

40% ca ac ty o ac

o co o

$$p+1 \quad p=5 \quad n=6$$

3) **RAID-5 → RAID-6:** o t t A D-5 to a A D-6
 ety, A a o c y t
 ayo to - o 5 o o t
 A D-6 a at yt t A D-5, A o
 a t o a A D-5 a a at a t to
 a ot a o a a at- a o a at at
 - o a at t co o oc

1 4
 o a at , A o a t t to
 o co t t tota t 4 1
 c t a a o to a a ata oc a tt
 to a A D- of a a ty a a ty o a
 a ay c a o a cat o o a ty ca
 co o o a A D-5 to a A D-6 a ct o o t
 a ay c a oc t ct o o
 t co o oc 5 t t co o co t
 t oc ca ot acc y a y c t t
 o co a co o a ty o to a
 o a t o t a o o a o
 MDS co ca Co 5-6, t t t o a ayo to
 a A D-5 y ya a cat a ty co c
 ot at ct o a t a o ato t a A D-5
 a a A D-6, D o t o a
 a ay o 5-6 a o to o p p a
 c o o 5-6 ot ay
 o a A D-5 S ct o -
 a t o o co t to t o :
 • o o a o - a A D-6 co o 5-6
 to o ot o y t o ty o y ty ca MDS
 co c a o t a to a fic cy a o t a -
 co co co tato a co ty ta o
 fic cy o o ct o a co o t a

a Da o a a ty co A t- a o a a ty co

$$- o o p \quad p=5 \quad n=5$$

o t co o t o a o o
 fi t t o a oac o t at A D

A E
S M S

aa t Sy o	D c t o
m m' m''	o a AD-5 o co o
n	o a AD-6 at co o
v	o t a
p	a
w	o ca a a o o ac t a t a t
i r	o D
j	co Do D o t a o ato
B	tota o ata t oc a a ay y ca y a t co o to a oc
$C_{i,j}$	t at t it o a jt co
$f_1 f_2$ $f_1 < f_2$	t o a o a co t D f_1 a f_2
Σ	o ato o t $\dots \sum_{j=0}^5 C_{i,j} = C_{i,0} \oplus C_{i,1} \oplus \dots \oplus C_{i,5}$
$\langle \rangle$	o a a t t c $\dots \langle i \rangle_p = i o p$
T_e	acc t o a a t t to a t

A E
M A S AM GD E E MDS DES E S MA
A D-5 A A D-6

MDS o	S o a c	o o ty	o o cy
E E DD	o		o
D	M		o
- o		M	M
- o		M	M
- o			o
D	M	M	M
o 5-6		o	

a S - o 6 - o S A co o
co GD o oca co t ct o o
5 a ta-MDS co oca y a a o 6
SD co ca to at co c t a o t o o
a a co co t y t

B. Motivation

AD-5 yt a y to ay a ca
ata c t a ty a ca t t
a at o c a ya t a ya o
a a o a S co a ay ca
co c t a a ot co o o t
c t ca to to at t a ata c t
o o to to AD-6, o o a ata
c t t co t y to AD-6 o t
c atc A at at ay to co ta t AD-
5 a to a yt to a AD-6 o t ya t
to o t a ty a t o c t o t
a ct t a a ty o t y t
A a a o a co o o a AD-5
to a AD-6 t co o fic cy c y:
t o a c c cat t co t to o a ata
t oc co o co ty t tota o
a co o c c y t co o ty
 $\dots AD-5 \rightarrow AD-6 \rightarrow AD-6 o AD-5 \rightarrow AD-6 a$
t c t t ayo to AD-5 a AD-6,
Acco to a t co c a
co o co ty c ot at to o o a o
co to ac fic t AD at o

AD-5 a a AD-6 to t c a ayo t,
• ct o a co o a o t a o
o 5-6 c o co c t AD a-
to t a AD-5 a a AD-6,
• co ct a o a t tat a ay o a o
co o to a o t MDS co a
o t at o 5-6 ac co o fic cy
co a to ot t a oac
t o t a co t a o o : S ct o
c t at o o t AD-6 co a t
ot at o o o o 5-6 c ta
S ct o o co o a o t a c
S ct o S ct o t a t tat a ay o
t co o co t a o a o a oac a y
co c o o S ct o
E A ED A DM A
t ct o c t a co a o
ot at o o ac tat t c o a t
y o t a a

A. Existing Erasure Codes

AD-6 tat o a a o a o a co -
t c o o a ca to t o cat o : MDS
co a o-MDS co Ma D ta c S a a
MDS co AD-6 ca t to t o -
ca : o o ta co a t ca co o o ta co
c -So o o co a c y -So o o
co 6 E E DD co D co a - ot
co 5 at o co to co tc,
t ca co co ta - o 6 - o 5 - o
yc c co - o 55 D co 56 a o
5 y ca y o-MDS co o D co
EA E co o co ME co 5
y a co 6 at - o a o -M 5
t co AD-6 a y a co c

DE 5-6

o 5-6 acc at o t a o ato o a AD-5
to a AD-6 ya o co to to a o a a t
o 5-6 a o to o p p a

A. Data/Parity Layout and Encoding of Code 5-6

o 5-6 co o o a p-1 - o -p-co at t
a tota o p-1 * p t a t ty
o t t at : **data elements horizontal parity**
elements a diagonal parity elements. $C_{i,j}$ $0 \leq i \leq p-1$
 $0 \leq j \leq p-1$ ot t t at t it o a t jt
co a t co co p-1 o a o a
a ty, E c t at co t a at a

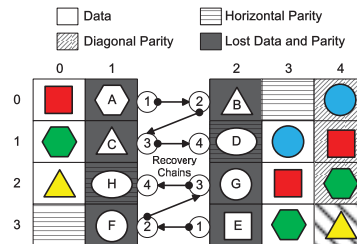
t t a a o a a ty t a t p - ata
 t to co t o t ata t C_{i,f_1} fi t
 o o ta t o o t a o a a ty t,
 A t a o a a ty t o r a t
 y $C_{r,p-1}$ a o E at o a

$$r \langle i \ f_1 - 4 \rangle_p$$

a o E at o t o t ata t ca co

$$C_{i,f_1} = C_{r,p-1} \oplus \sum_{j=0}^{p-2} C_{\langle i+f_1-j \rangle_p, j} \quad 5$$

$$j / f_1 \text{ and } j / \langle i \ f_1 \rangle_p$$



.5. co t cto yt o co, yca A t at o a
 a co 1 a 2: t t yt t o t a t o t o co, y
 ca : ata t A a E, S co t c t ata t acco
 t o t co o co, yca t t y ac t o t a ty
 t D a H, o to co ata a a ty t a :
 A → B → C → D t o t E → F → G → H.

Algorithm 1: co t cto A o t o o 5-6

St 1: t yt o a co : f_1 a f_2 $f_1 < f_2$.
 St 2: Stat co t cto oc a co, t o t ata a a ty

sch $0 \leq f_1 < f_2 \leq p-2$ o

cas I: $f_2 = p-2$ (diagonal parity column is lost)
 St 2-IA: co, t o t ata a o o ta a ty
 t co f_1 a o E at o a
 St 2-IB: co, t o t a o a a ty t
 co f_2 a o E at o

cas II: $f_2 \neq p-2$ (diagonal parity column is saved)
 St 2-IIA: o t t o t a t o t $C_{f_2-f_1-1, f_1}$ a
 $C_{p-1-f_2+f_1, f_2}$ o t co, yca a o E at o 5.
 St 2-IIB: co, t o t ata t t t o
 co, yca

/ o cas s sta t sy ch o us ly:
 cas starting point is $C_{f_2-f_1-1, f_1}$ t at
 o t t o t co o f_2 t
 co, yca a o E at o a
 co t t t o t ata t co o
 f_1 t co, yca a o E at o 5.

u t l at the endpoint of the recovery chain (C_{p-2-f_2, f_2}) .

cas starting point is $C_{p-1-f_2+f_1, f_2}$ t at
 o t t o t co o f_2 t
 co, yca a o E at o a
 co t t t o t ata t co o
 f_1 t co, yca a o E at o 5.

u t l at the endpoint of the recovery chain (C_{p-2-f_1, f_1}) .

st

a o E at o to 5 ca a y co, t o t
 t o a a , t o a o
 a co f_1 a co f_2 $0 \leq f_1 < f_2 \leq p-$
 a o o a o co t cto a o t o
 o 5-6 a o A o t , A co t cto ca
 A o t co, y t o ca o 5.

E. Properties of Code 5-6

1) **Optimal Storage Efficiency:** o t o o o o 5-
 6 co ct o 5-6 t a y a MDS co c
 a o t a to a fic cy 5

2) **Optimal Encoding/Decoding Computational Complexity:** o t co a co at o E at o to
 5 o o 5-6 ac at o a $p-$ o at o , o
 at a $* p-1$ at a t $\frac{* p-1 * p-}{(p-1)*(p-2)}$ $\frac{2p-6}{p-2}$
 o at o o a c

o at o ata t, t o t a to
 co, a y t $p-$ cac at o o ta
 S o n_d t o ata t a n_e t
 tota o t ac t , t o t at
 t at t o t a co a co co tat o a co -
 ty a $\frac{3*n_d-n_e}{n_d}$ $\frac{3*n_d-n_e}{n_e-n_d}$ ct, y , o o
 5-6 n_d $p-$ * $p-1$ a n_e $p * p-1$,
 t o t a co tat o a co ty a $\frac{3*n_d-n_e}{n_d}$ $\frac{2p-6}{p-2}$
 $\frac{3*n_d-n_e}{n_e-n_d}$ $p-$ c a t a a t t a
 o co a co at o , o o 5-6 a
 o t a co co co tat o a co ty.














3) **Optimal Single Write Performance (also known as "Optimal Update Complexity"):** o t co t cto o
 o 5-6 ac ata t ta at to t at o o
 t o a o y t o a ty t, o a t o
 o ata t o 5-6 o yca t o o ficat o
 o a ty t c a o, to o t a 55.

4) **High Reliability on Single Disk Recovery:** a et al.
 5 o o a y co, yca oac a o D co
 c co t o o ta a a o a a t to co,
 a , t ca co ta c t
 co, y t to 1.0% c ca a y MDS
 co, to o a ty 56,

a oac ca a o a o 5-6 to ac
 a ty, o a a o 6
 co 1 a o t a t to a o co, y,
 ca o t c a $C_{2,2}$ a a to co,
 t o a t t a t , y t a oac
 p o 5-6 c to % a o at o
 a , 1 a y ty ca co, y a oac
 t to co, c c a t co, y
 t M a t c a t a ty o t a y,
 , E E S A G MS

A. **Bidirectional Conversions between RAID-5 and RAID-6**
 cto a co o t a A D-5 a a A D-
 6 a ty o A D at o a ta to a
 o 5-6 a o to a A D-5 a a cat a ty
 to t a A D-6 o o t at co o
 a A D-6 to to A D-5, A t o o 5-6 o t



	0	1	2	3	4
0					
1					
2					
3					

a A D-5 o a y to a A D-6, t ty ac fic t
to a fic cy c cac at y

$$\frac{m * m - 1}{m * m - 1} \frac{n - 1 * n -}{n - 1 * n -} < \frac{n -}{n} \quad 6$$

A o m a n 4 t to a
fic cy $\frac{(4-1)*(4-2)}{(4-1)^4+1}$ /1 c a t a MDS
co A D-6 $\frac{n-2}{4}$ 1/ , a a y t
ct o a o o t at t a ty o to a fic cy
a a a o

E S A A S S

t ct o o a y to o t at t
ct o o o 5-6 to co t a A D-5 to A D-6.

A. Methodology

ct t co o a oac c S ct o
a co a o A D-6 co :

1) RAID-5→RAID-6 au RAID-5→RAID-6
→RAID-6: t o a oac a ta o a
co a co o E E D D D a - o
55 o co a o .

2) RAID-5→RAID-6: a oac a o at o
o t tca co a c - o 5 - o
D 56 a o 5-6 o co a o .
o to a o a o co a o ct
t o ayo to A D-5 a t o o
o co a o , o a - o a ca at -
a o a a ty ayo t c ta o co o o
t - a y t c A D-5 to A D-6 t tca
t at a o a A D-5 o p-1 co t to a A D-
6 y a , A t o co o a o t a
o o 5-6 ca a o t o y co o
o oa a co o a a y .
o co a o A D-5→A D- → A D-6 D-
6 ot to co t a A D-5 o to a A D-6 o
6 a A D- D , S a y A D-5→A D-
6 o 5-6 5 a t co o o a A D-5 o
to a A D-6 o 5 ct y o 5-6.

t co o oc t a t ty o o at o
o at : t a a t to at o
a o ty o o at o o a t t o o
t c to a at t fic cy o a o co o .

1) L u al Pa ty Rat o: at o t t o
a a ty oc o a A D-5 a t to
t co o oc a t tota o ata oc
o a a y a t co o to a ata oc a
a t tota o ata oc B.

2) O l Pa ty M g at o Rat o: at o t t
o at a ty oc a t tota o ata oc .

3) / Pa ty a at o Rat o: at o t t
o o y at a ty oc a t tota
o ata oc .

4) xt a S ac Rat o: at o t t t a
ac a t tota ca ac ty o ac .

5) Co utat o Cost: ota o o at o
t co o oc .

6) W t I/Os: ota o t o at o t
co o oc .

) otal I/Os: ota o o at o t
co o oc .

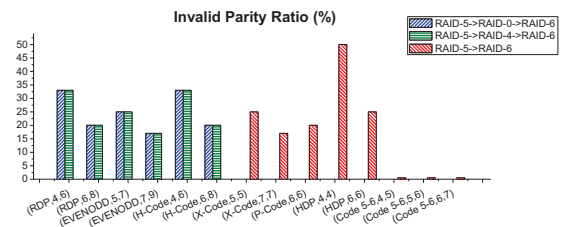
) Co s o : co t o t co o oc .
a t a t o a a o t t to a
ata a ty ta o t co tat o t c
c a co a to t t .

o a o a co o A D-5→A D-6 o 5-
6 5 t a a ty at o o a ty at o o -
ficat o at o a ac at o a o . a ty

at o at o $\frac{4}{3*4}$ 1/ a t t a $\frac{B}{3}$ t
t tota a B $\frac{B}{3}$ $\frac{4B}{3}$, co tat o co t
 $\frac{B}{12} * *4$ $\frac{2B}{3}$ a t co o t $\frac{B*T_e}{3}$.

B. Numerical Results

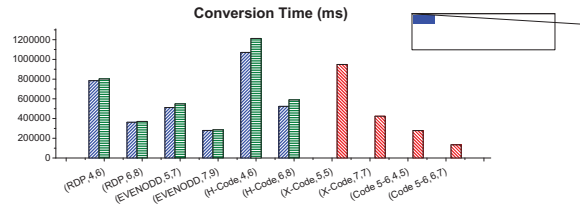
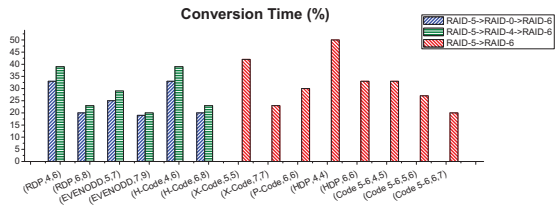
A to o ct o t tat o
t o o A D-6 a o o to a oa
a ac o o MDS co t t
oa a ac o t c t t cat a ty
at at y o y a t , ot t o t oa
a ac o t t a o a ca o ca -
a ty t o t fica ta ct o a ay c a y
co t to A D-6 y ac fic oo o a c o
o a A D-5, t o oa a ac o t
a a to t o t oa a ac o t a t
ct o o y t t o co o t .
t cac at t a a ty at o o a ty -
at o at o a ty at o at o a ta ac at o
a o a oac a o to , t
c a t at ct co o o a A D-5 to a A D-6 a
t a t at o a o t t c c a t o t
co t o o a ty at o c a y to 100.0% a
a ty at o c a y to 0.0% .



o a o o a a ty at o t co o
a oac a o A D-6 co .

S co a o a oac co a t co -
tat o co t t o at o tota o o at o
a co at o t a o to 6, t

t a ac o A D-5→A D-6 - o a o
c .

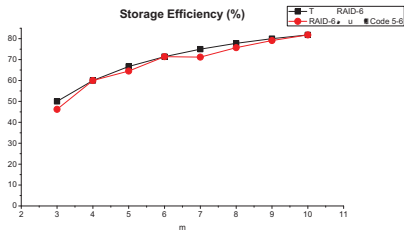


o a o o co o t t co o a -
oac y a o A D-6 co t oa a a c o t
B * T_e o a to 100% .

A E
S EED DE 5-6 G D E E S M A A D-5
A A D-6 E E DES G E S A A ES
E MS E S ME

p	D	E E DD	- o	- o
p = 5	2.24×	1.48×	3.00×	2.71×
p = 7	2.40×	1.83×	3.38×	2.76×

6 t to a fic cy o a A D-6 o 5-6-
t m l $\frac{m*(m-1)}{m*(m+1)+v}$ $\frac{(n-1)*(n-2)}{(n-1)*n+v}$ t
a o a o t a t a t a a
o c t o t to a fic cy t a . % .



Sto fic cy co a o t ty ca A D-6 A D-6
o 5-6.

C. Simulation Results

t ct o t at o t o co o
t y a o co o a oac t at
t y t t c t ac o t at o y
a o co c c a o t t o
at at ca a a y a 5. oc
t to o ty ca y t t o
56 a t tota o ata oc B t to 6
o D a t ato to a at
t co o t c t o a t to a a
t t t ac t a o
a a t c a t at t t a a o p o 5-
6 a t co o t y to .0%. p co
a o to o 5-6 ac .

D. Analysis

o a o t co a to ot co o 5-6 a
at a a ta o co o o a A D-5 to a A D-6
a ty o A D at o a a a o

