Lab04-Church's Thesis

CS363-Computability Theory, Xiaofeng Gao, Spring 2016

∗ Please upload your assignment to FTP or submit a paper version on the next class

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1. Suggest the natural definition of computability on domain $\mathbb{Q}$ (rational numbers).

2. Define $f(n)$ as the n-th digit in the decimal expansion of e. Use Church's Thesis to prove that f is computable. (e is the the base of the natural logarithm and can be calculated as the sum of the infinite series: $e = \sum_{n=0}^{\infty} \frac{1}{n!}$)

3. Suppose there is a two-tape Turing Machine M with alphabet $\Gamma = \{\triangleright, \triangleleft, \square, 1\}$ and state set $Q = \{q_s, q_1, q_2, q_h\}$. M has the following specifications. Transform M into a single-tape Turing Machine $\widetilde{M}$, and write down the new alphabet and specifications.

$$\begin{array}{rcl}
\langle q_s, \triangleright, \triangleright \rangle & \rightarrow & \langle q_1, \triangleright, S, R \rangle \\
\langle q_1, \triangleright, \square \rangle & \rightarrow & \langle q_2, 1, R, R \rangle \\
\langle q_2, 1, \square \rangle & \rightarrow & \langle q_2, 1, R, R \rangle \\
\langle q_2, \triangleleft, \square \rangle & \rightarrow & \langle q_h, \triangleleft, S, S \rangle
\end{array}$$

4. Design a three-tape TM M that computes the function $f(x, y) = x\%y$, where both m and n belong to the natural number set $\mathbb{N}$. The alphabet is $\{1, \square, \triangleright, \triangleleft\}$, where the input on the first tape is $x + 1$ "1"'s and $y + 1$ "1"'s with a "$\square$" as the separation. Below is the initial configurations for input (x, y). The result is the number of "1"'s on the output tape with the pattern of $\triangleright 111 \cdots 111 \triangleleft$. First describe your design and then write the specifications of M in the form like $\langle q_S, \triangleright, \triangleright, \triangleright \rangle \rightarrow \langle q_1, \triangleright, \triangleright, R, R, R \rangle$ and explain the transition functions in detail (especially the meaning of each state).

Initial Configurations

Tape 1:	$\triangleright$	1	1	$\cdots$	1	1	$\square$	1	1	$\cdots$	1	1	$\triangleleft$	
	$\uparrow$	$\leftarrow x+1$ squares $\rightarrow$						$\leftarrow y+1$ squares $\rightarrow$						
Tape 2:	$\triangleright$	$\square$	$\square$	$\cdots$ $\cdots$ $\cdots$							$\square$	$\square$	$\square$	
	$\uparrow$													
Tape 3:	$\triangleright$	$\square$	$\square$	$\cdots$ $\cdots$ $\cdots$							$\square$	$\square$	$\square$	
	$\uparrow$													