(v_2)\}).
\end{aligned}$$

This weight function ensures that two edges with different end-vertices have different weights. Note that $w(u, v) = w(v, u)$.

Definition 2.2 *Neighbor Set.* Node v is an *out-neighbor* of node u (and u is an *in-neighbor* of v) under an algorithm A, denoted $u \xrightarrow{A} v$, if and only if there exists an edge (u, v) in the topology generated by the algorithm. In particular, we use $u \rightarrow v$ to denote the neighbor relation in G. $u \overset{A}{\leftrightarrow} v$ if and only if $u \xrightarrow{A} v$ and $v \xrightarrow{A} u$. The *Out-Neighbor Set* of node u is $N_A^{out}(u) = \{v \in V(G) : u \xrightarrow{A} v\}$, and the *In-Neighbor Set* of u is $N_A^{in}(u) = \{v \in V(G) : v \xrightarrow{A} u\}$.

Definition 2.3 *Topology.* The topology generated by an algorithm A is a directed graph $G_A = (E(G_A), V(G_A))$, where $V(G_A) = V(G)$, $E(G_A) = \{(u,v) \in E(G) : u \xrightarrow{A} v\}$.

Definition 2.4 *Radius.* The radius, R_u, of node u is defined as the distance between node u and its farthest out-neighbor (in terms of Euclidean distance), i.e, $R_u = \max_{v \in N_A^{out}(u)}\{d(u,v)\}$.

Definition 2.5 *Connectivity.* In the topology generated by an algorithm A, node u is said to be *connected to* node v (denoted $u \Rightarrow v$) if there exists a path ($p_0 = u, p_1, \ldots, p_{m-1}, p_m = v$) such that $p_i \xrightarrow{A} p_{i+1}, i = 0, 1, \ldots, m-1$, where $p_k \in V(G_A), k = 0, 1, \ldots, m$. It follows that $u \Rightarrow v$ if $u \Rightarrow p$ and $p \Rightarrow v$ for some $p \in V(G_A)$.

Definition 2.6 *Bi-Directionality.* A topology generated by an algorithm A is *bi-directional*, if for any two nodes $u, v \in V(G_A)$, $u \in N_A^{out}(v)$ implies $v \in N_A^{out}(u)$. In other words, the topology generated by A is bi-directional if all edges in the topology are bi-directional.

Definition 2.7 *Bi-Directional Connectivity.* In the topology generated by an algorithm A, node u is said to be *bi-directionally connected to* node v (denoted $u \Leftrightarrow v$) if there exists a path ($p_0 = u, p_1, \ldots, p_{m-1}, p_m = v$) such that $p_i \overset{A}{\leftrightarrow} p_{i+1}, i = 0, 1, \ldots, m-1$, where $p_k \in V(G_A), k = 0, 1, \ldots, m$. It follows that $u \Leftrightarrow v$ if $u \Leftrightarrow p$ and $p \Leftrightarrow v$ for some $p \in V(G_A)$.

Deriving network topologies consisting of only bi-directional links facilitates link level acknowledgment in wireless networks, which is a critical operation for data transmissions and retransmissions over unreliable wireless media. Bi-directionality is also a key feature to the correctness of the floor acquisition mechanisms such as the RTS/CTS mechanism in IEEE 802.11.

Definition 2.8 *Addition and Removal.* The operation *Addition* is to add an extra edge (v,u) into G_A if $(u,v) \in E(G_A)$, $(v,u) \notin E(G_A)$, and $d(u,v) \leq r_v$. The operation *Removal* is to delete any edge $(u,v) \in E(G_A)$ if $(v,u) \notin E(G_A)$. Let G_A^+ and G_A^- denote the resulting topologies after applying *Addition* and *Removal* to G_A, respectively.

Both the *Addition* and *Removal* operations attempt to create a bi-directional topology by converting uni-directional edges into bi-directional or removing uni-directional edges. The resulting topology after *Addition* is not necessarily bi-directional, as it may attempt to increase the transmission power of a node v to a level that is beyond its maximum transmission power. The resulting topology after *Removal* is alway bi-directional, although it may not be strongly connected.

3. RELATED WORK AND WHY THEY CANNOT BE DIRECTLY APPLIED TO HETEROGENEOUS NETWORKS

Several topology control algorithms [Rodoplu and Meng 1999; Ramanathan and Rosales-Hain 2000; Li et al. 2001; Narayanaswamy et al. 2002; Kawadia and Kumar 2003; Borbash and Jennings 2002; Li et al. 2002; Li et al. 2003] have been proposed.

In this section, we first summarize these algorithm and then give examples to explain why they cannot be directly applied to heterogeneous networks.

3.1 Related Work

Rodoplu *et al.* [Rodoplu and Meng 1999] (denoted R&M) introduced the notion of *relay region* and *enclosure* for the purpose of power control. Instead of transmitting directly, a node chooses to relay through other nodes if less power will be consumed. It is shown that the network is strongly connected if every node maintains links with the nodes in its enclosure and the resulting topology is a minimum power topology. The major drawback of R&M is that it requires an explicit radio propagation model to compute the relay region, hence the resulting topology is sensitive to the model used in the computation.[1], Also, it assumes there is only one data sink (destination) in the network.

Ramanathan *et al.* [Ramanathan and Rosales-Hain 2000] presented two centralized algorithms to minimize the maximal power used per node while maintaining the (bi)connectivity of the network. They introduced two distributed heuristics for mobile networks. Both centralized algorithms require global information, and thus cannot be directly deployed in the case of mobility. On the other hand, the proposed heuristics cannot guarantee the preservation of network connectivity.

COMPOW [Narayanaswamy et al. 2002] and *CLUSTERPOW* [Kawadia and Kumar 2003] are approaches implemented in the network layer. Both hinge on the idea that if each node uses the smallest common power required to maintain network connectivity, the traffic carrying capacity of the entire network is maximized, the battery life is extended, and the MAC-level contention is mitigated. The major drawback is its significant message overhead, since each node has to run multiple daemons, each of which has to exchange link state information with the counterpart at other nodes.

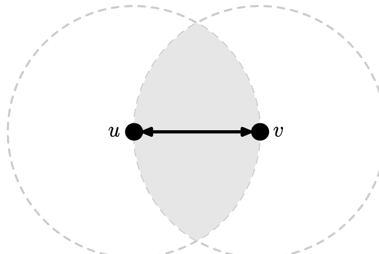
CBTC(α) [Li et al. 2001] is a two-phase algorithm in which each node finds the minimum power such that at least one (if any) node can be reached in every cone of degree α. The algorithm has been proved to preserve network connectivity if $\alpha < 5\pi/6$. Several optimization methods (that are applied after the topology is derived under the base algorithm) are also discussed to further reduce the transmitting power.

To facilitate the following discussion, we give the definition of the *Relative Neighborhood Graph* (RNG) below.

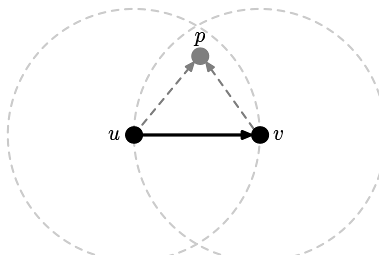
Definition 3.1 *Neighbor Relation in RNG.* For RNG [Toussaint 1980; Supowit 1983], $u \overset{RNG}{\longleftrightarrow} v$ if and only if there does not exist a third node p such that $w(u,p) < w(u,v)$ and $w(p,v) < w(u,v)$. Or equivalently, there is no node inside the shaded area in Figure 1(a).

Borbash and Jennings [Borbash and Jennings 2002] proposed to use RNG for topology initialization of wireless networks. Based on the local knowledge, each node makes decisions to derive the network topology based on RNG. The network topology thus derived has been reported to exhibit good overall performance in terms of power usage, interference, and reliability.

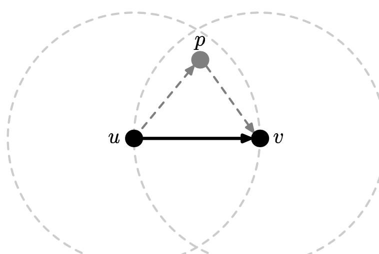
[1] In the simulation study presented in Section 6, we assume that the free-space model is used.



(a) *Relative Neighborhood Graph.*



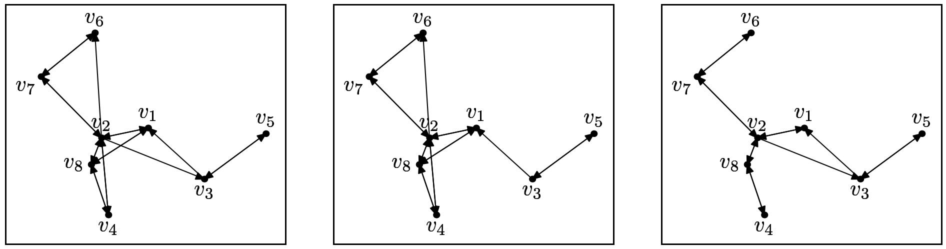
(b) *Modified Relative Neighborhood Graph* (to be defined in Section 3.2).



(c) *Directed Relative Neighborhood Graph* (to be defined in Section 4).

Fig. 1. The definition of RNG, MRNG and DRNG.

Li *et al.* [Li et al. 2002] presented the Localized Delaunay Triangulation, a localized protocol that constructs a planar spanner of the *Unit Disk Graph* (UDG). The topology contains all edges that are both in the unit-disk graph and the Delaunay triangulation of all nodes. It is proved that the shortest path in this topology between any two nodes u and v is at most a constant factor of the shortest path connecting u and v in UDG. However, the notion of UDG and Delaunay triangulation cannot be directly extended to heterogeneous networks.



(a) Original topology (without topology control) is strongly connected.

(b) Topology by $CBTC(\frac{2}{3}\pi)$ without optimization is not strongly connected: there is no path from v_1 to v_3.

(c) Topology by DLSS is strongly connected.

Fig. 2. An example that shows $CBTC(\frac{2}{3}\pi)$ may render disconnectivity in heterogeneous networks. There is no path from v_1 to v_3 due to the loss of edge (v_2,v_3), which is discarded by v_2 since v_1 and v_4 have already provided the necessary coverage.

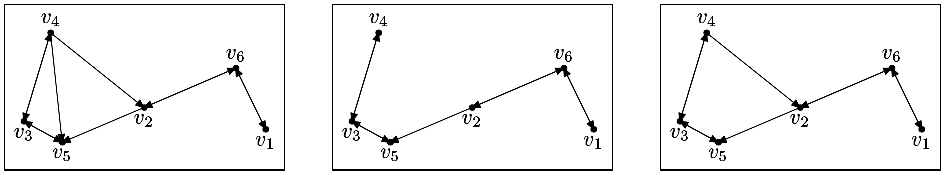
In [Li et al. 2003], we proposed LMST (Local Minimum Spanning Tree) for topology control in homogeneous wireless multi-hop networks. In this algorithm, each node builds its local minimum spanning tree independently and only keeps on-tree nodes that are one-hop away as its neighbors in the final topology. It is proved that (1) the topology derived under LMST preserves the network connectivity; (2) the node degree of any node in the resulting topology is bounded by 6; and (3) the topology can be transformed into one with bi-directional links (without impairing the network connectivity) after removal of all uni-directional links. Simulation results show that LMST can increase the network capacity as well as reduce the energy consumption.

Instead of adjusting the transmission power of individual devices, there also exist other approaches to generate power-efficient topology. By following a probabilistic approach, Santi *et al.* derived the suitable common transmission range which preserves network connectivity, and established the lower and upper bounds on the probability of connectedness [Santi et al. 2000]. In [Basagni et al. 2001], a "backbone protocol" is proposed to manage large wireless ad hoc networks, in which a small subset of nodes is selected to construct the backbone. In [J. Wu and Stojmenovic 2002], a method for calculating the power-aware connected dominating sets was proposed to establish an underlying topology for the network.

3.2 Why Existing Algorithms Cannot be Directly Applied to Heterogeneous Networks

Most existing topology control algorithms (except [Rodoplu and Meng 1999]) assume homogeneous wireless nodes with uniform transmission ranges. When directly applied to heterogeneous networks, these algorithms may render disconnectivity. In this subsection, we give several examples to motivate the need for new topology control algorithms for heterogeneous networks.

As shown in Figures 2(a)-(b) (note that in Figures 2–5 we use arrows to indicate

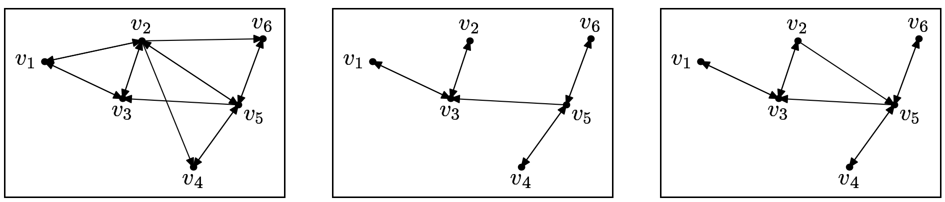


(a) Original topology (without topology control) is strongly connected.

(b) Topology by RNG is not strongly connected: there is no path from v_5 to v_2.

(c) Topology by DLSS is strongly connected.

Fig. 3. An example that shows RNG may render disconnectivity in heterogeneous networks. There is no path from v_5 to v_2 due to the loss of edge (v_4,v_2), which is discarded since $|(v_4, v_5)| < |(v_4, v_2)|$, and $|(v_2, v_5)| < |(v_4, v_2)|$.



(a) Original topology (without topology control) is strongly connected.

(b) Topology by MRNG is not strongly connected: there is no path from v_3 to v_5.

(c) Topology by DLSS is strongly connected.

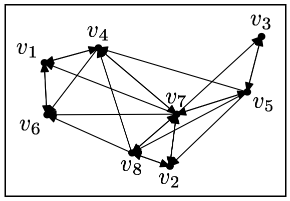
Fig. 4. An example that shows MRNG may render disconnectivity in heterogeneous networks. There is no path from v_3 to v_5 due to the loss of edge (v_2,v_5), which is discarded since $|(v_2, v_3)| < |(v_2, v_5)|$, and $|(v_5, v_3)| < |(v_2, v_5)|$.

the direction of the links), the network topology derived under $CBTC(\frac{2}{3}\pi)$ (without optimization) may not preserver the connectivity, when the algorithm is directly applied to a heterogeneous network. $CBTC(\frac{5}{6}\pi)$ also has the same problem.

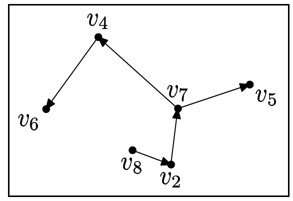
Similarly we show in Figure 3 (a)-(b) that the network topology derived under RNG may be disconnected when the algorithm is directly applied to a heterogeneous network. As RNG is defined for undirected graphs, one may tailor the definition of RNG for directed graphs.

Definition 3.2 *Neighbor Relation in MRNG.* For *Modified Relative Neighborhood Graph* (MRNG), $u \xrightarrow{MRNG} v$ if and only if there does not exist a third node p such that $w(u, p) < w(u, v), d(u, p) \leq r_u$ and $w(p, v) < w(u, v), d(v, p) \leq r_v$ (Figure 1(b)).

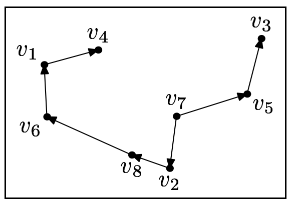
As shown in Figures 4(a)-(b), the topology derived under MRNG may still be disconnected. (We will give another variation of RNG in the next section that maintains network connectivity in heterogeneous networks.)



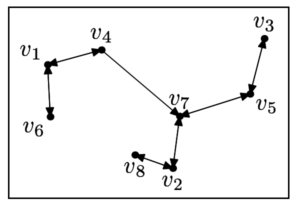
(a) Original topology (without topology control) is strongly connected.



(b) The local directed MST rooted at v_8.



(c) The local directed MST rooted at v_7.



(d) The resulting topology is not strongly connected: there is no path from v_7 to v_4.

Fig. 5. An example that shows the algorithm in which each node builds a local directed minimum spanning tree and only keeps the one-hop neighbors may result in disconnectivity.

One possible extension of LMST [Li et al. 2003] is for each node to build a local *directed* minimum spanning tree [Chu and Liu 1965; Edmonds 1967; Bock 1971] and keep only neighbors within one hop. Unfortunately, as shown in Figure 5, the resulting topology does not preserve the strong connectivity. In the next section, we will modify this approach to preserve the connectivity.

4. DRNG AND DLSS

In this section, we propose two localized topology control algorithms for heterogeneous wireless multi-hop networks with non-uniform transmission ranges: Directed Relative Neighborhood Graph (DRNG) and Directed Local Spanning Subgraph (DLSS). In both algorithms, the topology is derived by having each node build its neighbor set and adjust its transmission power based on locally collected information. Both algorithms are composed of three phases:

(1) *Information Collection*: each node locally collects the information of the neighborhood;

(2) *Topology Construction*: each node defines (in compliance with the algorithm) the proper set of neighbors for the final topology using the information in the neighborhood.

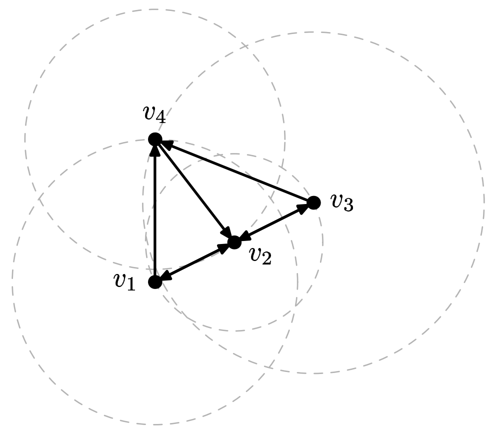


Fig. 6. An example that shows having each node broadcast a Hello message using its maximal transmission power may be insufficient for some nodes (e.g., node v_1) to know their reachable neighborhood. This figure also serves to show that given an arbitrary direct graph, it may be impossible to derive a bi-directional topology.

(3) *Construction of Topology with Only Bi-Directional Links* (Optional): each node adjusts its set of neighbors to make sure that all the edges are bi-directional.

4.1 Information collection

In the stage of information collection, every node collects the information of its neighborhood. In particular, each node needs to know all the edges in its neighborhood. In [Li et al. 2003], the *Reachable Neighborhood* was used for the local topology construction. We modify the definition for heterogeneous networks as follows:

Definition 4.1 *Reachable Neighborhood.* The reachable neighborhood N_u^R is the set of nodes that node u can reach using its maximal transmission power, i.e., $N_u^R = \{v \in V(G) : d(u,v) \le r_u\}$. For each node $u \in V(G)$, let $G_u^R = (V(G_u^R), E(G_u^R))$ be an induced subgraph of G such that $V(G_u^R) = N_u^R$.

For homogeneous networks, the *Reachable Neighborhood* N^R can be obtained locally if each node broadcasts periodically a Hello message using its maximal transmission power. The information contained in a Hello message should at least include the *id*, the maximal transmission power, and the position of the node. Here for ease of exhibition, we assume that each node is equipped with the capability to gather its location information via special hardware or localization service provided by the network (see, for example, [He et al. 2003] for a summary).

Note that our algorithms can still operate if the position information is not available, as only the knowledge of all the existing edges, $E(G^R)$, is required. $E(G^R)$ can be constructed locally as follows. First, each node periodically broadcasts, using its maximal transmission power, a very short Hi message which includes only its node *id* and its maximal transmission power. Upon receiving such a message from a neighbor node v, each node u estimates the length of the edge (u, v) based on the attenuation incurred in the transmission. Let the set of edges incident at u be denoted as $E_u^T = \{(u,v) : v \in N_u^R\})$. After node u collects the information on

Procedure: DLSS(u)
Input: G_u^R, the induced subgraph of G that spans the viewable neighborhood of u;
Output: $S_u = (V(S_u), E(S_u))$, the local spanning subgraph of G_u^R;
$V(S_u) := V$, $E(S_u) := \emptyset$;
begin
Sort all edges in $E(G_u^R)$ in the ascending order of weight (as defined in Definition 2.1);
for each edge (u_0, v_0) in the order
  if u_0 is not connected to v_0 in S_u
    $E(S_u) := E(S_u) \cup \{(u_0, v_0)\}$;
    if u_0 is connected to every other node in G_u^R
      exit;
    endif
  endif
end
end

Fig. 7. Algorithm description of DLSS.

E_u^T, it can then broadcast this information in an Edge message. Each node will be able to infer $E(G^R)$ based on the Edge messages received from all of its neighbors. Although this solution may incur more communication and computation overhead, and make our algorithms less "localized", it eliminates the need for the position information, and thus is better suited for wireless sensor networks where the cost and the energy consumption should be kept as low as possible.

Another issue that complicates the process of obtaining $E(G^R)$ and is unique in heterogeneous networks is that it may not be sufficient to have each node broadcast periodically a Hello message using its maximal transmission power, in order to obtain $E(G^R)$. For example, as shown in Figure 6, v_1 is unable to know the position of v_4 since v_4 cannot reach v_1. For ease of presentation, we assume for now that by the end of the first phase every node u obtains its $E(G^R)$. We will come back to this issue in Section 5.4.

4.2 Topology construction

After obtaining $E(G^R)$, the neighbor relation in both algorithms can be defined.

Definition 4.2 *Neighbor Relation in DRNG.* For *Directed Relative Neighborhood Graph* (DRNG), $v \xrightarrow{DRNG} u$ if and only if $v \in N_u^R$ and there does not exist a third node $p \in N_u^R$ such that $w(u,p) < w(u,v)$ and $w(p,v) < w(u,v), d(p,v) \leq r_p$ (see Figure 1(c)).

Definition 4.3 *Neighbor Relation in DLSS.* For *Directed Local Spanning Subgraph* (DLSS), $v \xrightarrow{DLSS} u$ if and only if $(u,v) \in E(S_u)$, where S_u is the output of DLSS(u) (Figure 7). Hence node v is a neighbor of node u if and only if node v is on node u's directed local spanning graph S_u, and is one-hop away from node u.

DLSS is a natural extension of LMST [Li et al. 2003] to heterogeneous networks. Instead of computing a directed local MST (which minimizes the total cost of all the edges in the subgraph, and is shown to be incapable of maintaining connectivity in heterogeneous networks in Section 3.2), each node u computes a directed local subgraph according to the algorithm in Figure 7 (which minimizes the maximum cost of all edges in the subgraph) and takes on-tree nodes that are one-hop away as its neighbors.

Each node broadcasts its own maximal transmission power in the Hello or Hi message. By measuring the receiving power of the messages, each node u can determine the specific power level required to reach each of its out-neighbors [Li et al. 2003]. Node u then uses the power level that can reach its farthest neighbor as its transmission power. This approach can be applied without knowing the actual propagation model.

4.3 Construction of topology with only bi-directional edges

As illustrated in Section 3.2 (e.g., Fig. 2(c)), some links in G_{DLSS} may be uni-directional. There may also exist uni-directional links in G_{DRNG}. We can apply either *Addition* or *Removal* to G_{DLSS} and G_{DRNG} to obtain bi-directional topologies. We will discuss some properties of these solutions in Section 5.2.

5. PROPERTIES OF DRNG AND DLSS

In this section, we prove the connectivity and bi-directionality of DLSS and DRNG and derive the bound of their node degree in Sections 5.1–5.3. Then we discuss in Section 5.4 how to deal with the problem (Figure 6) that arises in obtaining the reachable neighborhood $E(G^R)$ in heterogeneous networks, and in Section 5.5 how to relax the assumption of perfect omni-directional antenna patterns. We always assume G is strongly connected, i.e., $u \Rightarrow v$ in G for any $u, v \in V(G)$.

5.1 Connectivity

LEMMA 5.1. *For any edge $(u, v) \in E(G)$, we have $u \Rightarrow v$ in G_{DLSS}.*

PROOF. Let all the edges $(u, v) \in E(G)$ be sorted in the increasing order of weight, i.e., $w(u_1, v_1) < w(u_2, v_2) < \ldots < w(u_l, v_l)$, where l is the total number of edges in G. We prove by induction.

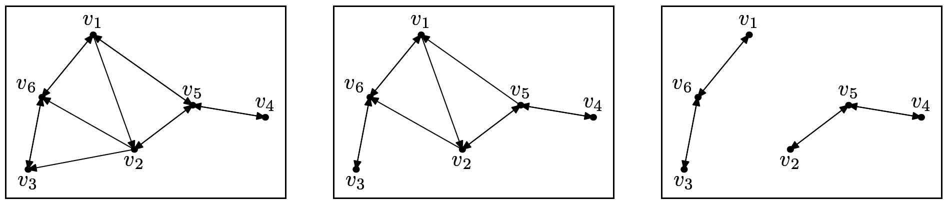
(1) *Basis*: The first edge (u_1, v_1) satisfies $w(u_1, v_1) = \min_{(u,v) \in E(G)}\{w(u, v)\}$. According to the algorithm in Figure 7, (u_1, v_1) and (v_1, u_1) will be included in G_{DLSS}, i.e., $u_1 \overset{DLSS}{\longleftrightarrow} v_1$.

(2) *Induction*: Assume the hypothesis holds for all edges $(u_i, v_i), 1 \leq i < k$, we prove $u_k \Rightarrow v_k$ in G_{DLSS}. If $u_k \overset{DLSS}{\longrightarrow} v_k$, then $u_k \Rightarrow v_k$. Otherwise in the local topology construction of v, before edge (u_k, v_k) was inserted into S_{u_k}, there must already exist a path $p = (w_0 = u_k, w_1, w_2, \cdots, w_{m-1}, w_m = v_k)$ from u_k to v_k, where $(w_i, w_{i+1}) \in E(S_{u_k}), i = 0, 1, \cdots, m-1$. Since edges are inserted in an ascending order of weight, we have $w(w_i, w_{i+1}) < w(u_k, v_k)$. Applying the induction hypothesis to each pair $(w_i, w_{i+1}), i = 0, 1, \cdots, m-1$, we have $w_i \Rightarrow w_{i+1}$. Therefore, $u_k \Rightarrow v_k$.

□

THEOREM 5.2 CONNECTIVITY OF DLSS. *G_{DLSS} preserves the connectivity of G, i.e., G_{DLSS} is strongly connected if G is strongly connected.*

PROOF. Suppose G is strongly connected. For any two nodes $u, v \in V(G)$, there exists at least one path $p = (w_0 = u, w_1, w_2, \cdots, w_{m-1}, w_m = v)$ from u to v, where



(a) Original topology (without topology control) is strongly connected.

(b) Topology by DLSS is strongly connected.

(c) Topology by DLSS with *Removal* is not strongly connected: there are 2 components.

Fig. 8. An example that shows DLSS with *Removal* may result in disconnectivity.

$(w_i, w_{i+1}) \in E(G), i = 0, 1, \cdots, m-1$. Since $w_i \Rightarrow w_{i+1}$ by Lemma 5.1, we have $u \Rightarrow v$. □

Lemma 5.3. *Given three nodes $u, v, p \in V(G_{DLSS})$ satisfying $w(u,p) < w(u,v)$ and $w(p,v) < w(u,v)$, $d(p,v) \le r_p$, then $u \nrightarrow v$ in G_{DLSS}.*

Proof. We only need to consider the case where $d(u,v) \le r_u$ since $d(u,v) > r_u$ would imply $u \nrightarrow v$. Consider the local topology construction of v. Before we insert (u,v) into S_u, the two edges (u,p) and (p,v) have already been processed since $w(u,p) < w(u,v)$ and $w(p,v) < w(u,v)$. Thus $u \Rightarrow p$ and $p \Rightarrow v$, which means $u \Rightarrow v$. Therefore, (u,v) should not be inserted into S_u according to the algorithm in Figure 7, i.e., $u \nrightarrow v$ in G_{DLSS}. □

Lemma 5.4. *The edge set of G_{DLSS} is a subset of the edge set of G_{DRNG}, i.e., $E(G_{DLSS}) \subseteq E(G_{DRNG})$.*

Proof. We prove by contradiction. Given any edge $(u,v) \in E(G_{DLSS})$, assume $(u,v) \notin E(G_{DRNG})$. According to the definition of $DRNG$, there must exist a third node p such that $w(u,p) < w(u,v), d(u,p) \le r_u$ and $w(p,v) < w(u,v), d(p,v) \le r_p$. By Lemma 5.3, $u \nrightarrow v$ in G_{DLSS}, i.e., $(u,v) \notin E(G_{DLSS})$. □

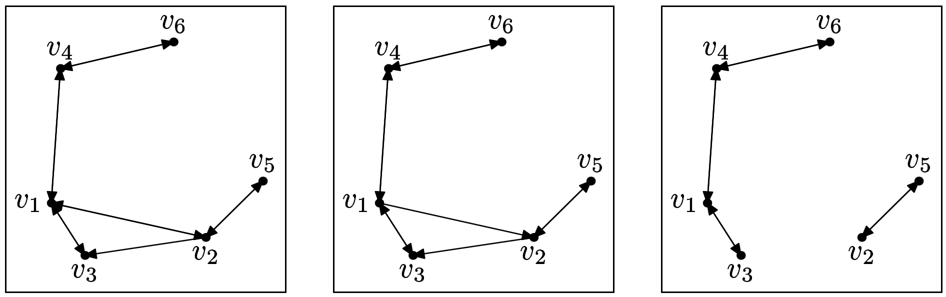
Theorem 5.5 Connectivity of DRNG. *If G is strongly connected, then G_{DRNG} is also strongly connected.*

Proof. This is a direct result of Theorem 5.2 and Lemma 5.4. □

5.2 Bi-directionality

Now we discuss the bi-directionality property of DLSS and DRNG. Since *Addition* may not always result in bi-directional topologies, we first apply *Removal* to topologies by DLSS and DRNG. It turns out the simple *Removal* operation may lead to disconnectivity. Examples are given in Figure 8–9 to show, respectively, that DLSS and DRNG with *Removal* may result in disconnectivity.

In general, G may not be bi-directional if the maximum transmission ranges are non-uniform. Since the maximal transmission range can not be increased, it may be impossible to find a bi-directional connected subgraph of G in some cases. An



(a) Original topology (without topology control) is strongly connected.

(b) Topology by DRNG is strongly connected.

(c) Topology by DRNG with *Removal* is not strongly connected: there are 2 components.

Fig. 9. An example that shows DRNG with *Removal* may result in disconnectivity.

example is given in *Figure 6*: v_1 can reach v_2 and v_4, v_2 can reach v_1 and v_3, v_3 can reach v_2 and v_4, and v_4 can reach v_2 only. *Addition* does not lead to bi-directionality since all edges entering or leaving v_4 are uni-directional with all nodes already transmitting with their maximal power. On the other hand, *Removal* will partition the network. In this example, although the graph G is strongly connected, its subgraph with the same vertex set cannot be both connected and bi-directional.

Now we show that bi-directionality can be ensured if the original topology is both strongly connected and bi-directional.

THEOREM 5.6. *If the original topology G is strongly connected and bi-directional, then G_{DLSS} and G_{DRNG} are also strongly connected and bi-directional after* Addition *or* Removal.

PROOF. We have $E(G^-_{DLSS}) \subseteq E(G^+_{DLSS})$, and $E(G^-_{DLSS}) \subseteq E(G^-_{DRNG}) \subseteq E(G^+_{DRNG})$ since $E(G_{DLSS}) \subseteq E(G_{DRNG})$. Therefore, we only need to prove that G^-_{DLSS} preserves the strong connectivity.

In the *Induction* step in Lemma 5.1, the only reason we cannot prove that $u_k \overset{DLSS}{\longleftrightarrow} v_k$ is that edge (v_k, u_k) may not exist. Given that G is bi-directional, we are able to prove that $u_k \overset{DLSS}{\longleftrightarrow} v_k$. Hence for any edge $(u, v) \in E(G)$, we have $u \Leftrightarrow v$ in G_{DLSS}. The removal of asymmetric edges in G_{DLSS} does not affect this property. Therefore, G^-_{DLSS} is still strongly connected. □

5.3 Degree Bound

It has been observed that any minimum spanning tree of a simple undirected graph in the plane has a maximum out-degree of 6 [Monma and Suri 1991]. However, this bound does not hold for directed graphs. An example is shown in Figure 11, where node u has 18 out-neighbors. In this section, we derive the bound on the node degree in the topology by DLSS and DRNG. First we define the *out-degree* and *in-degree* as follows:

Definition 5.7 *Degree.* The *out-degree* of a node u under an algorithm A, denoted

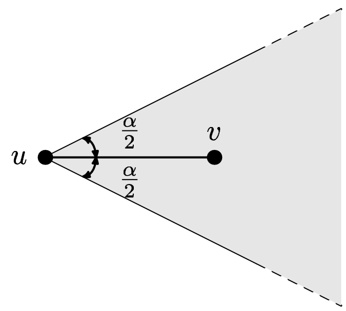


Fig. 10. The definition of $Cone(u, \alpha, v)$.

$deg_A^{out}(u)$, is the number of out-neighbors of u, i.e., $deg_A^{out}(u) = |N_A^{out}(u)|$. Similarly, the *in-degree* of a node u, denoted $deg_A^{in}(u)$, is the number of in-neighbors, i.e., $deg_A^{in}(u) = |N_A^{in}(u)|$.

Definition 5.8 *Disk.* $Disk(u, r)$ is the disk of radius r, centered at node u.

Definition 5.9 *Cone.* $Cone(u, \alpha, v)$ is the unbounded shaded region shown in Figure 10.

The following lemma is a direct result of the definition of DRNG.

LEMMA 5.10. *Given three nodes $u, v, p \in V(G_{DRNG})$ satisfying $w(u,p) < w(u,v)$ and $w(p,v) < w(u,v)$, $d(p,v) \leq r_p$, then $u \nrightarrow v$ in G_{DRNG}.*

The following corollary is a direct result of Lemma 5.3 and Lemma 5.10.

COROLLARY 5.11. *If v is an out-neighbor of u in G_{DLSS} or G_{DRNG}, and $d(u,v) \geq r_{min}$, then u can not have any other out-neighbor inside $Disk(v, r_{min})$.*

THEOREM 5.12. *For any node u in G_{DLSS} or G_{DRNG}, the number of out-neighbors that are inside $Disk(u, r_{min})$ is at most 6.*

PROOF. Let $N^{out}(u)$ be the set of out-neighbors of u in G_{DLSS} or G_{DRNG} that are inside $Disk(u, r_{min})$. Let the nodes in $N^{out}(u)$ be ordered such that for the ith node w_i and the jth node w_j $(j > i)$, $w(u, w_j) > w(u, w_i)$. By Lemma 5.3 and Lemma 5.10, we have $w(u, w_j) \leq w(w_i, w_j)$ (otherwise $u \nrightarrow w_j$). Thus $\angle w_i u w_j \geq \pi/3$, i.e., node w_j cannot reside inside $Cone(u, 2\pi/3, w_i)$. Therefore, node u cannot have neighbors other than node w_i inside $Cone(u, 2\pi/3, w_i)$. By induction on the rank of nodes in $N^{out}(u)$, the maximal number of out-neighbors that u can have is at most 6. □

THEOREM 5.13 OUT-DEGREE BOUND. *The out-degree of any node in G_{DLSS} or G_{DRNG} is bounded by a constant that depends only on r_{max} and r_{min}.*

PROOF. By Theorem 5.12, for any node u in G_{DLSS} or G_{DRNG}, there are at most 6 out-neighbors inside $Disk(u, r_{min})$. Also by Corollary 5.11, the set of disks $\{Disk(v, \frac{r_{min}}{2}) : v \in N^{out}(u), v \notin Disk(u, r_{min})\}$ are disjoint. Therefore, the total number of out-neighbors of u is bounded by:

$$c_1 = 6 + \left\lceil \frac{\pi[(r_{max} + \frac{r_{min}}{2})^2 - (\frac{r_{min}}{2})^2]}{\pi(\frac{r_{min}}{2})^2} \right\rceil = 4\lceil \beta(\beta+1) \rceil + 6,$$

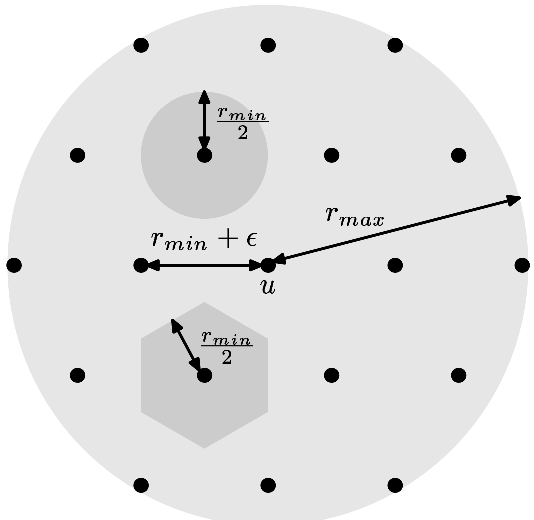


Fig. 11. An example that shows the out-degree in a heterogeneous network can be very large. The transmission range of u is r_{max} and the transmission range for all other nodes is r_{min}, where $r_{max} = 2(r_{min} + \epsilon)$, $\epsilon > 0$. All nodes are so arranged that the distance between any node and its closest neighbor is $r_{min} + \epsilon$. Therefore, the only links in the network are those from u to all the other nodes. Since relaying packets is impossible, u has to use its maximal transmission power and keeps all 18 neighbors.

where $\beta = \frac{r_{max}}{r_{min}}$. Actually we can observe that Figure 11 shows the scenario where the maximum out-degree of u is achieved if $\epsilon \to 0$. Therefore, we can further tighten the bound. Since the hexagonal area (as shown in Figure 11) centered at every neighbor of u is disjoint with each other, the total number of neighbors of u is bounded by:

$$c_2 = \left\lceil \frac{\pi(r_{max} + \frac{r_{min}}{\sqrt{3}})^2}{\frac{\sqrt{3}}{2} r_{min}^2} \right\rceil - 1 = \left\lceil \frac{2\pi}{\sqrt{3}}(\beta + \frac{1}{\sqrt{3}})^2 \right\rceil - 1.$$

□

THEOREM 5.14 IN DEGREE BOUND. *The in-degree of any node in G_{DLSS} or G_{DRNG} is bounded by 6.*

PROOF. Let $N^{in}(u)$ be the set of in-neighbors of u in G_{DLSS} or G_{DRNG}. Sort the nodes in $N^{in}(u)$ such that for the ith node w_i and the jth node w_j ($j > i$), $w(w_j, u) > w(w_i, u)$. By Lemma 5.3 and Lemma 5.10, we have $w(w_j, u) \leq w(w_i, w_j)$ (otherwise $w_j \nrightarrow u$). Thus $\angle w_i u w_j \geq \pi/3$, i.e., node w_j cannot reside inside $Cone(u, 2\pi/3, w_i)$. Therefore, node u cannot have in-neighbors other than node w_i inside $Cone(u, 2\pi/3, w_i)$. By induction on the rank of nodes in $N^{in}(u)$, the maximal number of in-neighbors that u can have is at most 6. □

The bound given in Theorem 5.12 is the same for DLSS and DRNG, but the out-degree of the same node in G_{DLSS} is always smaller than that in G_{DRNG} since $E(G_{DLSS}) \subseteq E(G_{DRNG})$. Although this given bound is quite large, the average out-degree of nodes is actually not as large. In particular, since $\sum_{v \in V} deg^{in}(v) = \sum_{v \in V} deg^{out}(v)$, we have $E[deg^{out}(v)] = \frac{1}{n}\sum_{v \in V} deg^{in}(v) \leq \frac{1}{n} \cdot 6n = 6$. We will also show in Section 6 that the average maximum out-degree is far less than the bound for networks with randomly distributed nodes.

Note that what has been discussed so far is the *logical* node degree, i.e., the number of logical neighbors. In practice, it is more important to consider the *physical* node degree, i.e., the number of nodes within the transmission radius. If omni-directional antennas are used, the physical degree cannot be bounded for an arbitrary topology. However, with the help of directional antennas, we can bound the physical degree given that the logical degree is bounded under DLSS (except in some extreme cases, e.g., a large number of nodes are of the same distance from one node). Essentially when transmitting to a specific neighbor, node u should adjust the direction and limit the transmission power so that no other nodes will be affected.

5.4 Obtaining the Reachable Neighborhood Information in Heterogeneous Networks

As mentioned in Section 4, having each node u broadcast its own position information to all the other nodes within r_u is not sufficient to ensure in a heterogeneous network that each node can obtain the information, $E(G^R)$, on its reachable neighborhood (Figure 6). This problem is common to any distributed/localized topology control algorithm since each node has to at least know the information of the reachable neighborhood to be able to preserve network connectivity.

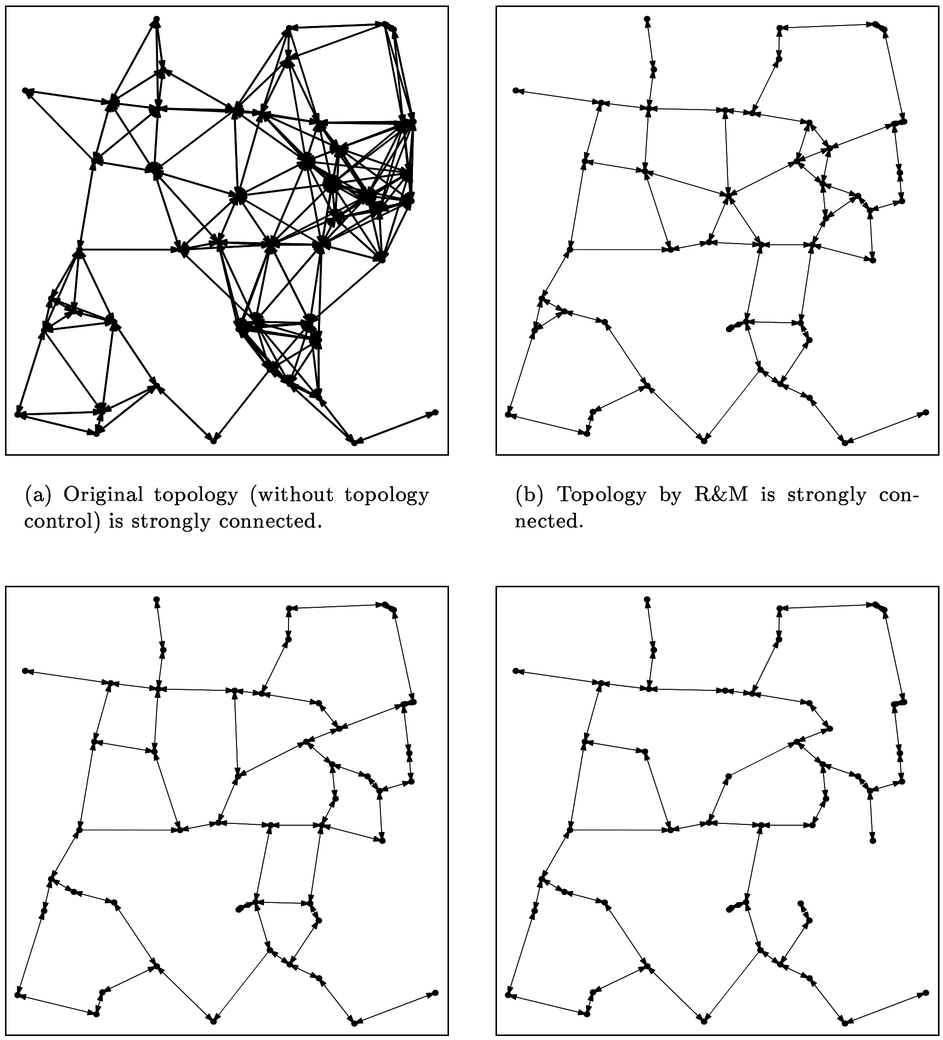
To solve this problem, we have to make an extra assumption about the original topology of the network. Consider a subgraph of G that has less edges: $G' = (V(G'), E(G'))$, where $E(G') = \{(u,v) : d(u,v) \leq \min(r_u, r_v), u, v \in V(G)\}$. Generally speaking, G' may not be strongly connected even if G is strongly connected. For each node to be able to obtain its reachable neighborhood, we add an additional assumption that G' should be strongly connected. For any edge $(u,v) \in E(G')$, since $d(u,v) \leq \min(r_u, r_v)$, we have $(v,u) \in E(G')$, which means G' is bi-directional. Define $N_u^{R'} = \{v \in V(G) : d(u,v) \leq \min(r_u, r_v)\}$, $r_u{}' = \max_{v \in N_u^{R'}}\{d(u,v)\}$, where $r_u{}' \leq r_u$ since for any $v \in N_u^{R'}$, $d(u,v) \leq r_u$. Let $r_{min}{}' = \min_{v \in V}\{r_v{}'\}$ and $r_{max}{}' = \max_{v \in V}\{r_v{}'\}$. By requiring each node u to broadcast its position and id to all other nodes within r_u, we are able to determine $N_u^{R'}$ and $r_u{}'$. We can then apply DRNG and DLSS to G' and prove that Theorems 5.2-5.13 still hold if the original topology is G'.

THEOREM 5.15. *Theorems 5.2, 5.5, 5.6, 5.12, 5.13, and 5.14 still holds if the original topology is G'.*

PROOF. We replace G, r_u, N_u^R, r_{min}, and r_{max} with G', $r_u{}'$, $N_u^{R'}$, $r_{min}{}'$ and $r_{max}{}'$ in the proof of Lemmas 5.1, 5.3, 5.4, and 5.10 and Theorems 5.2, 5.5, 5.6, 5.12, 5.13, and 5.14. By following the same line of arguments, we can prove that they still hold if the original topology is G'. □

THEOREM 5.16. *If the original topology is G' (which is a subgraph of G), G_{DLSS} and G_{DRNG} are bi-directional after* Addition *or* Removal.

PROOF. We apply Theorem 5.6 to G', for G' is bi-directional. □



(a) Original topology (without topology control) is strongly connected.

(b) Topology by R&M is strongly connected.

(c) Topology by DRNG is strongly connected.

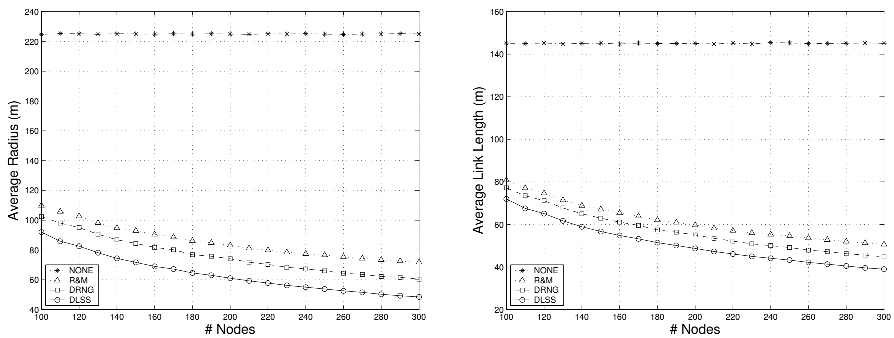
(d) Topology by DLSS is strongly connected.

Fig. 12. Topologies derived by R&M, DRNG, and DLSS.

5.5 Relaxing the Assumption of Perfect Omni-directional Antenna Patterns

Many topology control algorithms assume a Unit Disk Graph (UDG) model, i.e., the antenna pattern of a wireless device is a perfect disk. This is also the underlying assumption for algorithms that use explicit channel propagation models. Since the same models are applied to all directions, the antenna patterns have to be isotropic, which in turn implies that the transmission area is a perfect disk.

In the case of DLSS, the antenna pattern model influences the manner in which



(a) Average radius.

(b) Average link length

Fig. 13. Comparison of DLSS, DRNG and R&M with respect to average radius and average edge length.

the information of N_u^R can be collected. Given an arbitrary antenna pattern, we can simply employ the information dissemination technique in Section 4.1. It is obvious that the information dissemination technique does not rely on any specific antenna pattern, except that estimating the edge length becomes quite difficult. This is due to the fact that the antenna pattern is not necessarily isotropic, i.e., the power attenuation may vary in different directions. We are currently investigating how to address this problem.

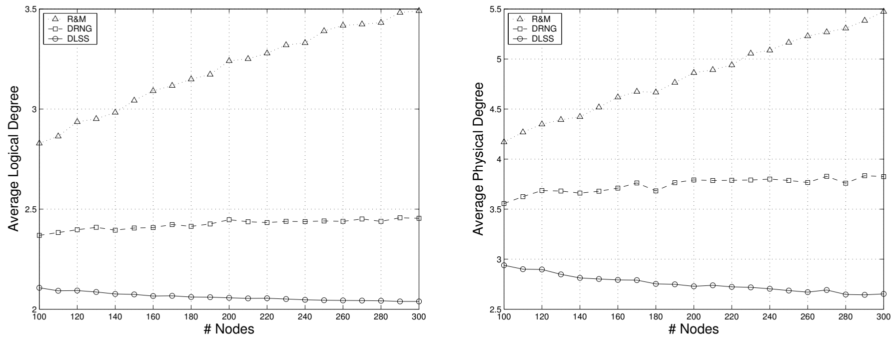
6. SIMULATION STUDY

In this section, we evaluate the performance of R&M, DRNG, and DLSS by simulations. Each data point reported below is an average of 50 simulation runs. All three algorithms are known to preserve network connectivity in heterogeneous networks.

In the first simulation, 50 nodes are uniformly distributed in a $1000m \times 1000m$ region. The transmission ranges of nodes are uniformly distributed in $[200m, 250m]$. Figure 12 gives the topologies derived using the maximal transmission power (labeled as NONE), R&M (under the two-ray ground model), DRNG, and DLSS for one simulation instance. As shown in Figure 12, R&M, DRNG and LMST all significantly reduce the average node degree, while maintaining network connectivity. Moreover, both DRNG and DLSS render less edges than R&M.

In the second simulation, we vary the number of nodes in the region from 100 to 300. The transmission ranges of nodes are uniformly distributed in $[200m, 250m]$. Figure 13 shows the average radius and the average link length for the topologies derived under NONE (no topology control), R&M, DRNG, and DLSS. DLSS outperforms the others, which implies that DLSS can provide better spatial reuse and nodes consume less energy to communicate with each other.

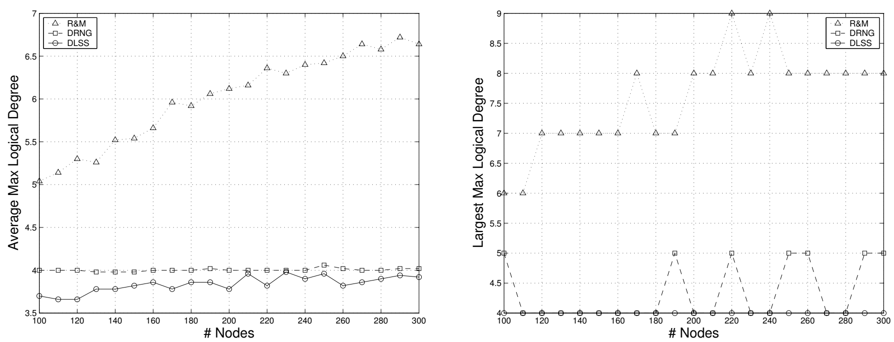
We also compare the out-degree of the topologies by different algorithms. The result of NONE is not shown because its out-degrees increase almost linearly with the number of nodes and are significantly larger than those under R&M, DRNG, and DLSS. Figure 14 shows the average logical/physical out-degree for the topolo-



(a) Average logical out-degree.

(b) Average physical out-degree.

Fig. 14. Comparison of R&M, DRNG and DLSS with respect to average out-degree.



(a) Average maximum logical out-degree.

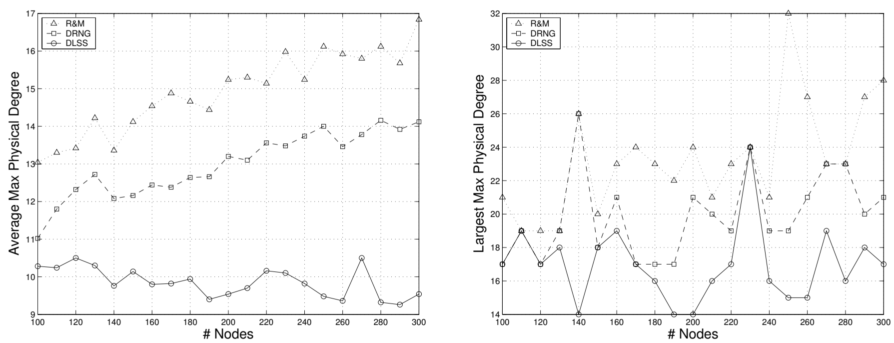
(b) Largest maximum logical out-degree.

Fig. 15. Comparison of R&M, DRNG and DLSS with respect to the maximum logical degree.

gies derived by R&M, DRNG, and DLSS. The average out-degrees under R&M and DRNG increase with the increase in the number of nodes, while that under DLSS actually decrease. Figure 15 shows the average maximum logical degree and the largest maximum logical out-degree for the topologies derived by R&M, DRNG, and DLSS. The largest maximum logical degree under DLSS is at most 4, and is well below the theoretical upper bound obtained in Theorem 5.13. Also, the topology derived under DLSS has much smaller out-degrees than the other topologies. Similar observations can be made in Figure 16 for physical degrees, except that physical degrees are in general larger than logical degrees for the same topology.

7. CONCLUSIONS

In this paper, we have proposed two localized topology control algorithms, Directed Relative Neighborhood Graph (DRNG) and Directed Local Spanning Subgraph



(a) Average maximum physical out-degree.

(b) Largest maximum physical out-degree.

Fig. 16. Comparison of R&M, DRNG and DLSS with respect to the maximum physical degree.

(DLSS), for heterogeneous wireless multi-hop networks in which each node may have different maximal transmission ranges. We show that as most existing topology control algorithms (except R&M [Rodoplu and Meng 1999]) do not consider the fact that nodes may have different maximal transmission ranges, they render disconnected network topologies when directly applied to heterogeneous networks. Then we devise DRNG and DLSS and prove that (i) both DRNG and DLSS preserve network connectivity; (ii) both DRNG and DLSS preserve network bi-directionality if *Addition* and *Remove* operations are applied to the topologies derived under these algorithms; and (iii) the out-degree of any node is bounded in the topology derived under DLSS or DRNG. The simulation study validates the superiority of DRNG and DLSS over R&M.

As part of our future research, we will (1) derive, given a topology in which each node transmits with different maximal transmission power, the probability that the topology is bi-directional with respect to the distribution and the density of nodes, and the distribution of the transmission ranges; and (2) jointly consider topology and MAC control and study the issue of how the MAC-level interference affects network connectivity and bi-directionality.

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