

Homework 4 - Typed

* If there is any problem, please contact TA.

Name: _____ Student ID: _____ Email: _____

Problem 1. (40 points)

Given the definition of $pred\ n$ (predecessor of n):

$$pred = \lambda n. \lambda f. \lambda x. n (\lambda g. \lambda h. h (g\ f)) (\lambda u. x) (\lambda u. u)$$

Please define following terms using lambda calculus:

1. sub $m\ n$ (subtraction)
2. iszero n
3. leq $m\ n$ (m is less or equal than n)
4. equal $m\ n$
5. factorial n (hint: try to define it using pair)

(You can directly use the definition in the slides and the last homework, like add, tru, etc.)

Problem 2. (20 points)

Prove the **exchange lemma**: If $\Gamma, x : t_1, y : t_2, \Gamma' \vdash e : t$, then $\Gamma, y : t_2, x : t_1, \Gamma' \vdash e : t$. (proof by induction on derivation of $\Gamma, x : t_1, y : t_2, \Gamma' \vdash e : t$).

Problem 3. (20 points)

Prove the **weakening lemma**: If $\Gamma \vdash e : t$ then $\Gamma, x : t' \vdash e : t$ (provided x not in $\text{Dom}(\Gamma)$).

Problem 4. (20 points)

Prove the **substitution lemma**: If $\Gamma, x : t' \vdash e : t$ and $\Gamma \vdash v : t'$ then $\Gamma \vdash e[v/x] : t$.

Remark: You need to use **LaTeX** to write your homework and **convert it into .pdf** file. Please upload both **.tex** and **.pdf** files on **Canvas**.

File name format: **HW_X_Name_StudentID.tex/HW_X_Name_StudentID.pdf**