CS383 Programming Languages

Quiz 2

1. “succ(n) nat” is a judgement or judgement form ?

a. judgement

b. Judgement form

- Whether sth is a natural number
- Judgement= instance,
  particular object or objects having that property
- Judgement form=abstract structure (schema)
  （can’t be ‘more abstract’）

2. Which one of the following is NOT a rule of definition of judgement form a = b + c?

a. $\dfrac{b \to 0 \quad c \to 0 \quad a \to 0}{a = b + c}$

b. $\dfrac{b \to 0 \quad c \to 0 \quad a = b + c}{a \to 0}$

c. $\dfrac{a = b + c}{succ(a) = succ(b) + c}$

d. $\dfrac{a = b + c}{succ(a) = b + succ(c)}$

$$\frac{}{add'\ Z\ Z\ Z}add'Z$$

$$\frac{add'\ n_1\ n_2\ n_3}{add'\ (Sn_1)\ n_2\ (Sn_3)}add'-l$$

$$\frac{add'\ n_1\ n_2\ n_3}{add'\ n_1\ (Sn_2)\ (Sn_3)}add'-r$$

3. A top-down derivation of a judgement starts from ?

a. Proper rules
b. Axioms
c. Premises
d. Conclusion

- Opposite(due to tree representation)

4. Which one of the following is NOT a part of doing an inductive proof?

a. Clearly state the induction hypothesis.
b. Make a proper inductive definition.
c. Clearly state what you are doing induction on.
d. Show one case for each rule in the inductive definition.

Theorem 2: If n nat, then either even n or odd n.

Proof: By induction on the derivation of n nat.

Clearly state what you are doing induction on.

Case: $\dfrac{}{Z \quad nat}\, Z$

even Z (By rule evenZ)

Case: $\dfrac{n \quad nat}{S\, n \quad nat}\, S$

Show one case for **each rule** in the inductive definition.

(1) even n or (2) odd n(By I.H.)

Need to prove: even (S n) or odd (S n)

Clearly state the induction hypothesis.(property you try to prove)

Assuming (1):

odd (S n) (By (1) and rule oddS)

Assuming (2):

even (S n) (By (2) and rule evenS)

QED.

5. If the structure of your induction hypothesis is "***If X or Y then A***", which of the following things is proper for you to assume and prove?

a. Assume X or Y, prove A

b. Assume X and Y, prove A

c. Assume X prove A, or Assume Y prove A

d. Assume X prove A, and Assume Y prove A

6. If the structure of your induction hypothesis is "***If X and Y then A***", which of the following things is proper for you to assume and prove?

a. Assume X or Y, prove A
b. Assume X and Y, prove A
c. Assume X prove A, or Assume Y prove A
d. Assume X prove A, and Assume Y prove A