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Capacity of Wireless Ad Hoc Networks Using Practical Directional Antennas

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***Abstract*—Capacity is one fundamental problem in wireless Ad hoc networks. Deploying directional antennas to wireless networks can reduce interference among concurrent transmissions and increase spatial reuse, while the technology of multi-channel can separate concurrent transmissions. Therefore, combining these two technologies into one wireless network is capable of great improvement on the network capacity. Recent studies proposed a multi-channel network architecture that equips each wireless node with multiple directional antennas, which is called MC-MDA network. The capacity in MC-MDA network is derived under arbitrary and random placements. However, they only used a simplified directional antenna model. For approaching the more accurate capacity in real scenario, in this paper, we consider a hybrid antenna model, which takes the effect of side lobe and back lobe into account. We derive the capacity upper-bounds of MC-MDA networks in arbitrary and random network with the hybrid model to find the effect of back lobe and side lobe. We show that the network capacity is closely related to s the ratio of the radiuses of side lobe to main lobe. The capacity decreases when s increases. Moreover, we compare the network capacity of MC-MDA using the simplified antenna model with our results. When considering the factor of interference constraint, the capacity gain using the hybrid antenna model is $\sqrt{\frac{\theta^2}{\theta^2+(4\pi^2-\theta^2)s}}$ over the one with the simplified model.**

***Keywords*—Ad hoc networks, capacity, directional antennas, multiple channels, multiple interfaces**

I. Introduction

Capacity computing is one of the most important problems in wireless Ad hoc networks, which provide the theoretical guideline to technical development. Since Gupta and Kumar [1] proposed the method of evaluating the capacity performance on wireless networks, many research studies have been carried out on this topic. They computed the upper and lower bounds of the network capacity based on different technologies and tried to find out the method to improve the throughput.

In recent years, a great number of works reported on the multi-channel networks and the application of directional antennas. A node equipped with multiple network interfaces can proceed multiple simultaneous transmissions and receptions in multiple channels. The analytical results in [2] discovered that the capacity of multi-channel networks have different bounds depending on the ratio of the number of interfaces m to channels c. At the same time, the characteristics of directional antennas in wireless networks were investigated quite intensively. Based on the directions of transmission, different pairs of nodes located in each other's vicinity can communicate simultaneously. This can reduce interference among concurrent transmission and increase spatial reuse of the wireless channel. Nowadays, emphases are placed on the integration of the multi-channel and directional antennas, which is more beneficial. H.-N Dai *et al.* [14] identified the capacity of multi-channel Ad hoc using directional antennas which they defined as MC-MDA network.

However, few studies considered the real antennas radiation pattern when calculating the network capacity. Most of them used a sector model to formulate the transmit area of directional antennas. On the contrary, the effect side lobe and back lobe is inevitable under recent technology. Thus, their results may be not so close to the theoretical value. In order to obtain a more accurate capacity to reality, in this paper, we derive the capacity in MC-MDA network with a hybrid directional antennas model.

In our hybrid model, the side lobe and back lobe is formulated as a circle, while the main lobe is still a sector. A parameter s is used to described the ratio of the radiuses of side lobe to main lobe. After calculating the receive-based interference area with practical antenna model for each combination of antenna modes, we derive the capacity upper-bounds of MC-MDA networks in arbitrary and random network with the receiver-based interference model. To observe more clearly, we also compare the network capacity of MC-MDA using the simplified antenna model with the one using the hybrid model. We show that the network capacity is closely related to s,i.e., the theoretical result exists gap between sector model and hybrid model.

The rest of the paper is organized as follows. In Section II, we do the literature of related work. In Section III, we present the contributions and main results. The hybrid model and interference model are formulated in Section IV. The throughput capacity of arbitrary networks and random networks are derived in Section V and Section VI, respectively. In Section VII, we analyze the performance. We conclude and discuss our future work in Section VI.

II. Related Work

Most of the studies related to this paper focused on the improvement of the network capacity.

Some researches presented theoretical and experimental studies on the capacity of multi-channel wireless networks.

Gupta and Kumar [1] started the analysis of network capacity, which guided Kyasanur and Vaidya [2] to adapt the multi-channel technology to wireless networks where nodes may not have a dedicated interface per channel. [3][4][5][6] proved that multi-channel wireless networks can achieve performance improvement over single-channel networks, though the nodes are equipped with omnidirectional antennas.

Recent results showed that the capacity of wireless networks can be enhanced by using directional antennas instead of omnidirectional antennas. S. Yi *et al.* [7] applied directional antennas to the single-channel wireless Ad-hoc networks, so that transmission and reception of nodes can both be directional. Some other works such as [8][9][10][11][12][13] concentrated on MAC protocols of Ad hoc wireless networks with directional antennas.

The combination of the technology of multi-channel and directional antennas brought us a new domain of investigation on capacity improvement. H.-N Dai *et al.* [14] identified the capacity of multi-channel Ad hoc using directional antennas which they defined as MC-MDA network. In this paper, we also attempt to get the upper bound of MC-MDA network capacity, but under the practical directional antennas model instead.

All of the researches mentioned above used simplified antennas model where the effect of side lobe and back lobe is ignored except [7]. Little attention has been paid on the real antenna model before, which does limit the upper bound of capacity in fact. S. Yi *et al.* [7] defined a hybrid antennas model to achieve a better model of real directional antennas for the first time. They evaluated the capacity of single-channel Ad hoc wireless networks using directional antennas with sender-based interference model. Nevertheless, our work aims to detect the upper bound of MC-MDA network capacity with the hybrid antennas model, so that we can estimate the impact of side lobe and back lobe on the network performance.

III. Contributions and Main Results

A. Contributions

In summary, the contributions of this paper are as follows:

TABLE I
NOTATIONS

s	The ratio of the radius of the side lobe and the main lobe in the hybrid antenna mode.
λ	Each node sends λ bits per second.
n	The number of nodes.
c	The number of available channels.
m	The number of interfaces at each node where each interface is associated with a directional antenna.
W	The total data rate by using all channels. Each channel can support the data rate $\frac{w}{c}$.
θ	The beamwidth of a directional antenna.
$MIN_O(f(n), g(n))$	is equal to $f(n)$, if $f(n) = O(g(n))$.

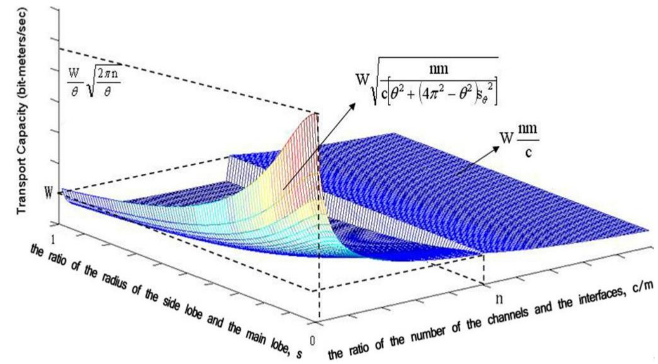

Fig. 1. The upper bound for the transport capacity of arbitrary networks (figure is not to scale), $\theta = 60^o$

- For our best knowledge, it is the first work that addresses the problem of the network capacity with the consideration of radius of the side lobe and back lobe of antennas.
- We calculate the receive-based interference area with practical antenna model for any combination of antenna modes.
- The upper bound on the capacity of multi-channel wireless networks under both arbitrary and random networks with practical antenna model is derived.
- From our results, we verify that the capacity decreases when the radius of side lobe and back lobe s increases. When s is smaller, the impact on the network capacity is greater. For arbitrary networks, the effect of side lobe and back lobe is unignorable.

To get our results, we consider a static multi-channel wireless network containing n nodes. Each node is equipped with the same directional antennas, which have the same beam width θ (generally less than π). We also take the side lobe and back lobe of the antenna into account. We list all the notations in Table I.

B. Results

We conclude the results in this paper as follows.

1) Arbitrary Networks: The transport capacity of the network is that the network transports one bit-meter per second when one bit has been transported a distance of one meter within one second [1]. The transport capacity of arbitrary networks is presented as follows.

- When $\frac{c}{m}$ is $O(n)$, the transport capacity of the network is $O(W\sqrt{\frac{nm}{c(\theta^2+(4\pi^2-\theta^2)s)}})$ bit-meters/sec.
- When $\frac{c}{m}$ is $\Omega(n)$, the transport capacity of the network is $O(\frac{Wnm}{c})$ bit-meters/sec.

We illustrate the upper bound for the throughput capacity of arbitrary networks in Fig.1.

2) Random Networks: The aggregate throughput capacity of the whole network is measured in bits/sec [1]. We provide the upper bound of the throughput capacity of random networks as follow.

- When $\frac{c}{m}$ is $O(\log n)$, the throughput capacity of the network is $O(W\sqrt{\frac{n}{\log n}})$ bits/sec.

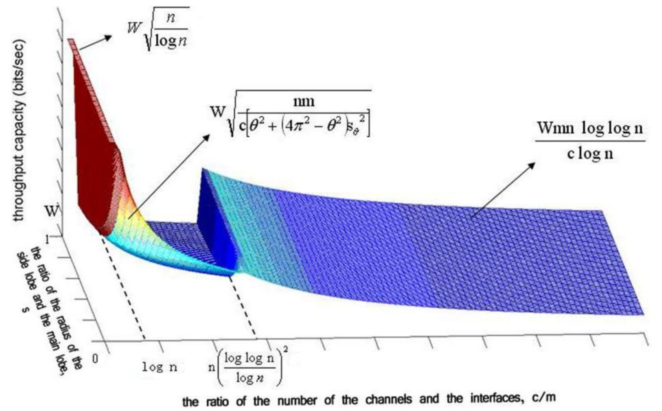

Fig. 2. The upper bound for the transport capacity of random networks (figure is not to scale), $\theta = 60^o$, front view

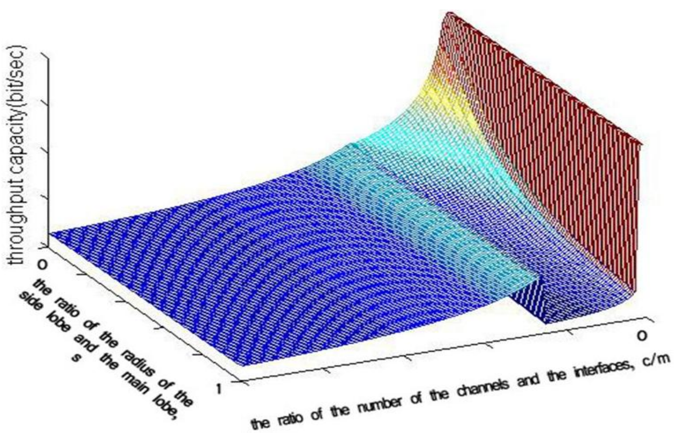

Fig. 3. The upper bound for the transport capacity of random networks (figure is not to scale), $\theta = 60^o$, back view

- When $\frac{c}{m}$ is $\Omega(\log n)$ and also $O(n(\frac{\log\log n}{logn})^2)$, the throughput capacity of the network is $O(W\sqrt{\frac{nm}{c(\theta^2+(4\pi^2-\theta^2)s)}})$ bits/sec.
- When $\frac{c}{m}$ is $\Omega(n(\frac{\log\log n}{logn})^2)$, the throughput capacity of the network is $O(W\sqrt{\frac{n}{\log n}})$ bits/sec.

We illustrate the upper bound for the throughput capacity of random networks in Fig.2 and Fig.3.

Compared to the results in [14], our results verify that the effect of side lobe and back lobe of directional antennas cannot be ignored in capacity evaluating.

IV. Model

A. Antenna Model

Previous works usually study the network capacity based on the simplified directional antenna model, where side lobes and back lobes are ignored. However, we focus on the practical directional antenna so that we use a hybrid antenna model[7] in this paper. According to [8], when the main beamwidth is more than 40^o, the effect of side lobes and back lobes are so considerable that we can take it into account. Moreover, the upper bound of the capacity is constrained by them in reality.

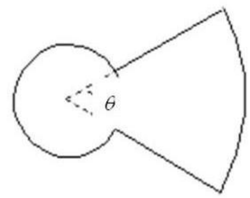

Fig. 4. Antenna model

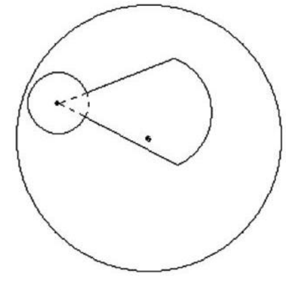

Fig. 5. Interference model

Besides, the goal of this paper is to analyze the difference of the network capacity with and without side lobes and back lobes.

We simulate the real directional antenna with a hybrid antenna model. The beamforming patterns of the model are a mix of omnidirectional and directional antenna models. In the model as shown in Fig.4, the main lobe of beamwidth θ is characterized as a sector. Side lobes and back lobes form a circle. We define parameter s as the ratio of the radius of the circle and the sector, which is generally less than 1. The probability that the antenna beam is pointed to a certain direction is $\frac{\theta}{2\pi}$. Hence a node can receive from transmitters within the area of both the circle and the sector.

B. Interference Model

Since the concurrent transmission can cause interference, the nodes should be separated to avoid collision in the intersection of the transmission zones. Thus, we assume there is an interference area that can ensure the successful transmission.

We use the receiver-based interference model which depends on the protocol model proposed by Gupta and Kumar [1]. The transmission from node to X_i over X_j a channel is successful if for every other node X_k simultaneously transmitting over the same channel, the following condition holds. Since X_k is within the beam of X_j, we get

$$d(k,j) \geq (1+\Delta)d(i,j), \Delta > 0 \quad (1)$$

where $d(k,j)$ is the distance between X_k and X_j , and the guard zone Δ is a parameter that ensures that concurrently transmitting nodes are sufficiently far away from the receiver to prevent excessive interference.

C. Interference Area

Now we come to the calculation of the area of the interference zone with hybrid antenna model for each combination of antenna modes.

For omnidirectional transmission and omnidirectional reception, the interference zone area A_{OO} is πr^2, where r is the radius of the omnidirectional antenna radiation pattern.

For omnidirectional transmission and directional reception shown in Fig.5, the interference zone area A_{OD} is the area of

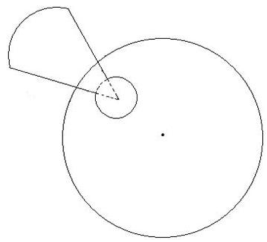

Fig. 6. Interference model for directional transmission and omnidirectional reception

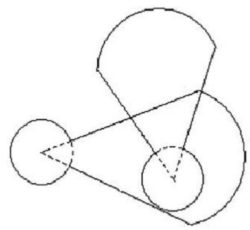

Fig. 7. Interference model for directional transmission and directional reception

the hybrid antenna radiation pattern, which is calculated as:

$$\begin{aligned} A_{OD} &= \pi(sr)^2 + \frac{\theta}{2\pi} \cdot \pi r^2 - \frac{\theta}{2\pi} \cdot \pi(sr)^2 \\ &= \pi r^2 \cdot \frac{\theta + 2\pi s^2 - \theta s^2}{2\pi} \\ &= r^2 \cdot \frac{\theta + s^2(2\pi - \theta)}{2} \end{aligned} \tag{2}$$

where r is the radius of the main lobe of directional antennas.

For directional transmission and omnidirectional reception shown in Fig.6, if the receiver is within the transmission range of other senders, the transmission may be interfered. So we calculate the interference area as:

$$\begin{aligned} A_{DO} &= \pi r^2 \cdot (P\{|T_x - R_v| \le sr\} \\ &\quad + P\{|T_x - R_v| > sr\} \cdot P\{T_x \to R_v\}) \\ &= \pi r^2 \cdot (s^2 + (1 - s^2) \cdot \frac{\theta}{2\pi}) \end{aligned} \tag{3}$$

where $P\{x\}$ is probability of x, and $T_x \to R_v$ is used to describe that the main lobe of T_x pointing to R_v.

For directional transmission and directional reception shown in Fig.7, which is used in our analysis of network capacity in this paper, we divide the reception area into two parts to simplify the calculation: one is a small circle with radius sr and the other is an annulus sector with radius r. Since the beamwidth of all the nodes is θ. The conditional interference area is:

$$\begin{aligned} A_{DD} &= \pi(sr)^2 + \frac{\theta}{2\pi}(\pi r^2 - \pi(sr)^2) \cdot P\{T_x \to R_v\}) \\ &= \pi r^2 (s^2 + \frac{\theta^2}{4\pi^2} - \frac{\theta^2 s^2}{4\pi^2}) \\ &= \pi r^2 (s^2 + (1 - s^2)\frac{\theta^2}{4\pi^2}) \end{aligned} \tag{4}$$

Compared with the sender-based interference areas in [7], the receiver-based results are just the inverse, i.e. the receiver-based A_{DO} is the same as the sender-based A_{OD}.

V. Transport Capacity for Arbitrary Networks

We derive different upper bounds of throughput capacity for the networks using directional antennas from two factors, i.e., the interference and the limited transmission on an interface. The minimum bound of them is an upper bound on the network capacity. As for the hybrid antenna model, the transport capacity for arbitrary networks is closely related to s.

A. Interference Constraint

According to the channel model in [1], we make assumption that there are n nodes arbitrarily located in a disk of unit area on the plane and each node has m interfaces.

The network transports λnT bits over T seconds. If the average distance between the source and destination of a bit is L, then a transport capacity of λnL bit-meters per second is achieved.

We consider any time period of length T. In this time interval, consider a bit b, $1 \le b \le \lambda nT$. We assume that bit b traverses $h(b)$ hops on the path from its source to its destination, where the h^{th} hop traverses a distance of r_b^h. Since the distance traversed by a bit from its source to its destination is at least equal to the length of the line joining the source and the destination, we have

$$\sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} r_b^h \ge \lambda nTL \tag{5}$$

Let us define H to be the total number of hops traversed by all bits in T, i.e. $H = \sum_{b=1}^{\lambda nT} h(b)$. Therefore, the number of bits transmitted by all nodes in T (including bits relayed) is equal to H. There are c channels in the networks where each node has m interfaces, and each interface transmits over a channel with rate $\frac{W}{c}$. Moreover, transporting a bit across one hop requires two interfaces, one each at the transmitting and the receiving nodes. So, the total number of bits that can be transmitted by all nodes over all interfaces is at most $\frac{WTnm}{2c}$. Hence, we have

$$H \le \frac{WTnm}{2c} \tag{6}$$

It is shown in [2] that each hop consumes a disk of radius $\frac{\Delta}{2}$ times the length of the hop around each receiver, i.e. r_b^h. When we use directional antennas at both transmitter and receiver ends, from the Eqn. (4), we can get the conditional interference zone area $\frac{\Delta^2}{4}\pi(r_b^h)^2(s^2 + \frac{\theta^2}{4\pi^2}(1 - s^2))$. Therefore,

$$\begin{aligned} \sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} \frac{\Delta^2}{4}\pi(r_b^h)^2(s^2 + \frac{\theta^2}{4\pi^2}(1 - s^2)) &\le WT \\ \sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} \frac{(r_b^h)^2}{H} &\le \frac{4WT}{\Delta^2 H\pi(s^2 + \frac{\theta^2}{4\pi^2}(1 - s^2))} \end{aligned} \tag{7}$$

Since the expression on the left hand side is convex, we have,

$$(\sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} \frac{r_b^h}{H})^2 \le \sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} \frac{(r_b^h)^2}{H} \tag{8}$$

From Eqn. (7) and Eqn. (8),

$$\sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} r_b^h \le \sqrt{\frac{16\pi WTH}{\Delta^2(\theta^2 + (4\pi^2 - \theta^2)s^2)}} \tag{9}$$

Substituting for H from Eqn. (6), and using Eqn. (5) we have,

$$\lambda nL \leq \frac{W}{\Delta}\sqrt{\frac{8\pi nm}{c(\theta^2+(4\pi^2-\theta^2)s^2)}} \quad (10)$$

B. Interface Constraint

The maximum number of bits that can be transmitted simultaneously over all interfaces is also one of the important elements of capacity constraints. Since there are totally mn interfaces in the network, and each interface can support at most $\frac{W}{c}$ bits/sec. So if all the interfaces are working, the whole network can support $\frac{Wmn}{c}$ bits/sec. Meanwhile, the maximum distance that a bit can travel in the network is $O(1)$ meters. Thus, the bound of the network is $O(\frac{Wmn}{c})$ bit-meters/sec.

Combining the bound of interference constraint with interface constraint, we take $MIN_O(W\sqrt{\frac{nm}{c(\theta^2+(4\pi^2-\theta^2)s^2)}}, \frac{Wmn}{c})$ as the result, which has been described in Section III.B.

VI. Throughout Capacity for Random Networks

In random networks, the nodes are randomly placed and the traffic patterns are randomly chosen. Capacity for random networks is mainly constrained by network connectivity, interference, and destination bottleneck. We derive the three bounds respectively and combine them into the final result.

A. Connectivity Constraint

Successful transmission from any source to destination depends on great connectivity of the network, which limits the network capacity to some extent. As a basic constraint, the random network should ensure to be connected which means that a network is connected *whp*, i.e. with probability no less than $(1-\frac{1}{n})$. Since the nodes are randomly placed, each node should keep the range of the transmission more than a certain value so that the number of transmission is limited. For this constraint, Gupta and Kumar [1] found that the upper bound of a random network using directional antennas at both the transmitter and the receiver is $O(W\sqrt{\frac{n}{\log n}})$ bits/sec. This bound is also applicable to multi-channel networks.

B. Interference Constraint

Since a random multi-channel wireless network is a special kind of arbitrary networks, the upper bound of capacity for arbitrary networks is applicable in random networks. Therefore, multi-channel random networks are also affected by interference of concurrent transmission. The upper bound of the arbitrary networks under interference constraint is $O(W\sqrt{\frac{nm}{c(\theta^2+(4\pi^2-\theta^2)s^2)}})$ bit-meters/sec derived in Section V.A. The average distance between each source and destination in a random network is $\Theta(1)$ meter. Thus, the throughput capacity of random network is $O(W\sqrt{\frac{nm}{c(\theta^2+(4\pi^2-\theta^2)s^2)}})$ bits/sec.

C. Destination Bottleneck Constraint

The maximum data flow that a node can receiver as destination is limited, which constrains the network capacity as well. The capacity of a wireless network is also constrained by the maximum number of bits that can be transmitted simultaneously over all interfaces in the network. Consider a node X which is the destination of the maximum number, $D(n)$, of flows. Each node has m interfaces which can transmit $\frac{W}{c}$ bits/sec, so a node can achieve at most $\frac{Wm}{c}$ bits/sec. According to [2], the maximum number of flows $D(n)$ is $\Theta(\frac{\log n}{\log\log n})$. Hence, the data rate of the flow with the minimum rate is at most $\frac{Wm}{cD(n)}$ bits/sec, which implies that capacity for the random networks is at most $O(\frac{Wmn\log\log n}{c\log n})$ bits/sec.

Considering the bound of connectivity constraint, interference constraint and destination bottleneck constraint, we take $MIN_O(W\sqrt{\frac{n}{\log n}}, W\sqrt{\frac{nm}{c(\theta^2+(4\pi^2-\theta^2)s^2)}}, \frac{Wmn\log\log n}{c\log n})$ as the result, which has been described in Section III.B.

VII. Performance

Using directional antenna in the multi-channel Ad hoc wireless networks can largely increase network connectivity and reduce radio interference, thereby improving the network performance greatly.

We compare the performance of our results with that of [14] in the following two tables, through which we can analyze more clearly.

For arbitrary networks, listed in Table II, when $\frac{c}{m}$ is $O(n)$, the transport capacity for arbitrary networks has a capacity gain $\sqrt{\frac{\theta^2}{\theta^2+(4\pi^2-\theta^2)s}}$ of over an MC-MDA network using simplified antennas model in [14]. The network capacity depends on s the ratio of the radiuses of side lobe to main lobe. Since m is not more than $\frac{2\pi c}{\theta}$, the minimum value of $\frac{c}{m}$ is $\frac{\theta}{2\pi}$, which is smaller than one. Therefore, there always exist some values of $\frac{c}{m}$ that satisfy the condition of $O(n)$,

TABLE II
Comparison of Transport Capacity for Arbitrary Networks

$\frac{c}{m}$	Simplified antennas model	Hybrid antennas model
$O(n)$	$\Theta(\frac{W}{\theta}\sqrt{\frac{nm}{c}})$	$O(W\sqrt{\frac{nm}{c(\theta^2+(4\pi^2-\theta^2)s^2)}})$
$\Omega(n)$	$\Theta(\frac{Wnm}{c})$	$O(\frac{Wnm}{c})$

TABLE III
Comparison of Transport Capacity for Random Networks

$\frac{c}{m}$	Simplified antennas model	Hybrid antennas model
$O(\log(n))$	$\Theta(\frac{W}{\theta^2}\sqrt{\frac{n}{\log n}})$	$O(W\sqrt{\frac{n}{\log n}})$
$\Omega(\log n)$ and $O(n(\frac{\log\log n}{\log n})^2)$	$\Theta(\frac{W}{\theta}\sqrt{\frac{nm}{c}})$	$O(W\sqrt{\frac{nm}{c(\theta^2+(4\pi^2-\theta^2)s^2)}})$
$\Omega(n(\frac{\log\log n}{\log n})^2)$	$\Theta(\frac{Wnm\log\log n}{c\log n})$	$O(\frac{Wnm\log\log n}{c\log n})$

which means that the network capacity must be concerned with s in some condition. The capacity decreases with s increasing. When s tends to zero, the upper bound is just the same as the corresponding condition in [14]. Meantime, the network capacity is at its peak of $\frac{W}{\theta}\sqrt{\frac{2n\pi}{\theta}}$, when m reaches its maximum value $\frac{2\pi}{\theta}$. When s tends to 1, the antenna mode is equal to the omnidirectional antennas. As a result, the capacity reaches minimum. So, our results are more general compared with [14]. When $\frac{c}{m}$ is $\Omega(n)$, the transport capacity of the network is independent with s.

For random networks, listed in Table III, the capacity is concerned with s only when $\frac{c}{m}$ is $\Omega(\log n)$ and also $O(n(\frac{\log\log n}{\log n})^2)$. In this condition, we find the same capacity gain over MC-MDA network that we get in the arbitrary network under interference constraint.

It is indicated that in the range within the bounds under the interference constraint, the impact of side lobes and back lobes on capacity is unignorable. On the other hand, the upper bound is not related to s in the rest of the range, because other constraints are tighter than the interference.

From the results obtained so far, it seem that when the radius of side lobes and back lobes are smaller, the effect on the network capacity is greater. When s is in the different range from zero to one, the speed of decrease of capacity is also different. From Fig.1, Fig.2 and Fig.3, we believe that the capacity decreased faster when s is smaller, considering the capacity is associated with s. In reality, s is usually small so that the theoretical upper bound of capacity proposed before is hardly achieved.

VIII. Conclusion

In this paper, we have derived the upper-bounds of MC-MDA networks in arbitrary and random networks using hybrid antenna model that includes the effect of side lobe and back lobe. We have also calculated the receive-based interference area with practical antenna model for each combination of antenna modes. We compared our results with [14], which focused on the capacity of MC-MDA using the simplified antennas model, and evaluated the effect of sidelobe and back lobe on the network performance. The results indicate that the capacity decreases when s the ratio of the radiuses of side lobe to main lobe increases under the interference constraint. The capacity gain using the hybrid antenna model is $\sqrt{\frac{\theta^2}{\theta^2+(4\pi^2-\theta^2)s}}$ over the one with the simplified model in [14]. The capacity decreases faster when s is larger. For arbitrary networks, the network capacity must be concerned with s in some condition.

In reality, s is usually so large that its impact on the network capacity is worth studying. It should be noted that this preliminary study has not detected the effect of side lobe and back lobe of real antennas on the networks capacity thoroughly. The results can not be used to determine the exact data about the bound of capacity. However, we have got the general rule of it. One of our future works may include considering other factors that eliminate side lobe and back lobe with the capacity calculation under hybrid antennas model. Another future work is to develop practical protocols for wireless Ad hoc networks to approaching the upper bound capacity when transmitting with directional antennas.

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