Exploring the Multicast Lifetime Capacity of WANETs with Directional Multibeam Antennas

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***Abstract*—We explore the multicast lifetime capacity of energy-limited wireless ad hoc networks using directional multibeam antennas by formulating and solving the corresponding optimization problem. In such networks, each node is equipped with a practical smart antenna array that can be configured to support multiple beams with adjustable orientation and beamwidth. The special case of this optimization problem in networks with single beams have been extensively studied and shown to be NP-hard. In this paper, we provide a globally optimal solution to this problem by developing a general MILP formulation that can apply to various configurable antenna models, many of which are not supported by the existing formulations. The multicast lifetime capacity is then quantitively studied by simulations. The experimental results show that using two-beam antennas can exploit most lifetime capacity of the networks for multicast communications.**

Keywords-wireless ad hoc network; multicast; directional multibeam antenna

I. INTRODUCTION

Wireless ad hoc networks (WANETs) are expected to be deployed in a wide range of civil and military applications where temporary network connectivity is needed. The communicating nodes in these networks might be distributed randomly and are assumed to have packet-forwarding capability in order to communicate with each other over a shared and limited radio channel. Multicasting plays an important role in typical WANETs for applications where close collaboration of network hosts is required to carry out a given task. Because energy supplied by batteries is likely to be scarce and non-renewable resource in theses networks, it is essential to develop efficient multicast protocols that are optimized to maximize the operating lifetime. Recent use of directional antennas in wireless communication has further enabled new approaches for energy saving in WANETs. This is because directional communications can save transmission power by concentrating RF energy where it is needed [1, 2]. Some recently proposed optimal algorithms, *e.g.* in [6-10], with polynomial time complexity show this optimization problem belonging to P problem in networks with omnidirectional antennas. However, the same optimization problem in networks with directional antennas has been proven NP-hard [12]. Some exact and heuristic algorithms can be found in [11] and [1, 2, 10, 12], respectively.

The lifetime of a multicast session is typically defined as the duration of the network operation time until the battery depletion of the first node in the network (*e.g.* [3-12]), although other definitions, like the time before a percentage of live nodes in the network, are possible. All existing solutions are based on a single-beam antenna model. However, the following observation shows that such antenna type may not flexible enough for energy efficient communications. Let us consider a broadcast scenario using the conventional single-beam antenna, in which a transmitting node needs to reach a set of downlink receiving nodes that are far separated in different directions. To exploit the single transmission property, a large beamwidth must be applied to cover all its desired receiving nodes. In this situation, the energy-saving feature of directional antennas has to be somewhat weakened. The extreme case of beam configuration would make directional antenna degenerate into omnidirectional antenna. This may result in the quick energy depletion of the transmitting nodes. On the other hand, the approach using single narrow-beam but multiple transmissions (*i.e.* on the cost of extra bandwidth) is usually impractical due to its lack of scalability in resource-constrained WANETs. The above observation and consideration lead us to look at the multibeam antennas that can mitigate this inefficiency significantly. The advances of the practical techniques on multiple beam antenna [13] and smart antenna array [14] make the above consideration meaningful and thus inspire us to study on this optimization problem further.

In order to explore the lifetime capacity of multicast in WANETs, we shall develop a MILP (mixed integer linear programming) model for this optimization problem under a very general directional multibeam antenna model. To our best knowledge, such formulation is the first work that can accommodate so many various antenna configurations, including the multibeam antennas, that all existing work [11, 15, 16] for similar problems cannot. Many application scenarios can be solved based on this formulation using branch-and-cut or cutting planes techniques. Some numerical results using Integer Programming solver shall be presented for some network examples. The optimal solutions can be used to assess the performance of heuristic algorithms. Through a simulation study, we have also evaluated the tradeoffs between the costs, in terms of the maximum number of beams that the networks should support for each node, and the lifetime improvements by using the multibeam antenna technology. The experimental results show that using two-beam directional antennas can exploit most lifetime capacity of the networks for both multicast and broadcast communications.

The remainder of this paper is organized as follows. Section 2 describes our system model. Section 3 formulates this optimization problem as a MILP model. Simulation studies for the tradeoffs analysis between the equipment costs and performance improvements are given in Section 4. Section 5 summarizes our findings.

II. SYSTEM MODEL

A. Directional Multibeam Antenna

Broadcasting is an inherent characteristic of wireless transmission because signal propagation occurs in all directions if networking units are equipped with omnidirectional antennas. In such a networking environment, a certain transmission power corresponds to an area of coverage, and a single transmission delivers a message to all nodes within the area. It has an obvious energy-saving benefit for broadcast and multicast applications. The directional antenna, on the other hand, permits energy savings by concentrating RF transmission energy to where it is needed. In particular, the directional multibeam antennas provide us an additional dimension to exploit the energy-saving features in WANETs.

The antennas can be broadly classified into two typical categories, *i.e. continuous* and *discrete*, based on the values that antenna beamwidth and antenna orientation (*i.e.* the direction of antenna boresight) can take. The *discrete* type antenna can only take a finite number of values for its beamwidth or orientation, while the *continuous* type antenna can take any value in a given continuous domain for both beamwidth and orientation. The omnidirectional antenna can be seen as a *discrete* type since the beamwidth can only take the value 2π. In the practical antenna design [14], the *continuous* antenna is based on the concept of antenna arrays. The antenna equipped for each node consists of a set of elements whose outputs are combined to give an overall antenna pattern that can be spread into the space and be steered to desired user's directions without physically moving any of the individual elements, by varying the phase and amplitude of the individual element outputs.

We consider a directional multibeam antenna model. The antenna equipped for each node v can support a set of beams $B_v = \{1, 2, \ldots, |B_v|\}$, each beam-$i$ ($i \in B_v$) with a beamwidth θ_v^i between the angles $\theta_{\min}$ and $\theta_{\max}$, and at most K beams active at a time. We apply a beam discretization technique to accommodate our model for both antenna types. It is trivial to enumerate all possible beams for the discrete type. Although there are technically an infinite number of beam configurations for the continuous type, it is sufficient to determine the solution of our optimization problem under the beam configurations such that any antenna beam can achieve an *exact coverage* for a set of nodes that are continuously distributed over the adjacent space. For a general case where a node has λ neighbors, we can derive that the number of possible beams is at most $\lambda(\lambda-1)+1$.

In order to describe the orientation of each beam configuration, a matrix with binary elements c_{vu}^i, $i \in B_v$ and $(v, u) \in A$, is recorded such that by setting $c_{vu}^i = 1$ to represent the neighboring node u can be *potentially* covered by beam-i of node v. Here, the potential coverage means the neighboring node can always be reached if the RF power at the transmitting node is sufficiently high. Note that both the beam discretization and the corresponding orientation matrix can be obtained immediately once the topology of the network is given.

B. Newtork Model

We consider the RF power consumption at a single beam first. The transmitted power required for a beam with a beamwidth θ to support a link between two nodes separated by a distance r is proportional to r^α and the beamwidth θ, where the propagation loss exponent α typically takes on a value between 2 and 4. Although the minimum beamwidth $\theta_{\min}$ could be arbitrarily small in our model, we constrain the transmitted power within the range $p_{\min}$ and $p_{\max}$ for the reality consideration. Without loss of generality, we further assume that all receivers have the same signal detection threshold and such threshold is normalized to one, resulting in the RF transmission power p to be

$$p(r,\theta) \equiv \max(p_{\min}, r^\alpha \cdot \frac{\theta}{2\pi}) \le p_{\max} . \qquad (1)$$

A wireless ad hoc network can be modeled by a simple directed graph $G(N, A)$. It has a finite node set N with $|N| = n$ nodes and an arc set A corresponding to the unidirectional wireless communication links. The RF power required on each arc is given by the function (1) which is dependent on the antenna configuration of the node. A source-initiated multicast is consists of a source node s and a set of destination nodes D. All the nodes $M = \{s\} \cup D$ involved in the multicast form a multicast tree rooted at the node s, *i.e.* a rooted tree T_s, with a tree node set $N(T_s)$ and a tree arc set $A(T_s)$.

Let e_v be the energy supply associated with node v. We use binary symbols $x_v^i = 1$ and $z_{vu}^i = 1$ to denote if beam-i ($i \in B_v$) is active and if node u can be covered by beam-i, respectively. Given a supporting multicast tree T_s with a feasible antenna beam configuration at each node (*i.e.* the values of x_v^i and z_{vu}^i are all properly set), we can obtain the lifetime of node v as

$$t_v = \frac{e_v}{\sum_{i \in B_v} p(r_v^i, \theta_v^i) \cdot x_v^i} , \qquad (2)$$

in which $p(r_v^i, \theta_v^i)$ is the RF power required for beam-i and r_v^i is the maximum distance to reach all links (v, u) separated by r_{vu} within the coverage of beam-i, *i.e.*

$$r_v^i \equiv \max_{u:(v,u) \in A(T_s)} (r_{vu} \cdot z_{vu}^i) . \qquad (3)$$

The lifetime of the multicast communication can thus be expressed as follows.

$$t(T_s) = \min_{v \in N(T_s)} (t_v) = \min_{v \in N(T_s)} \left(\frac{e_v}{\sum_{i \in B_v} p(r_v^i, \theta_v^i) \cdot x_v^i} \right) \qquad (4)$$

III. Optimal Algorithm for Maximum-lifetime Multicast using Multi-beam Antennas

In order to investigate the lifetime capacity of WANETs with directional multibeam antennas, we use mixed integer linear programming to determine an optimal multicast tree and beam configurations for each transmitting nodes in the tree. Such technique has been applied to some similar energy-aware multicast problems in the recent literature [11, 15, 16]. However, those MILP models are based on either omni-directional antenna model [15, 16] or directional single-beam antenna model [11] and none of them can be easily extended for the multibeam antennas. In this section, we investigate the MILP formulation of the maximum-lifetime multicast problem under the general antenna model discussed in Section 2A.

A. Objective Function

Let Ω_M be the family of the trees that includes all multicast nodes in M. Objective of the multicast lifetime maximization problem is to find a multicast tree with the maximum lifetime t^* defined as follows.

$$t^* = \max_{T_s \in \Omega_M} \min_{v \in N(T_s)} t_v = 1 \Big/ \min_{T_s \in \Omega_M} \max_{v \in N(T_s)} \frac{1}{t_v} \quad (5)$$

If we consider the reciprocal of the node lifetime as the weight, *i.e.* $\omega_v = 1/t_v$, the bottleneck weight ω of the multicast tree T_s,

$$\omega \equiv \max_{v \in N(T_s)} \omega_v , \quad (6)$$

can be used as the objective function of the optimization problem. Therefore, the original problem is equivalent to minimize the variable ω. We shall see later that using the objective function in form of (6), instead of (4), can facilitate the formulation of linear constraints.

TABLE I. Description of Optimization Variables

Variable	Description
z_{vu}^i	A binary decision variable which is equal to one if the arc (v, u) in the sub-graph T_s* of G is covered by beam-i of node v, and zero otherwise.
p_v^i	A nonnegative continuous variable which represents the transmission power at beam-i of node v.
x_v^i	A binary variable which is equal to one if beam-i of node v is active in order to cover some/all of its child nodes in the multicast tree T_s*, and zero otherwise.
f_{vu}^d	A nonnegative continuous variable that represents the ficticious flow produced by the multicast initiator s going through arc (v, u) and terminated at a destination node d.

B. Optimization Variables

In order to formulate the problem, we define a set of optimization variables in Table 1. The main idea is to extract a sub-graph T_s^* from the original graph G using the solutions of these optimization variables, such that T_s^* is a multicast tree rooted at node s with maximum lifetime. At each node v in the tree, the beam-i should be active for the multicast if and only if $x_v^i = 1$. The configuration of beam-i is defined by the beamwidth θ_v^i and the transmission power p_v^i , which are obtained from the beam discretization process and the solution of the MILP model, respectively. Finally, T_s^* is a multicast tree of G with maximum lifetime t^*, which is the reciprocal of the optimal solution ω^*.

C. Constraint Formulation

To complete the MILP model with the objective function that is to minimize ω, it remains to construct a set of linear constraints that should define a multicast tree with feasible beam configurations in the context of directional multibeam antennas.

1) Bottleneck Constraints

These constraints guarantee that the objective function ω is equal to the bottleneck weight of the final multicast tree. In other words, its values should be equal to or greater than the lifetime reciprocal of any node in the tree, *i.e.*

$$\omega \ge \sum_{i \in B_v} \frac{p_v^i}{e_v} \quad \forall v \in N .$$

2) Multibeam Constraints

Recall that each antenna can support K beams at most. This can be translated into the following constraints using the optimization variables x_v^i .

$$\sum_{i \in B_v} x_v^i \le K \quad \forall v \in N$$

Note that x_v^i is not an independent variable. In fact, binary variables x_v^i and z_{vu}^i are in a close relationship such that beam-i ($i \in B_v$) of node v is active if and only if there exist at least one link (v, u) which must be covered by this beam in the final beam configuration, *i.e.*

$$x_v^i = 1 \Leftrightarrow \exists (v,u) \in A, z_{vu}^i = 1 \quad \forall v \in N, \forall i \in B_v . \quad (7)$$

This statement is equivalent to the following linear constraints.

$$x_v^i \le \sum_{u:(v,u) \in A} z_{vu}^i \quad \forall v \in N, \forall i \in B_v ,$$

$$(n-1) \cdot x_v^i \ge \sum_{u:(v,u) \in A} z_{vu}^i \quad \forall v \in N, \forall i \in B_v .$$

3) Power Constraints

The transmission power p_v^i of the node v using beam-i must satisfy

$$p_v^i = p(r_v^i, \theta_v^i) \cdot x_v^i . \quad (8)$$

In the following, we shall linearize (8) to a set of constraints that involve the variables defined in Table 1 only. First of all, the transmitted power p_v^i for any active beam should be between the minimum and maximum power levels, while it must be zero for non-active beams. These requirements can be satisfied by the following constraints.

$$p_v^i \ge p_{\min} \cdot x_v^i \quad \forall v \in N, \forall i \in B_v ,$$

$$p_v^i \le p_{\max} \cdot x_v^i \quad \forall v \in N, \forall i \in B_v .$$

min: ω		(11)
s.t. $\omega \geq \sum_{i\in B_v} p_v^i / e_v$	$\forall v \in N$	(12)
$\sum_{i\in B_v} x_v^i \leq K$	$\forall v \in N$	(13)
$\sum_{u:(v,u)\in A} z_{vu}^i / (n-1) \leq x_v^i \leq \sum_{u:(v,u)\in A} z_{vu}^i$	$\forall v \in N, \forall i \in B_v$	(14)
$p_{\min} \cdot x_v^i \leq p_v^i \leq p_{\max} \cdot x_v^i$	$\forall v \in N, \forall i \in B_v$	(15)
$p_v^i \geq z_{vu}^i \cdot r_{vu}^\alpha \cdot \theta_v^i / 2\pi$	$\forall (v, u) \in A, \forall i \in B_v$	(16)
$z_{vu}^i \leq c_{vu}^i$	$\forall (v, u) \in A, \forall i \in B_v$	(17)
$\sum_{u\in N\setminus\{v\}} f_{uv}^d - \sum_{u\in N\setminus\{v\}} f_{vu}^d = b_v^d$	$\forall v \in N, \forall d \in D$	(18)
$f_{vu}^d \leq \sum_{i\in B_v} z_{vu}^i$	$\forall (v, u) \in A, \forall d \in D$	(19)
$x_v^i, z_{vu}^i \in \{0,1\}$	$\forall (v, u) \in N, \forall i \in B_v$	(20)

Figure 1. MILP model for the maximum-lifetime multicast problem

We then consider the more difficult requirements that are defined by Equations (8), (1) and (3). Suppose that the two constraints above are already satisfied and the arc (v, u) is chosen as a tree arc of T_s^*. The additional constraints must allow the power level p_v^i being sufficiently high to reach node u if beam-i is used to cover arc (v, u). We construct these requirements by the linear constraints below.

$$p_v^i \geq \frac{\theta_v^i}{2\pi} \cdot r_{vu}^\alpha \cdot z_{vu}^i, \forall (v,u) \in A, \forall i \in B_v$$

4) Orientation Constraints

These constraints require that for any (v, u) in the final tree T_s^*, node u must be located within the beam orientation of node v. By investigating the following necessary condition involving variables z_{vu}^i and constants c_{vu}^i :

$$z_{vu}^i = 1 \Rightarrow c_{vu}^i = 1 \quad \forall (v,u) \in A, \forall i \in B_v \,, \tag{9}$$

we can provide the equivalent constraints as follows.

$$z_{vu}^i \leq c_{vu}^i \quad \forall (v,u) \in A, \forall i \in B_v$$

5) Flow Conservation Constraints

The formulation using single-commodity flow can be found in the existing work, *e.g.* in [11, 15]. A more standard and elegant formulation using multiple-commodity flow f_{vu}^d can also achieve the flow conservation constraints:

$$\sum_{u\in N\setminus\{v\}} f_{uv}^d - \sum_{u\in N\setminus\{v\}} f_{vu}^d = b_v^d \quad \forall v \in N, \forall d \in D \,,$$

in which the demand vector b_v^d is defined as

$$b_v^d \equiv \begin{cases} -1 & v = s \\ 0 & v \in N \setminus \{s,d\} \\ 1 & v = d \end{cases}. \tag{10}$$

Note that these variables f_{vu}^d only represent *fictitious* flow, instead of meaning multiple copies of same data to be sent to a set of children from a relay node. In fact, the constraints below allow the extracted sub-graph T_s^* preserving the tree structure, *i.e.* a directed acyclic graph spanning all destination nodes, in which a source node has no incoming arcs and each other node v has exactly one incoming arc.

$$f_{vu}^d \leq \sum_{i\in B_v} z_{vu}^i \quad \forall (v,u) \in A, \forall d \in D$$

Our derivations on the linear constraints can now allow us to complete the MILP formulation as summarized in Fig. 1 for the maximum-lifetime multicast problem.

IV. PERFORMANCE EVALUATION

In this section, we would like to explore the multicast lifetime capacity of WANETs using multi-beam directional antennas by evaluating the optimal algorithm based on MILP given in Fig. 1, which shall be solved by CPLEX [17]. In order to guide the practical deployment of multi-beam technology in WANETs, we shall also evaluate the tradeoffs between the hardware costs, in terms of maximum number of beams that the networks can support for each node, and the lifetime improvement by using multiple beam antennas.

TABLE II. PARAMETER VALUES FOR SIMULATION

Parameters	Values
n	20
m	10 and 20
$\theta_{\min}$	15°, 30°, 60°, 90°, 180° and 360°
($p_{\max}$, $p_{\min}$)	(10, 0.1)
($e_{\max}$, $e_{\min}$)	(500, 10)
K	1, 2 and 3
α	2

TABLE III. PERFORMANCE IMPROVEMENTS USING MULTIPLE BEAM ANTENNAS IN 20-NODE NETWORKS

K	*m*	θ_{min} = 15°	θ_{min} = 30°	θ_{min} = 60°	θ_{min} =90°	θ_{min} = 360°
2	10	(2.45, 0.750)	(1.67, 0.231)	(1.20, 0.077)	(1.06, 0.011)	(1.00, 0.000)
	20	(2.61, 0.905)	(1.71, 0.461)	(1.22, 0.056)	(1.06, 0.009)	(1.00, 0.000)
3	10	(2.66, 0.893)	(1.70, 0.366)	(1.24, 0.068)	(1.06, 0.011)	(1.00, 0.000)
	20	(2.81, 0.878)	(1.79, 0.410)	(1.22, 0.056)	(1.06, 0.009)	(1.00, 0.000)

In our network settings, a number of nodes are randomly generated within a square region 10×10. The energy supply at each node is uniform distributed across e_{min} and e_{max}. The maximum number of active beams includes K = 1, 2 and 3, which are sufficient to learn the performance trend as we shall see later. The simulation parameters are summarized in Table 2, in which the units of parameters are all consistent with each other. We randomly generated 50 network examples for each network setting and we present here the average over those examples.

Let t_K denotes the tree lifetime obtained when the maximum beam number is set as K. We use the metric t_K/t_1 to facilitate the comparisons of performance improvements under various values of K to the traditional case using single-beam antennas over a wide range of network examples. Table 3 summarizes the simulation results on 20-node network examples under various multicast group sizes and minimal antenna beamwidths. We list mean and variance of the performance metrics in a format (mean, variance) for each K = 2 and 3 in the table. We observe that, for all the cases, using two-beam directional antennas can improve the multicast lifetime significantly, while the additional improvements are very marginal when increasing the antenna array elements to support more beams. In particular, for smart array antennas that can tune narrow beams, *e.g.* θ_{min} = 15°, the communication time can be averagely doubled using two-beam antennas compared to the traditional single-beam antennas.

V. CONCLUSION

In this paper, we have systematically studied the fundamental capacity issue associated with multicast lifetime in WANETs with directional multibeam antennas. Our new proposed MILP formulation provides exact algorithm for small sized networks. To our best knowledge, such formulation is the first work that can accommodate both continuous and discrete multibeam antenna types. Many application scenarios can be solved efficiently based on this formulation using branch-and-cut or cutting planes techniques. We have also found from the experiments that using two-beam directional antennas can exploit most lifetime capacity of the networks for multicast communications.

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