Viewing

Part I (History and Overview of Projections)

Lecture Topics

- History of projection in art
- Geometric constructions
- Types of projection (parallel and perspective)

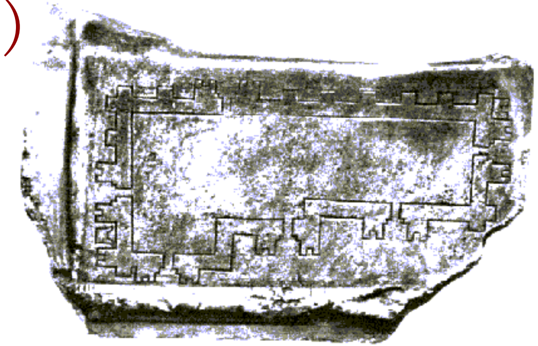
Drawing as Projection (Turning 3D to 2D)

- Painting based on mythical tale as told by Pliny the Elder: Corinthian man traces shadow of departing lover
- Projection through use of shadows
- Detail from ***The Invention of Drawing* (1830)** – Karl Friedrich Schinkle

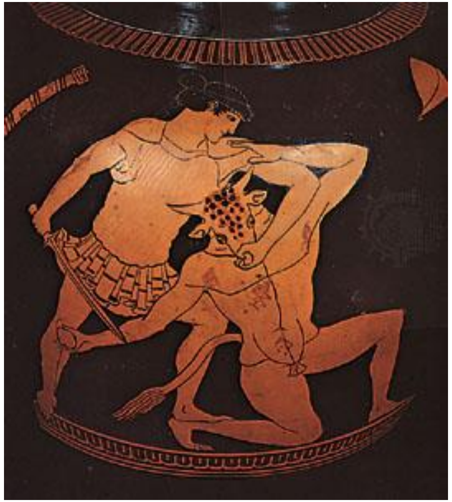
William J. Mitchell, *The Reconfigured Eye*, Fig 1.1

Early Forms of Projection (1/2)

- Plan view (*parallel*, specifically orthographic, projection) from Mesopotamia (2150 BC): Earliest known technical drawing in existence
- Greek vase from the late 6th century BC: Shows signs of attempts at *perspective foreshortening*!
  - Note relative sizes of thighs and lower legs of minotaur

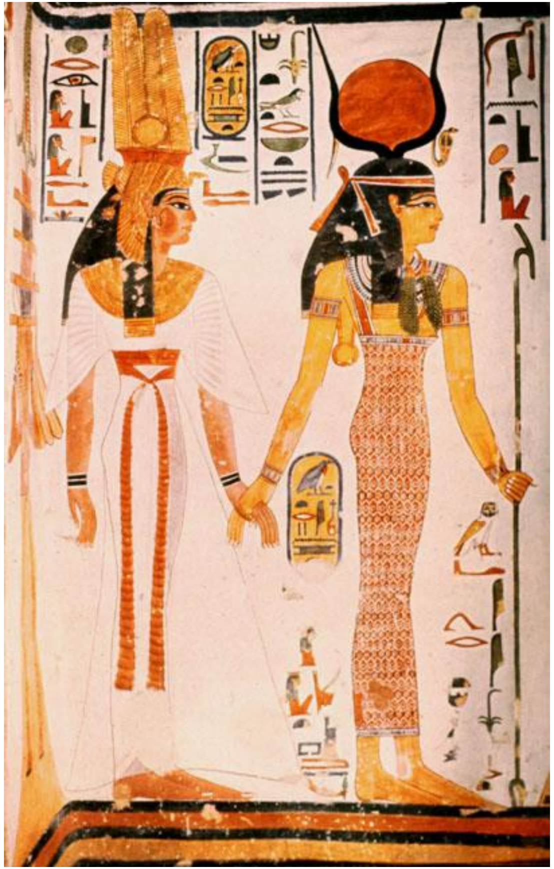


Ingrid Carlbom
Planar Geometric Projections and Viewing Transformations
Fig. 1-1



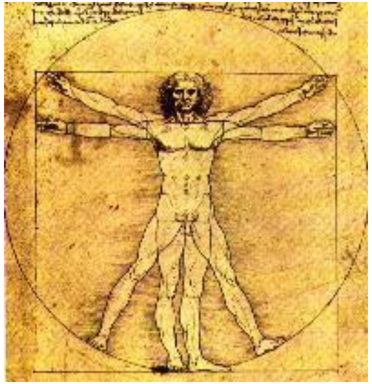
Early Forms of Projection (2/2)

- Ancient Egyptian Art
  - Multiple Viewpoints
  - Parallel Projection (no attempt to depict perspective foreshortening)
- Tomb of Nefertari, Thebes (19th Dyn, ~1270 BC), *Queen Led by Isis.* Mural
- Note how depiction of body implies a front view but the feet and head imply side view (early cubism!)



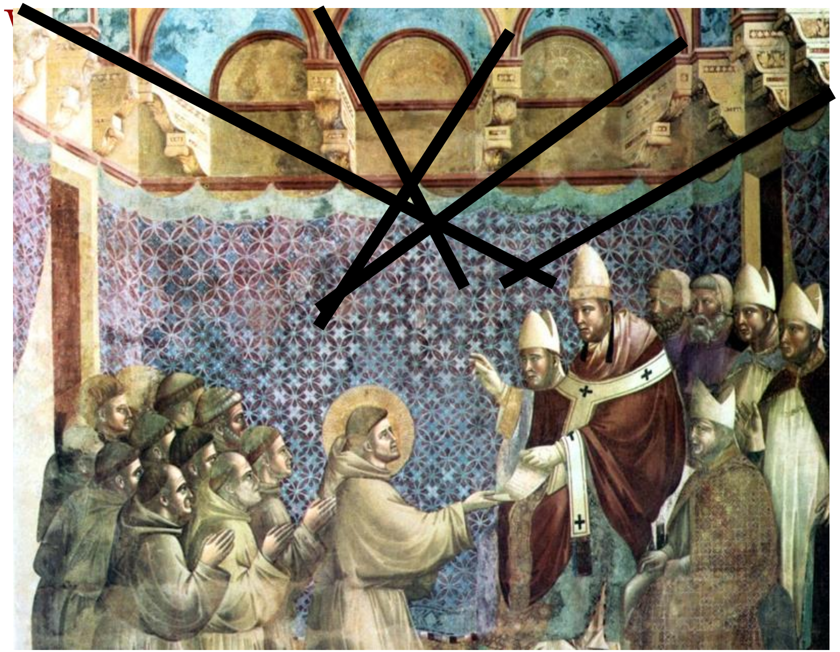
The Renaissance

- Starting in the 13th century (AD): New emphasis on importance of individual viewpoint, world interpretation, power of observation (particularly of nature: astronomy, anatomy, etc.)
  - Masaccio, Donatello, DaVinci
- Universe as clockwork: rebuilding the universe more systemically and mechanically
  - Tycho Brahe and Rudolph II in Prague (detail of clockwork), c. 1855
  - Copernicus, Kepler, Galileo, Newton...: from earth-centric to heliocentric model of the (mechanistic) universe whose laws can be discovered and understood



Early Attempts at Perspective

- In art, an attempt to represent 3D space more realistically
- Earlier works invoke a sense of 3D space but not systematically
  - Parallel lines converge, but no single vanishing point (where parallel lines converge)



Giotto

Franciscan Rule Approved

Assisi, Upper Basilica, c.1288-1292

Brunelleschi and Vermeer

- Brunelleschi invented systematic method of determining perspective projections (early 1400s). He created demonstration panels with specific viewing constraints for complete accuracy of reproduction.
  - The perspective is accurate only from one POV
- Vermeer and others created *perspective boxes* where a picture, when viewed through viewing hole, had correct perspective
- Vermeer on the web:
  - http://www.grand-illusions.com/articles/mystery_in_the_mirror/
  - http://essentialvermeer.20m.com/
  - http://brightbytes.com/cosite/what.html

Perspective Box
Samuel van Hoogstraten
National Gallery, London

Perspective Box of a Dutch Interior
Samuel van Hoogstraten
National Gallery, London

Brunelleschi's Method

- Brunelleschi was reported to have determined the accuracy of his paintings by making a hole in the vanishing point, examining the reflection in a mirror and comparing the line convergence to the real model
- The realism of his paintings are evidence that Brunelleschi had some systematic method for determining perspective projections, although the procedure he used was never documented
- His illusion inspired other artists to explore linear perspective

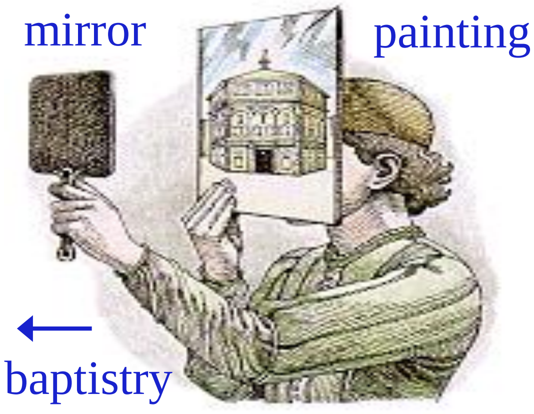
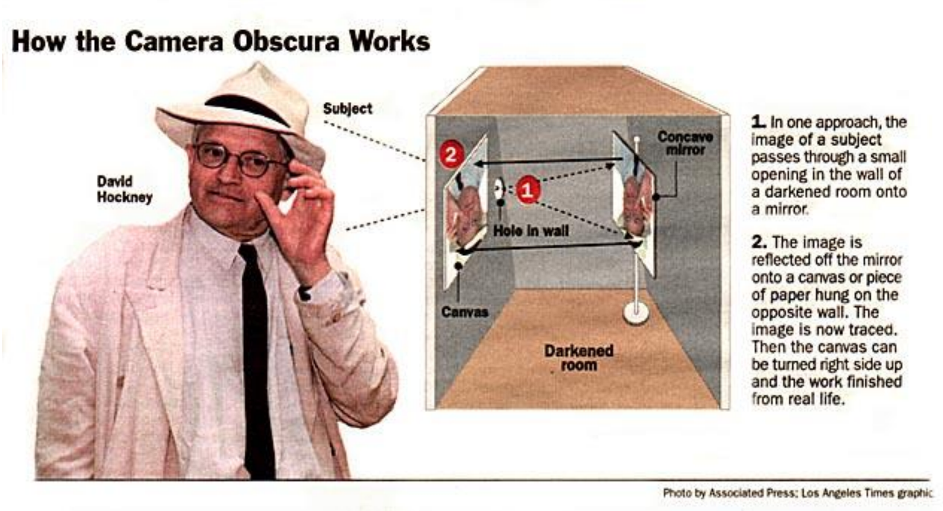


Image credit: COGS011 (Perception, Illusion and Visual Art, William Warren)

A Similar Idea: Camera Obscura

- Artist David Hockney proposed that many Renaissance artists, including Vermeer, might have been aided by *camera obscura*, raising a big controversy
- David Stork, a Stanford optics expert, refuted Hockney's claim in the heated 2001 debate about the subject. Also wrote "Optics and Realism in Renaissance Art" to disprove Hockney's theory
- More recently, in "Tim's Vermeer" Inventor Tim Jenison paints a Vermeer using mirrors
  - Directed by Teller, written by Penn Jillette and Teller



Hockney, D. (2001) *Secret Knowledge: Rediscovering the Lost Techniques of the Old Masters*. New York: Viking Studio.
Stork, D. (2004) Optics and Realism in Renaissance Art. *Scientific American 12*, 52-59.

James Burke: "Masters of Illusion"

- https://www.youtube.com/watch?v=Cp5iqYawEw8

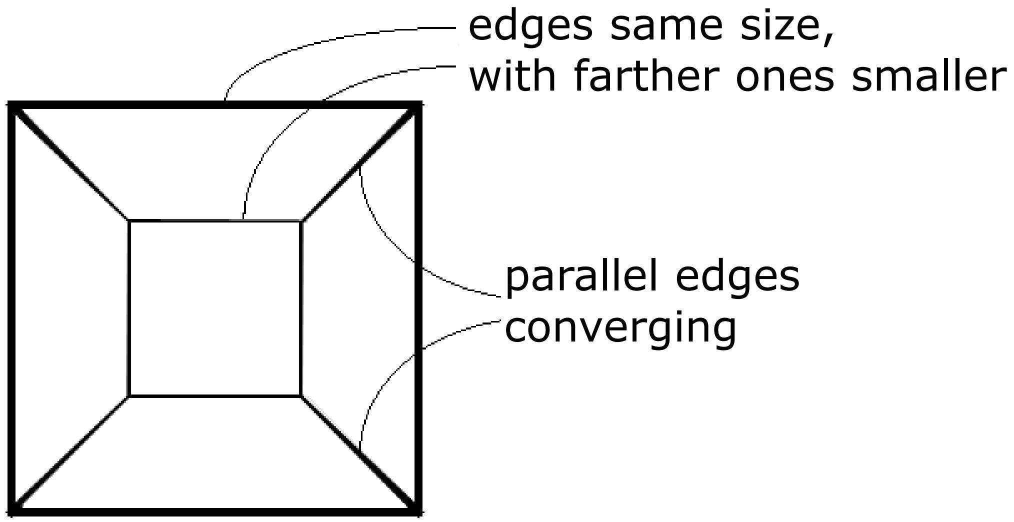
Forced Perspective Art

- http://www.youtube.com/watch?v=uzNVo8NbpPI

Rules of Linear Perspective

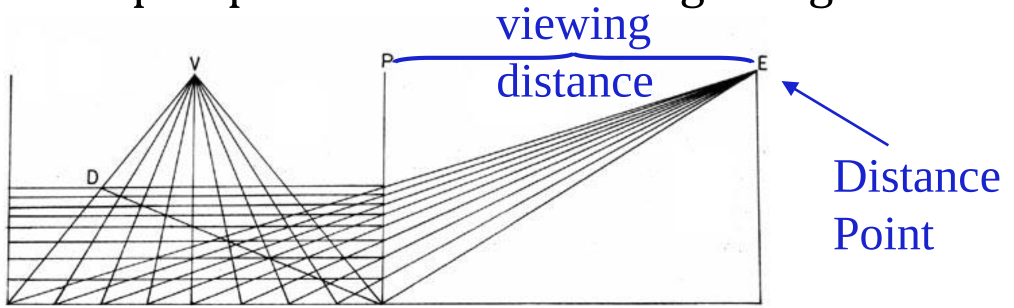
- Driving ideas behind linear perspective:
- Parallel lines converge (in 1, 2, or 3 axes) to *vanishing point*
- Objects farther away are more *foreshortened* (i.e., smaller) than closer ones
- Example: perspective cube

Great depth cue, but so are stereo/binocular disparity, motion parallax, shading and shadowing, etc..

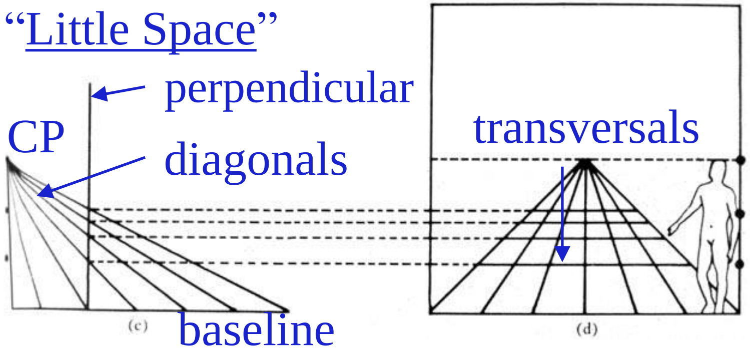


Linear Perspective (Vanishing Points)

- Both Da Vinci and Alberti created accurate geometric ways of incorporating linear perspective into a drawing using the concept vanishing points



Da Vinci's Method

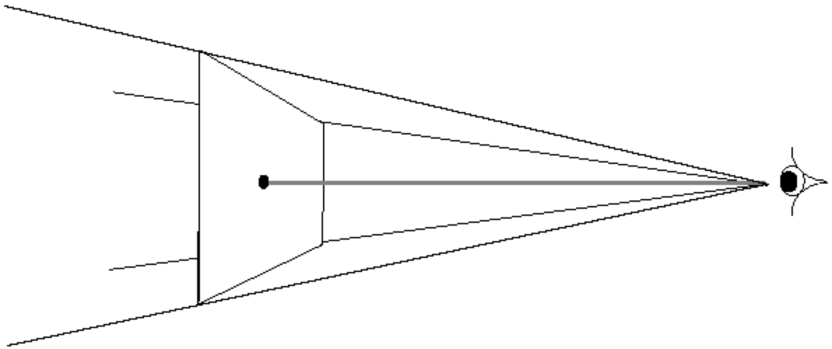


Alberti's Method

Image credit: COGS011 (Perception, Illusion and Visual Art, William Warren)

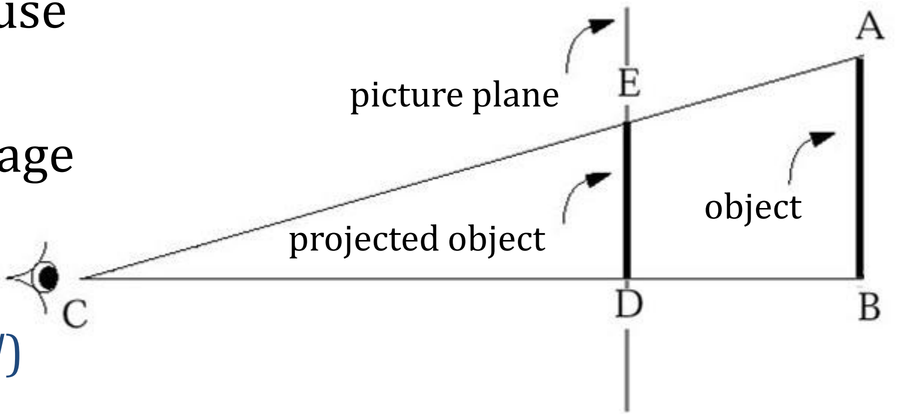
Alberti on Linear Perspective (View Points)

- Published first treatise on perspective, *Della Pittura*, in 1435
- "A painting **[the projection plane]** is the intersection of a visual pyramid **[view volume]** at a given distance, with a fixed center **[center of projection]** and a defined position of light, represented by art with lines and colors on a given surface **[the rendering]**." (Leono Battista Alberti (1404-1472), *On Painting*, pp. 32-33)
- A different way of thinking about perspective from the vanishing point



Triangles and Geometry (1/2)

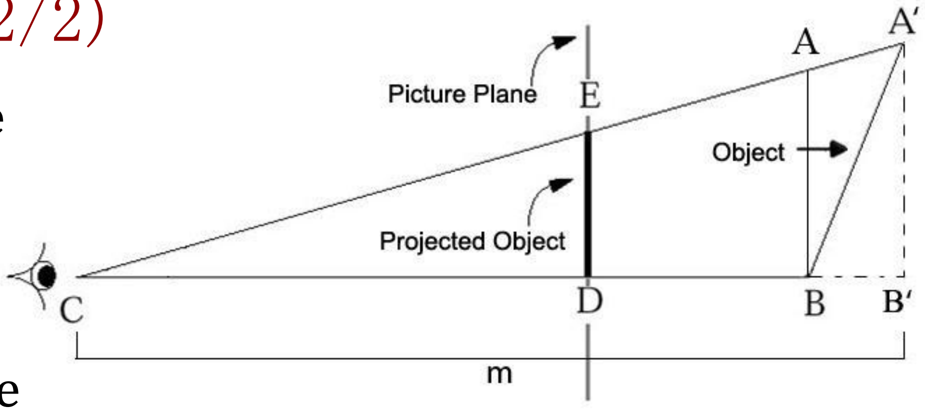
- Idea of "visual pyramid" implies use of geometry of similar triangles
- Easy to project object onto an image plane based on:
  - height of object (||*AB*||)
  - distance from eye to object (||*CB*||)
  - distance from eye to picture (projection) plane (||*CD*||)
  - relationship ||*CB*|| : ||*CD*|| as ||*AB*|| : ||*ED*||; *solve for* ||*ED*||



||*CB*|| : ||*CD*|| as ||*AB*|| : ||*ED*||

Triangles and Geometry (2/2)

- The general case: the object we're considering is not parallel to the picture plane
- Use the projection of ***CA'*** onto the unit vector ***CB/||CB||*** to determine the vector ***CB'***, then use prior similar triangle technique
  - Remember, the dot product of a vector ***a*** with a **unit** vector ***b*** is the projection of ***a*** onto ***b*** (scalar)
- ***||CB'|| : ||CD||*** as ***||A'B'|| : ||ED||***



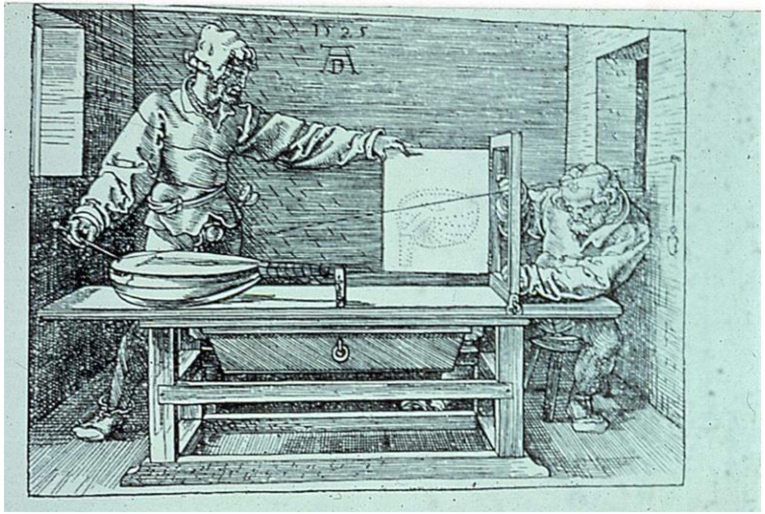
- So if ***U*** is the unit vector in the direction of ***CB*** (i.e. ***U = CB/||CB||***), we get:

$$\boldsymbol{CB'} = \|\boldsymbol{CB'}\| * \boldsymbol{U}$$
$$= (\boldsymbol{CA'} \bullet \boldsymbol{U}) * \boldsymbol{U}$$

- ***U***: direction, ***||CB'||***: magnitude

Dürer Woodcut

- Concept of similar triangles described both geometrically and mechanically in widely read treatise by Albrecht Dürer (1471-1528).
- Refer to chapter 3 of the book for more details.



Albrecht Dürer

Artist Drawing a Lute

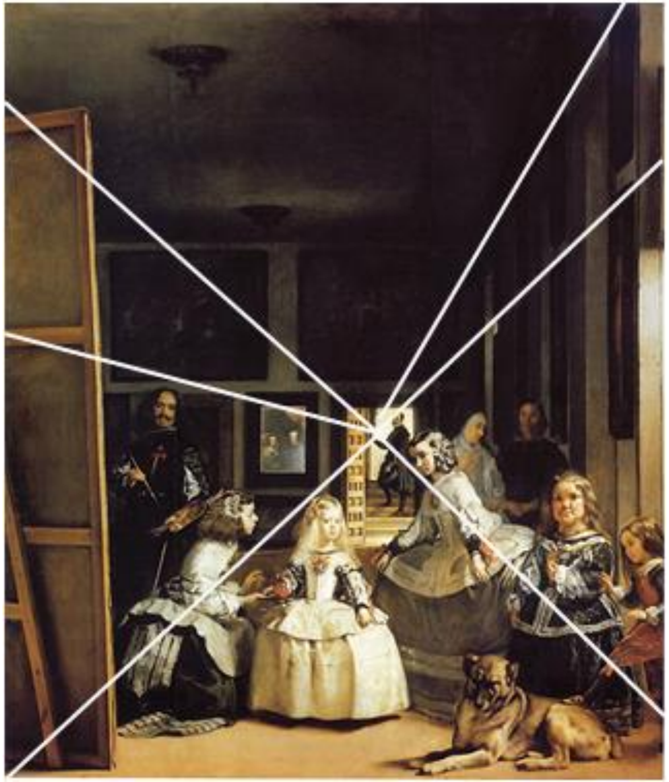
Woodcut from Dürer's work about the Art of Measurement. 'Underweysung der messung', Nurenberg, 1525

Art of Perspective (1/5)

Robert Campin - *The Annunciation Triptych* (ca. 1425)

Art of Perspective (2/5)

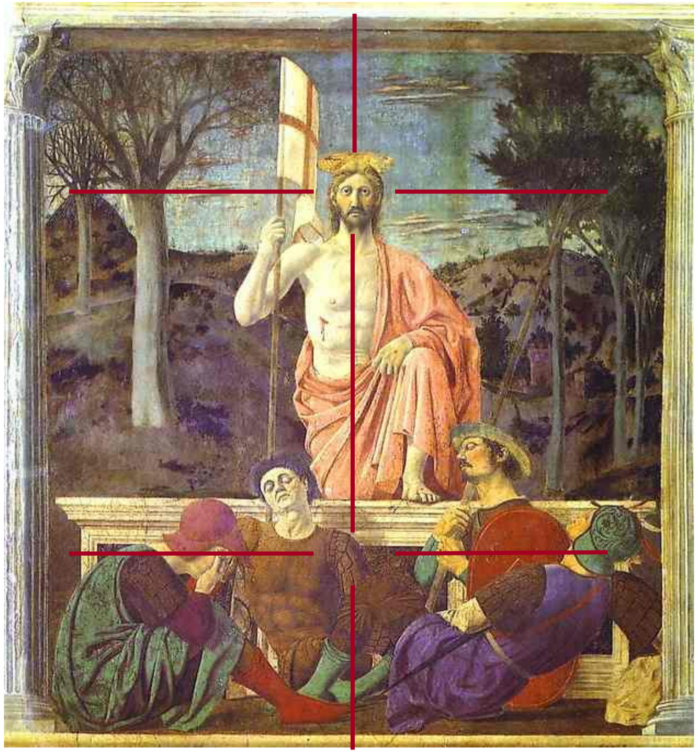
- Point of view influences content and meaning of what is seen
- Are royal couple in mirror about to enter room? Or is their image a reflection of painting on far left?
- Analysis through computer reconstruction of the painted space: royal couple in mirror is reflection from canvas in foreground, not reflection of actual people (Kemp pp. 105-108)



Diego Velázquez, *Las Meninas* (1656)

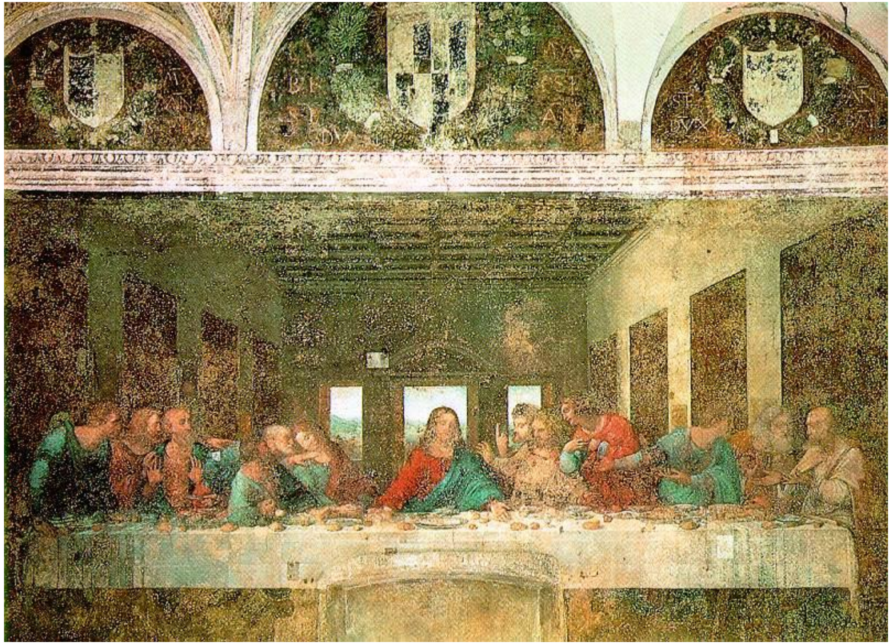
Art of Perspective (3/5)

- Perspective can be used in unnatural ways to control perception
- Use of two viewpoints concentrates viewer's attention alternately on Christ and sarcophagus

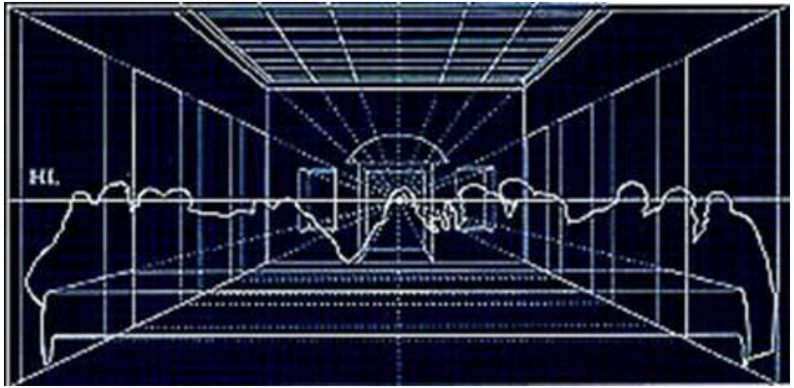


Piero della Francesca, *The Resurrection* (1460)

Art of Perspective (4/5)



Leonardo da Vinci, *The Last Supper* (1495)

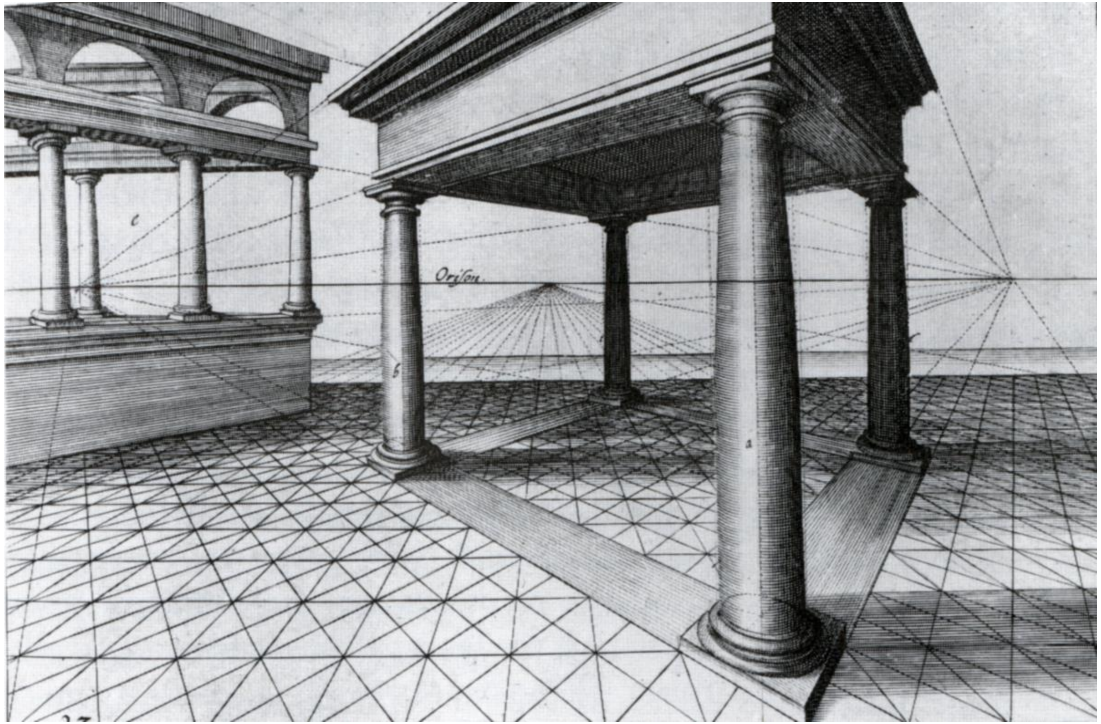


"Mr. King provides a lively account of Leonardo's continual hunt for faces he might sketch, and speculates about the identity of the models (including himself) that he might have used to create the faces of Jesus and the apostles. He also writes about how Leonardo presumably started the painting by hammering a nail into the plaster to mark "the very center of the mural, the point on which all lines and all attention would converge: the face of Christ," and how he used perspective and his knowledge of geometry and architecture to map out the rest of the painting."

Ross King, 'Leonardo and "The Last Supper" '

Art of Perspective (5/5)

Several vanishing points, two point perspective



Vredeman de Vries,

Perspective 23 (1619)

Kemp p.117

http://www.gurari.com/architecture2.php?collection_id=39

Types of Projection

- Different methods of projecting objects to the screen can have a large impact on the viewer's interpretation of the scene
- Here, two objects are displayed in very different ways to highlight certain features

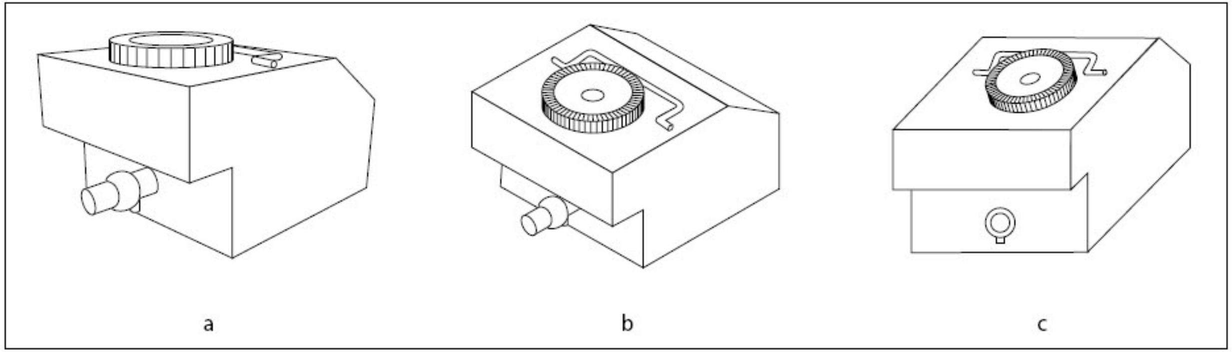
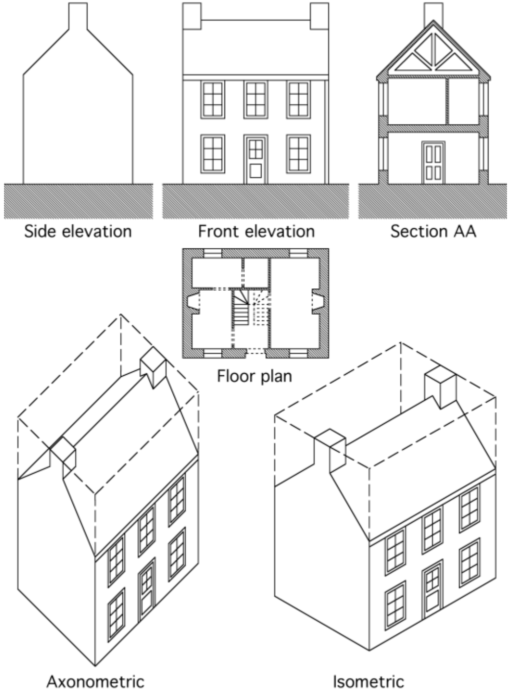
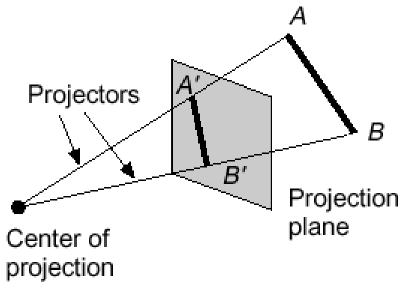


Figure 2-17. (a) Perspective, (b) isometric, and (c) oblique drawings.

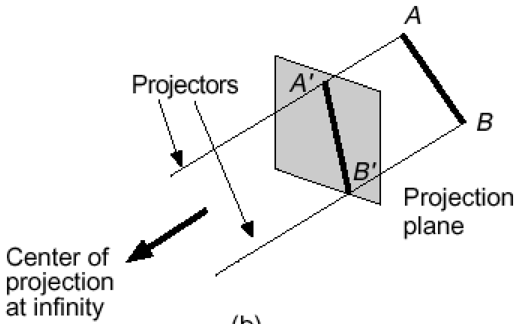


Main Classes of Planar Geometrical Projections

- (a) Perspective: determined by *center of projection* (in our diagrams, the "eye")
  - Simulates what our eyes or a camera sees
- (b) Parallel: determined by *direction of projection* (projectors are parallel—do not converge to "eye" or COP). Alternatively, COP is at ∞
  - Used in engineering and architecture for measurement purposes
- In general, a projection is determined by where you place the projection plane relative to principal axes of object (relative angle and position), and what angle the projectors make with the projection plane

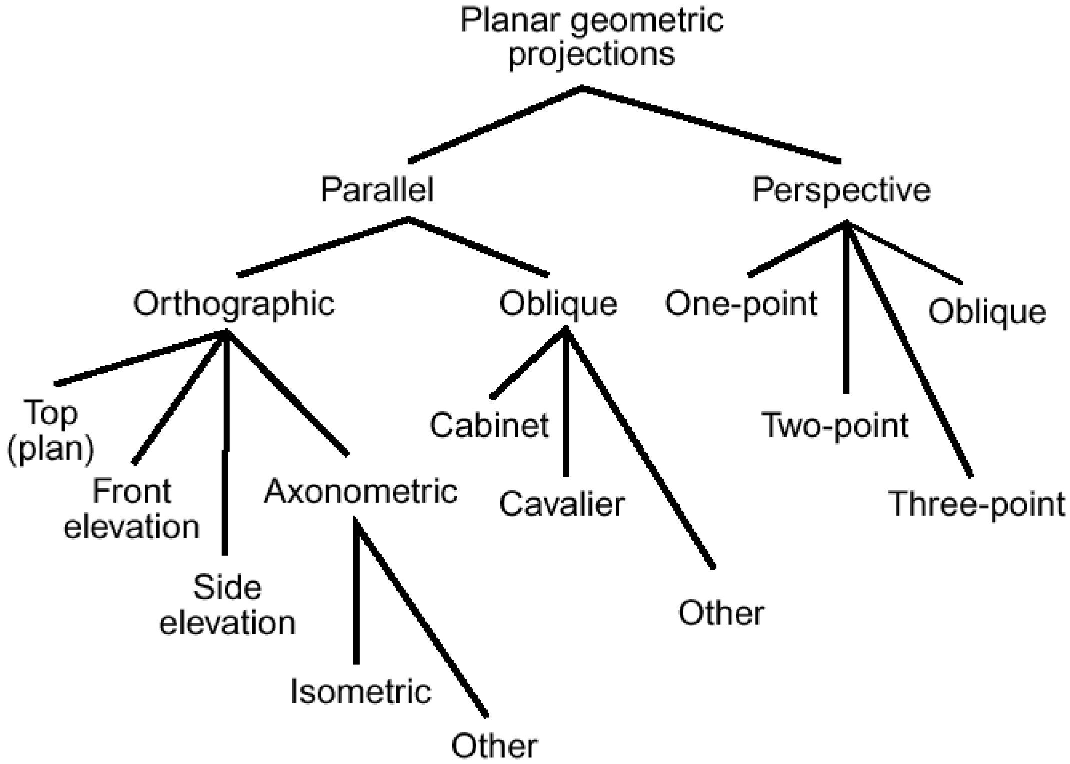


(a)



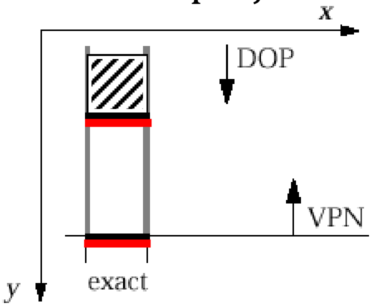
(b)

Logical Relationship Between Types of Projections



Overview of Parallel Projections

- Assume object face of interest lies in principal plane, i.e. parallel to *xy*, *yz*, or *xz* planes. (DOP = direction of projection, VPN = view plane normal)

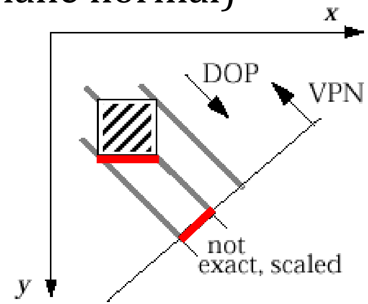


Multiview Orthographic

VPN $\parallel$ a principal axis

DOP $\parallel$ VPN

Shows single face, exact measurements

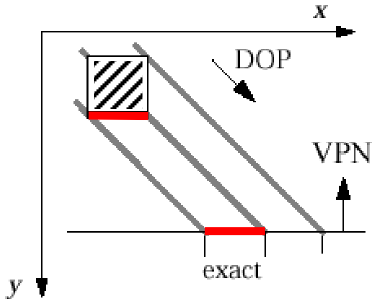


Axonometric

VPN $\nparallel$ a principal axis

DOP $\parallel$ VPN

adjacent faces, none exact, uniformly foreshortened (function of angle between face normal and DOP)



Oblique

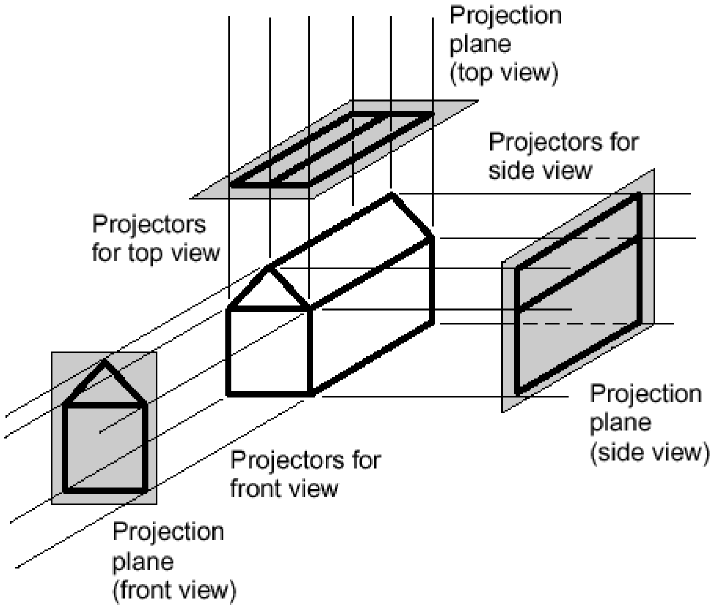
VPN $\parallel$ a principal axis

DOP $\nparallel$ VPN

adjacent faces, one exact, others uniformly foreshortened

Multiview Orthographic (Parallel)

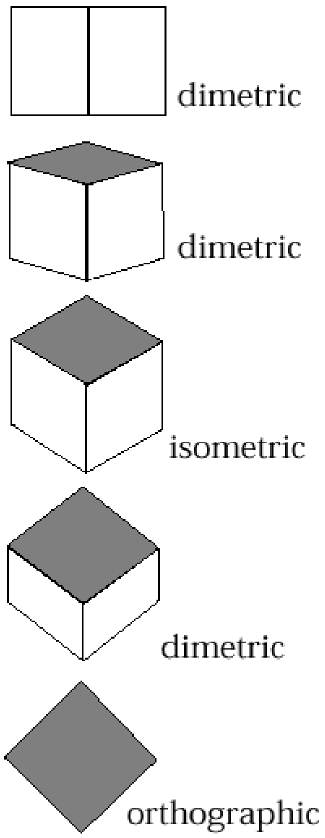
- Used for:
  - Engineering drawings of machines, machine parts
  - Working architectural drawings
- Pros:
  - Accurate measurement possible
  - All views are at same scale
- Cons:
  - Does not provide “realistic” view or sense of 3D form
- Usually need multiple views to get a three-dimensional feeling for object



Axonometric (Parallel)

- Same method as multiview orthographic projections, except projection plane not parallel to any of **coordinate planes**; parallel lines equally foreshortened

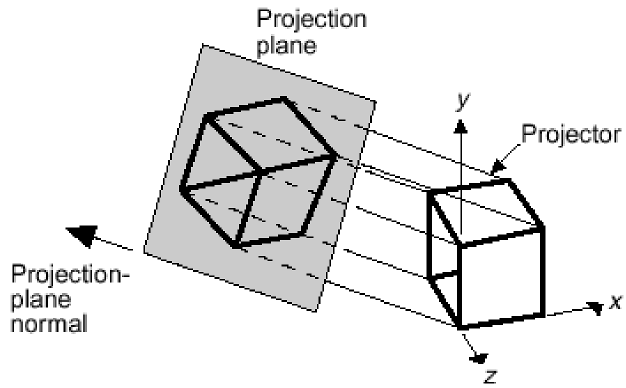
- ***Isometric***: Angles between all three principal axes equal (120º). Same scale ratio applies along each axis
- ***Dimetric:*** Angles between two of the principal axes equal; need two scale ratios
- ***Trimetric:*** Angles different between three principal axes; need three scale ratios



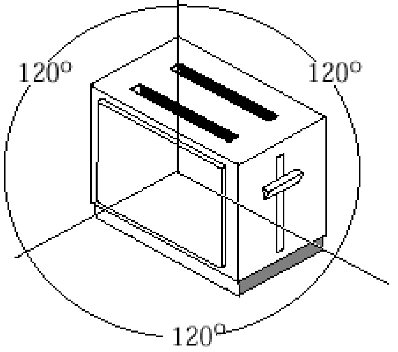
Carlbom Fig.3-8

Isometric Projection

- Used for:
  - Catalogue illustrations
  - Patent office records
  - Furniture design
  - Structural design
  - 3D Modeling in real time (Maya, AutoCad, etc.)
- Pros:
  - Don't need multiple views
  - Illustrates 3D nature of object
  - Measurements can be made to scale along principal axes
- Cons:
  - Lack of foreshortening creates distorted appearance
  - More useful for rectangular than curved shapes



Construction of an isometric projection: projection plane cuts each principal axis by 45°

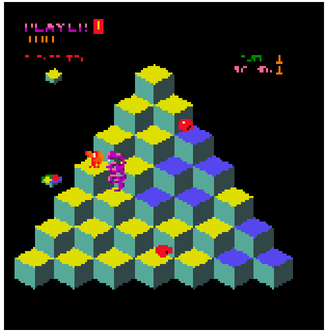


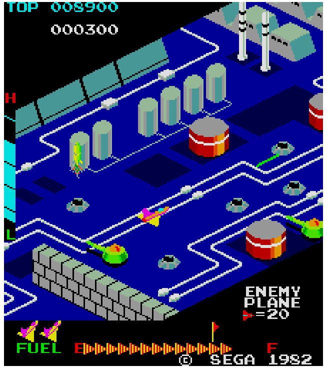
Example

Carlbom Fig.2.2

Axonometric Projection in Games

- Video games have been using isometric projection for ages.
  - It all started in 1982 with *Q*Bert* and *Zaxxon* which were made possible by advances in raster graphics hardware.
- Still in use today when you want to see things in distance as well as things close up (e.g. strategy, simulation games).
  - StarCraft II, Transistor
- While many games technically use axonometric views, the general style is still referred to isometric or, inappropriately, “2.5D”/ “three quarter”.

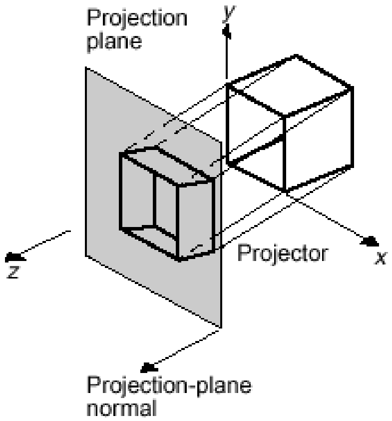




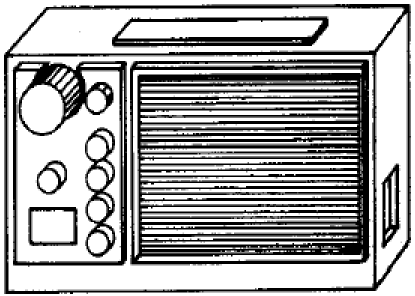
Oblique Projection (Parallel)

- Projectors at oblique angle to projection plane; view cameras have accordion housing, can adjust the angle of the lens relative to the projection plane
- Pros:
  - Can present exact shape of one face of an object (can take accurate measurements): better for elliptical shapes than axonometric projections, better for "mechanical" viewing
  - Lack of perspective foreshortening makes comparison of sizes easier
  - Displays some of object's 3D appearance
- Cons:
  - Objects can look distorted if careful choice not made about position of projection plane (e.g., circles become ellipses)
  - Lack of foreshortening (not realistic looking)

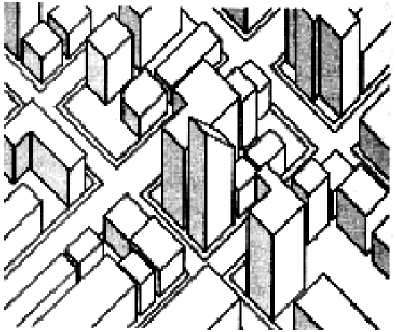
Examples of Oblique Projections



Construction of oblique parallel projection



Front oblique projection of radio
(Carlbom Fig. 2-4)

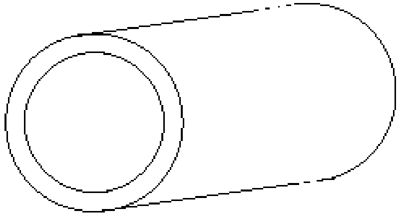


(Carlbom Fig. 2-6)

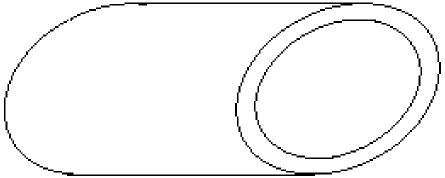
Plan oblique projection of city

Rules for Constructing Oblique Views

- Rules for placing projection plane for oblique views: projection plane should be chosen according to one or several of following:
  - Parallel to most irregular of principal faces, or to one which contains circular or curved surfaces
  - Parallel to longest principal face of object
  - Parallel to face of interest



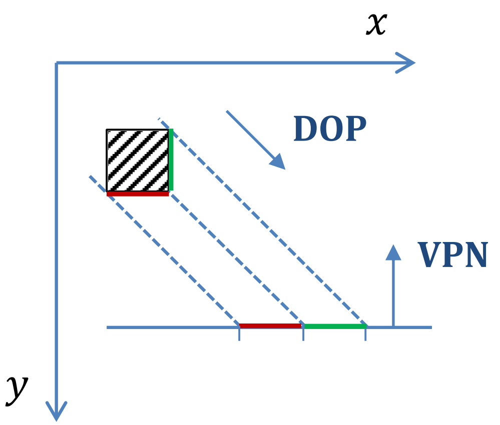
Projection plane parallel to circular face



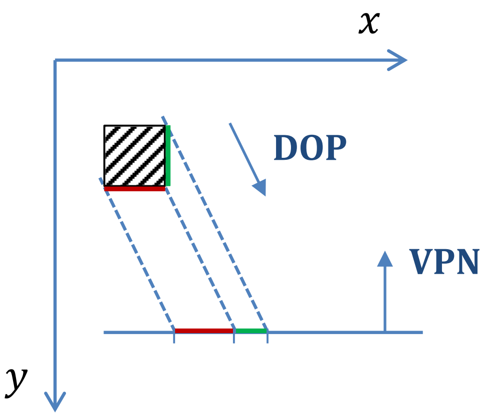
Projection plane not parallel to circular face

Main Types of Oblique Projections

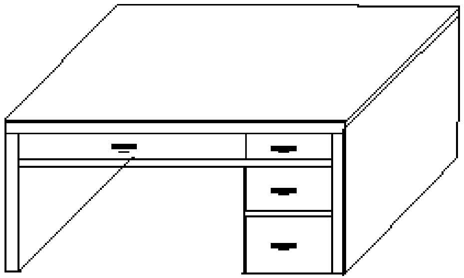
- *Cavalier:* Angle between projectors and projection plane is 45. Perpendicular faces projected at full scale.



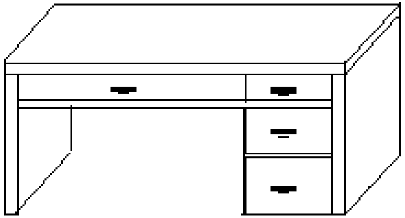
- *Cabinet:* Angle between projectors and projection plane: $tan^{-1}(2) = 63.4^o$. Perpendicular faces projected at 50% scale



A Desk in Parallel

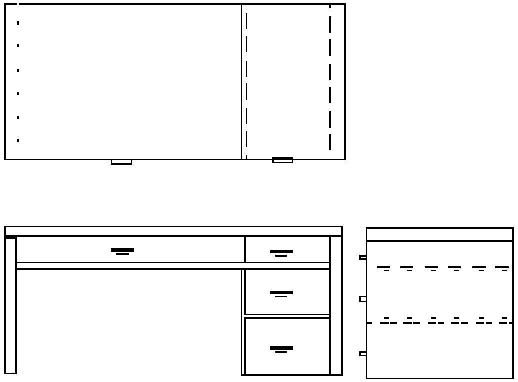


Cavalier



Cabinet

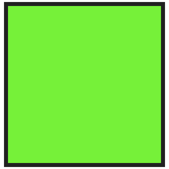
Carlbom Fig. 3-2



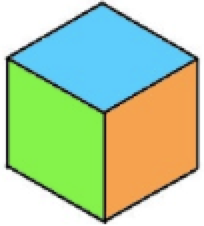
Multiview Orthographic

Summary

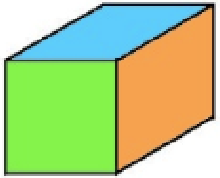
- Three main types of parallel projections:
  - **Orthographic**: projectors orthogonal to projection plane, single face shown



  - **Axonometric**: projection plane rotated relative to principle axes, reveals multiple faces

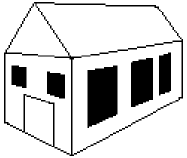


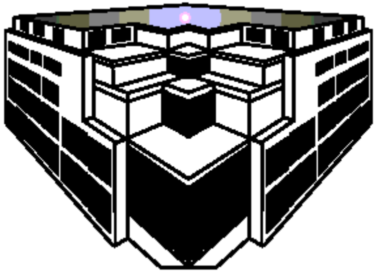
  - **Oblique**: projectors intersect projection plane at oblique angle, revealing multiple faces, often more skewed representation, with a plane of interest undistorted



Perspective Projections

- Used for:
  - Fine Art
  - Human visual system…
- Pros:
  - Gives a realistic view and feeling for 3D form of object
- Cons:
  - Does not preserve shape of object or scale (except where object intersects projection plane)
- Different from a parallel projection because
  - Parallel lines not parallel to the projection plane converge
  - Size of object is diminished with distance
  - Foreshortening is not uniform
- Two understandings: **Vanishing Point and View Point**
- There are also oblique perspective projections (same idea as parallel oblique), we'll see an example next lecture

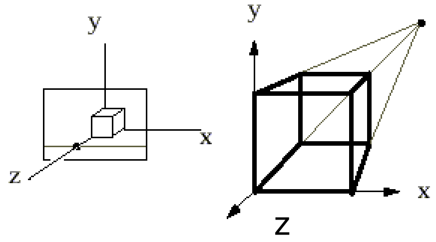




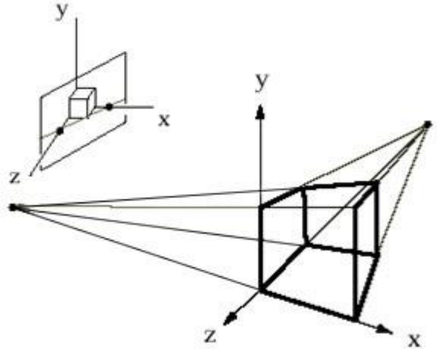
If we were viewing this scene using parallel projection, the tracks would not converge

Vanishing Points (1/2)

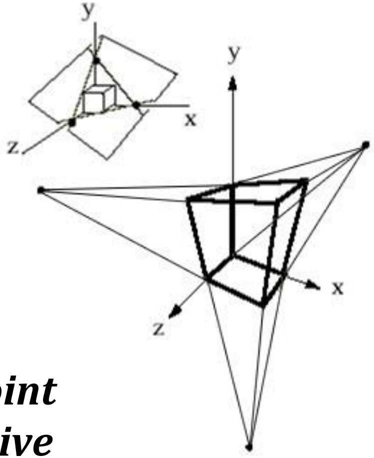
- Lines extending from edges converge to common vanishing point(s)
- For right-angled forms whose face normals are perpendicular to the *x, y, z* coordinate axes, number of vanishing points equals number of principal coordinate axes intersected by projection plane



One Point Perspective
(*z*-axis vanishing point)



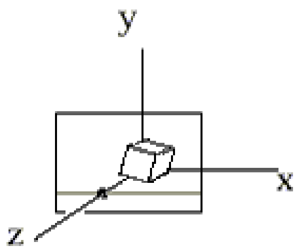
Two Point Perspective
(*z* and *x*-axis vanishing points)



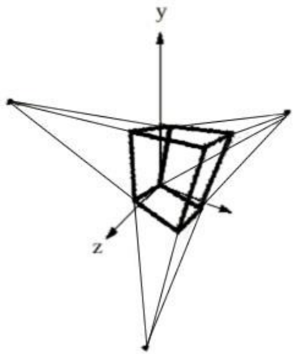
Three Point Perspective
(*z*, *x*, and *y*-axis vanishing points)

Vanishing Points (2/2)

- What happens if same form is turned so its face normals are *not* perpendicular to *x*, *y*, *z* coordinate axes?
  - New viewing situation: cube is rotated, face normals no longer perpendicular to any principal axes.
  - Although projection plane only intersects one axis (z), three vanishing points created.
  - Can still achieve final results identical to previous situation in which projection plane intersected all three axes.



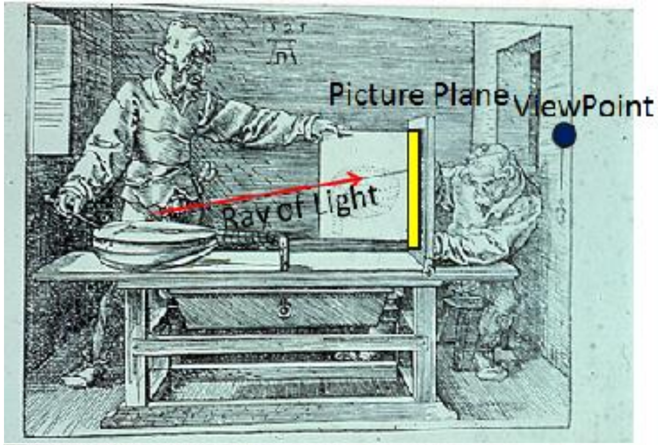
Unprojected cube depicted here with parallel projection

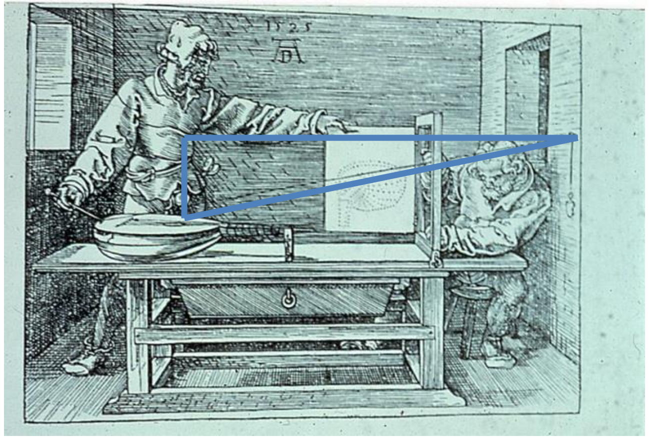


Perspective drawing of the rotated cube

The Single Viewpoint

- Art employs the vanishing point idea while computer graphics uses the view point concept, where your view point is the location of the virtual camera (eye)
- Rays of light reflecting off of an object converge to the point of the viewer's eye
- Lines representing light intersect the picture plane thus allowing points in a scene to be projected along the path of light to the picture plane (basis for ray tracing…stay tuned!)
- Concept of similar triangles described earlier applies here

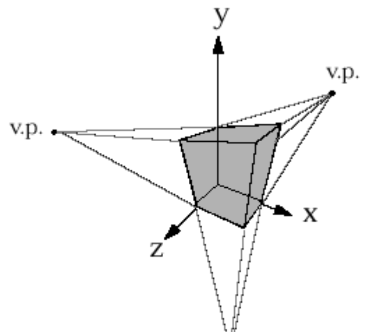




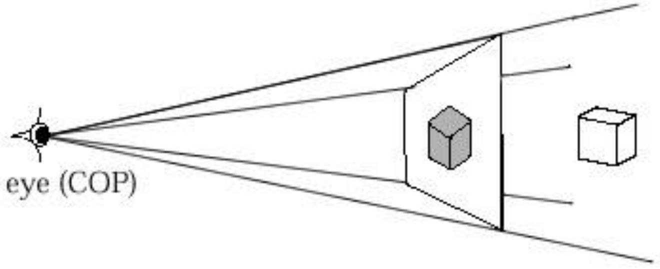
Vanishing Points and the View Point (1/4)

- We've seen two pyramid geometries for understanding perspective projection:

1. Perspective image is result of foreshortening due to convergence of some parallel lines toward vanishing points.

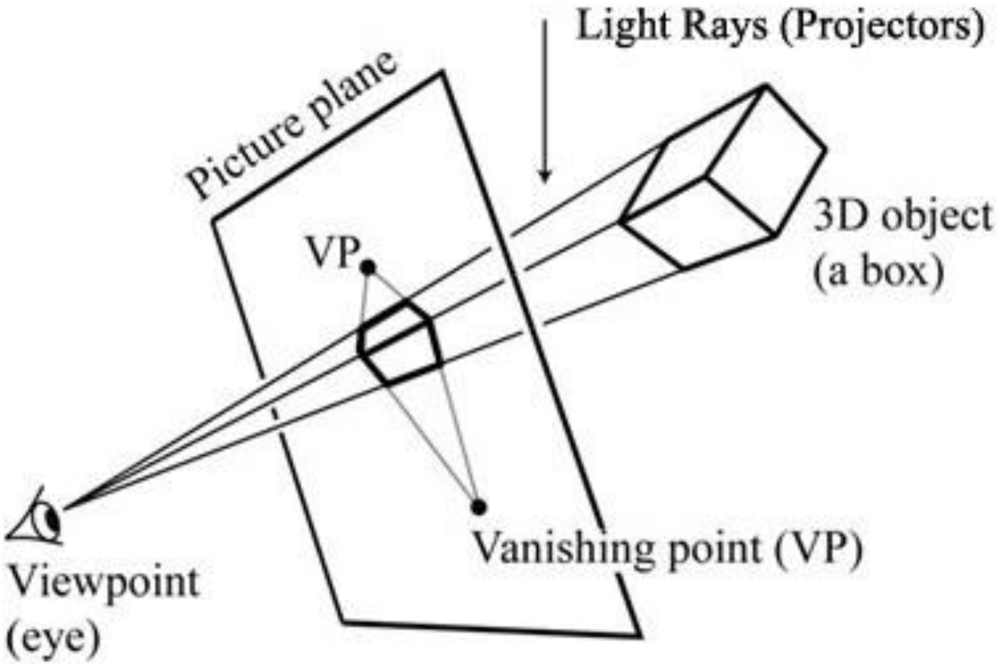


2. Perspective image is intersection of a plane with light rays from object to eye (COP)

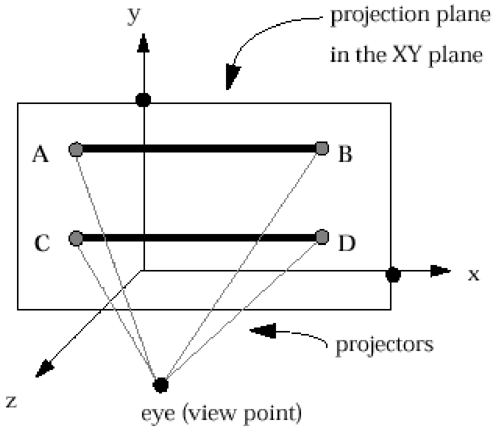


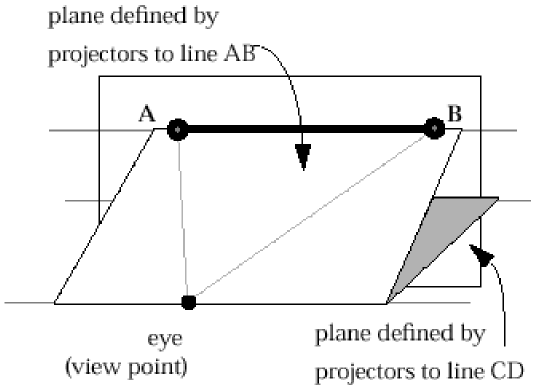
Vanishing Points and the View Point (2/4)

- We can combine the two:



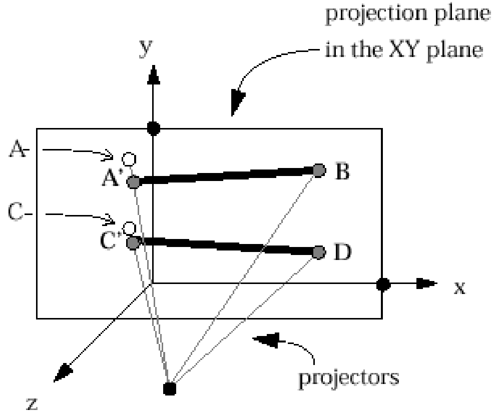
Vanishing Points and the View Point (3/4)



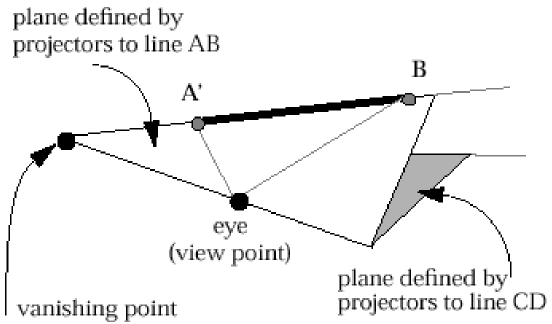


- Project parallel lines *AB*, *CD* on *xy* plane
- Projectors from eye to *AB* and *CD* define two planes, which meet in a line that contains the view point, or eye
- This line **does not** intersect projection plane (*XY*) because it's parallel to it. Therefore, there is no vanishing point

Vanishing Points and the View Point (



- Lines *AB* and *CD* (this time with *A* and *C* behind the projection plane) projected on *xy* plane: *A'B* and *C'D*
- Note: *A'B* not parallel to *C'D*
- Projectors from eye to *A'B* and *C'D* define two planes which meet in a line which contains the view point
- This line **does** intersect projection plane
- Point of intersection is vanishing point



Next Time: Projection in Computer Graphics