

Lab11-Reducibility

CS363-Computability Theory, Xiaofeng Gao, Spring 2016

* Please upload your assignment to FTP or submit a paper version on the next class

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1. Recall that $A \otimes B = \{\pi(a, b) \mid a \in A, b \in B\}$. Prove the following statements.
 - (a) For any sets A, B , if $B \neq \emptyset$ then $A \leq_m A \otimes B$.
 - (b) $A \equiv_m A \otimes \mathbb{N}$ for any set A ,
 - (c) $A \equiv_m A \otimes B$ if $A \neq \mathbb{N}$ and B is a non-empty recursive set.
2. Let \mathbf{a}, \mathbf{b} be m-degrees.
 - (a) Show that the least upper bound of \mathbf{a}, \mathbf{b} is uniquely determined; denote this by $\mathbf{a} \cup \mathbf{b}$;
 - (b) Show that if $\mathbf{a} \leq_m \mathbf{b}$ then $\mathbf{a} \cup \mathbf{b} = \mathbf{b}$;
 - (c) Show that if \mathbf{a}, \mathbf{b} are r.e., then so is $\mathbf{a} \cup \mathbf{b}$;
 - (d) Let $A \in \mathbf{a}$ and let \mathbf{a}^* denote $d_m(\overline{A})$. (Check that \mathbf{a}^* is independent of the choice of $A \in \mathbf{a}$.) Show that $(\mathbf{a} \cup \mathbf{a}^*)^* = \mathbf{a} \cup \mathbf{a}^*$.
3. Show that the following sets all belong to the same m-degree:
 - (a) $\{x \mid \phi_x = 0\}$,
 - (b) $\{x \mid \phi_x \text{ is total and constant}\}$,
 - (c) $\{x \mid W_x \text{ is infinite}\}$.