SMASHER: A Strategy-Proof Combinatorial Auction Mechanism for Heterogeneous Channel Redistribution*

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ABSTRACT

Auction is believed to be an effective way to solve or relieve the problem of radio spectrum shortage, by dynamically redistributing idle wireless channels of primary users to secondary users. However, to design a practical channel auction mechanism, we have to consider five challenges, including strategy-proofness, channel spatial reusability, channel heterogeneity, bid diversity, and social welfare maximization. Unfortunately, none of the existing works fully considered the five design challenges. In this paper, we present the first in-depth study on the problem of dynamic channel redistribution by jointly considering the five design challenges, and present SMASHER, which is a Strategy-proof coMbinatorial Auction mechaniSm for HEterogeneous channel Redistribution. Our analyses show that SMASHER achieves both strategy-proofness and approximately efficient social welfare.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design – *Wireless Communication*

General Terms

Algorithm, Design, Economic

Keywords

Channel Allocation; Combinatorial Auction

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*This work was supported in part by the State Key Development Program for Basic Research of China (Grant No. 2012CB316201), in part by China NSF grant 61272443, 61133006, 61073152, and in part by Shanghai Science and Technology fund 12PJ1404900 and 12ZR1414900. The opinions, findings, conclusions, and recommendations expressed in this paper are those of the authors and do not necessarily reflect the views of the funding agencies or the government.

MobiHoc'13, July 29–August 1, 2013, Bangalore, India.

1. INTRODUCTION

The last two decades have witnessed a rapid development of wireless communication technology. Unfortunately, naturally limited radio spectrum is becoming a more and more serious bottleneck of the ongoing growth of wireless applications and services. Most of the countries have specific departments to regulate spectrum usage, *e.g.*, Federal Communications Commission (FCC) in the US and Radio Administration Bureau (RAB) in China. They statically allocate spectrum to wireless application service providers on a long term basis for large geographical regions. Such static management leads to low spectrum utilization in the spatial and temporal dimensions. Large chunks of radio spectrum are left idle most of the time at a lot of places, while new wireless applications are starving for the radio spectrum. Therefore, an open and market-based framework is highly needed to dynamically redistribute the radio spectrum, and thus improve the utilization of the radio spectrum.

Auctions are the most well-known market-based mechanisms to redistribute resources [3]. Since 1994, FCC has conducted a series of auctions for the licenses of radio spectrum. While FCC auctions target at large wireless service providers, our focus is on small wireless applications, such as community wireless networks or home wireless networks.

There exist many challenges in designing a practical channel auction mechanism. We list five major challenges:

- *Strategy-Proofness*: In strategy-proof auction mechanisms, simply submitting truthful channel demands (*e.g.*, valuation of the channels) maximizes each participant's utility. Since the participants are normally rational and selfish, they always tend to strategically manipulate the auction, if doing so can increase their utilities. Therefore, it discourages truthfully behaving participants from joining the auction, if strategy-proofness is not guaranteed.
- *Spatial Reusability*: Spatial reusability differentiates the wireless channels from conventional goods. Two wireless users can use the same wireless channel simultaneously, if they are well-separated.
- *Channel Heterogeneity*: Channel heterogeneity comes from both *spatial heterogeneity* and *frequency heterogeneity*. On one hand, the availability and quality of a channel vary at different locations. On the other hand, channels with different central frequency may have different propagation and penetration characteristics.
- *Bid Diversity*: Wireless devices may be equipped with multiple radios, each of which can work on a distinguished channel at the same time. Consequently, a

wireless user may request multiple bundles of channels, according to her quality of service requirement. Buyers have higher opportunities to obtain channels by submitting diverse bids, which makes the channel redistribution more flexible. Therefore, it is necessary to allow users to express diverse demands for channels.

- *Social Welfare*: The objective of any auction is to maximize social welfare, which is the sum of the auction winners' valuations of the allocated goods.

A number of related works (*e.g.*, [1, 2, 4–6, 8–10]) exist in the literature. Unfortunately, none of these works fully consider the five design challenges. Some of strategy-proof channel auction mechanisms (*e.g.*, VEARITAS [8], TRUST [9], SMALL [5]) consider channel spatial reusability, but only work when the trading channels are homogenous. Two recent works TAHES [2] and CRWDP [1] consider channels' heterogeneity, but TAHES restricts each user to bid for a single channel while CRWDP ignores the spatial reusability of channels.

In this paper, we conduct an in-depth study on the problem of dynamic channel redistribution by jointly considering the five design challenges, and present SMASHER, which is a Strategy-proof coMbinatorial Auction mechaniSm for HEterogeneous channel Redistribution. SMASHER is a novel combinatorial auction mechanism for indivisible heterogeneous channel redistribution, and achieves both strategy-proofness and approximately efficient social welfare.

We make the following contributions in this paper:

- First, we present a general model of combinatorial auction for heterogeneous channel redistribution. The auction model is powerful enough to express channel spatial reusability and heterogeneity, as well as bid diversity.

- Second, we introduce the concept of *virtual channel* to capture the conflict of channel usage among different auction participants. By using virtual channels, we transform the problem of heterogeneous channel allocation to a classic multi-unit combinatorial auction.

- Third, we propose SMASHER, which is a combinatorial auction mechanism for heterogeneous channel redistribution, achieving both strategy-proofness and approximately efficient social welfare.

2. PRELIMINARIES AND PROBLEM FORMULATION

In this section, we present the auction model for the problem of heterogeneous channel allocation.

2.1 Auction Model

We consider a static scenario, in which there is a primary spectrum user, called "seller", who wants to lease out her temporarily unused wireless channels, and some secondary users (*e.g.*, WiFi access points), called "buyers", who want to lease channels to provide services to their customers at certain quality of service (QoS). We consider that the channels for leasing are *heterogeneous*, and thus the buyers have their own preference over the channels due to spatial variance (*e.g.*, background noise, temperature, and landform). Since wireless devices can be equipped with multiple radios, the buyers may request more than one channel according to their requirements of QoS. Considering the diversity of QoS demand and heterogeneity of the channels, we allow the buyers to submit multiple channel requests, among which one of the requests can be granted. We assume that the buyers have uniform valuation over any of their channel requests, because the buyers' requirement of QoS can be satisfied if one of their requested bundles is allocated. Different from the allocation of traditional goods, wireless channels can be spatially reused, meaning that well-separated buyers can work on the same channel simultaneously, if they do not have interference between each other.

We model the process of heterogeneous channel redistribution as a sealed-bid combinatorial auction, in which buyers simultaneously submit their demands for channels to a trustworthy auctioneer, such that no buyer can know other participants' information. The auctioneer makes the decision on channel allocation and the charge to each winner. We denote the set of orthogonal and heterogeneous channels for leasing by $\mathbb{C} \triangleq \{c_1, c_2, \ldots, c_m\}$, and the set of buyers by $\mathbb{N} \triangleq \{1, 2, \ldots, n\}$. We list useful notations in our model of combinatorial channel auction as follows:

Channel Request R_i: Each buyer $i \in \mathbb{N}$ submits a vector of requested channel bundles $R_i \triangleq \left(S_i^1, S_i^2, \ldots, S_i^K\right)$ to the auctioneer. Any channel bundle $S_i^j \subseteq \mathbb{C}, 1 \leq j \leq K$ can satisfy her QoS. We assume that buyer's request is strict, meaning that the buyer is only interested in winning a whole bundle S_i^j in her request vector. We call a buyer, who submits a request vector of K channel bundles, and is interested in winning one of the bundles, as K-minded buyer. If $K = 1$, then the buyer is single-minded. Note that our auction model is a generalization of existing models with single-minded buyers (*e.g.*, [1, 2]). We denote the channel request vector $\vec{\mathbb{R}}$ of all the buyers as $\vec{\mathbb{R}} \triangleq (R_1, R_2, \ldots, R_n)$.

Valuation v_i: Each buyer $i \in \mathbb{N}$ has an uniform valuation v_i over any requested channel bundles in R_i. Here, v_i is the private information of the buyer i. This is also known as *type* in mechanism design. We denote the valuation vector $\vec{\mathbb{V}}$ of all the buyers as $\vec{\mathbb{V}} \triangleq (v_1, v_2, \ldots, v_n)$.

Bid b_i: Each buyer $i \in \mathbb{N}$ submits a bid b_i to the auctioneer, meaning that if she wins any channel bundle S_i^j, she would like to pay no more than b_i for it. Here, the bid b_i may not necessarily be equal to her valuation v_i. Let vector $\vec{\mathbb{B}}$ represent the bids of all the buyers $\vec{\mathbb{B}} \triangleq (b_1, b_2, \ldots, b_n)$.

Clearing price p_i: The auctioneer charges each winning buyer $i \in \mathbb{N}$ a clearing price p_i. The loser in the auction is free of any charge. We use vector $\vec{\mathbb{P}} \triangleq (p_1, p_2, \ldots, p_n)$ to represent the clearing prices of all the buyers.

Utility u_i: The utility of a buyer $i \in \mathbb{N}$ in the auction is defined as the difference between her valuation on the bundle of channels she wins and her clearing price p_i:

$$u_i \triangleq v_i - p_i. \tag{1}$$

We consider that the buyers are rational and selfish, thus their goals are to maximize their own utilities. In contrast to the buyers, the auctioneer's objective is to maximize *social welfare*. Here social welfare is defined as follows:

DEFINITION 1 (SOCIAL WELFARE). *The social welfare in a channel auction is the sum of winning buyers' valuations on their allocated bundles of channels,* i.e.*,*

$$SW \triangleq \sum_{i \in W} v_i, \tag{2}$$

where W is the set of winners.

In this paper, we assume that buyers do not collude with each other and do not cheat about their channel bundles, while leaving these problems to our future works.

3. MULTI-UNIT COMBINATORIAL CHANNEL AUCTION

Different from existing works on strategy-proof channel allocation, we introduce a novel concept of *virtual channel* to represent the conflict of channel usage among the buyers. By introducing virtual channels, we transform the problem of heterogeneous channel allocation to a classic multi-unit combinatorial auction.

3.1 Virtual Channel

We introduce *virtual channel* to capture the interference among the buyers on different channels. Specifically, a virtual channel $vc_{i,j}^k$ denotes that the buyer i and the buyer j may cause interference between each other on channel c_k, and thus they cannot work on channel c_k simultaneously. Since virtual channel $vc_{i,j}^k$ represents the exclusive usage of channel c_k between the buyer i and j, its quantity is set to 1. When virtual channel $vc_{i,j}^k$ is added to the requested bundle(s) that contains channel c_k from the buyer i and j, at most one of the requests containing channel c_k from the two buyers can be granted. Consequently, the exclusive usage of channel c_k between the buyer i and j is guaranteed. We present the definition of virtual channel as follows.

DEFINITION 2 (VIRTUAL CHANNEL). *There is a virtual channel $vc_{i,j}^k$, if the buyer i and buyer j are within the interference range of each other on channel c_k.*

In most of existing works on channel auction, a single conflict graph is used to represent the interference among buyers. However, in the case of heterogeneous channels, each channel may have a distinctive conflict graph. Let $G_k \triangleq (O_k, E_k)$ denote the conflict graph on channel c_k, where $O_k \subseteq \mathbb{N}$ is the set of buyers who can access channel c_k, and each edge $(i,j) \in E_k$ represents the interference between the buyer i and j on channel c_k.

Since conflict graph is commonly assumed to be available in wireless networks, we construct the virtual channel from conflict graph. We create a virtual channel $vc_{i,j}^k$, if there is an edge between the buyer i and j in conflict graph G_k, and append $vc_{i,j}^k$ to the requested bundle(s) containing channel c_k from the buyer i and j, while remaining the corresponding bids unchanged. Let $\mathbb{VC}$ be the set of virtual channels and $\vec{\mathbb{R}}'$ be the vector of updated requests with virtual channels.

3.2 Multi-Unit Combinatorial Auction

Given the virtual channel introduced in last section, we are ready to transform the problem of heterogenous channel allocation to a classic multi-unit combinatorial auction.

The goods in the multi-unit combinatorial auction are the channels and virtual channels. The quantities of each channel $c_k \in \mathbb{C}$ and virtual channel $vc_{i,j}^k \in \mathbb{VC}$ are n and 1, respectively. Let $x\left(i, S_i'^j\right) = 1$ denote that the channel set $S_i'^j$ is granted to the buyer i; otherwise, $x\left(i, S_i'^j\right) = 0$. The process of winner determination can be modeled as a binary program. The objective is to maximize the social welfare. We use b_i, instead of v_i, because the strategy-proof mechanism shown in later sections will guarantee that bidding truthfully is the dominate strategy of each buyer $i \in \mathbb{N}$.

Objective:

$$Maximize \qquad \sum_{i \in N} \sum_{j=1}^{K} x(i, S_i'^j) \times b_i$$

Subject to:

$$\sum_{i \in N} \sum_{S_i'^j \in \mathbb{R}_i', S_i'^j \ni c_k} x\left(i, S_i'^j\right) \le n \qquad \forall c_k \in \mathbb{C} \tag{3}$$

$$\sum_{i \in N} \sum_{S_i'^j \in \mathbb{R}_i', S_i'^j \ni vc_k} x\left(i, S_i'^j\right) \le 1 \qquad \forall vc_k \in \mathbb{VC} \tag{4}$$

$$\sum_{j=1}^{K} x\left(i, S_i'^j\right) \le 1 \qquad \forall i \in \mathbb{N} \tag{5}$$

$$x\left(i, S_i'^j\right) \in \{0,1\} \quad \forall i \in \mathbb{N}, 1 \le j \le K \tag{6}$$

If the optimal social welfare can be achieved by solving the above binary program, then the celebrated VCG mechanism can be applied to calculate the clearing price that can ensure the strategy-proofness of the auction mechanism. Unfortunately, the above winner determination problem can be proven to be NP-hard by reducing to the *exact cover* problem. Considering the computational intractability of the winner determination problem, we integrate a greedy allocation algorithm with a novel pricing mechanism to provide a strategy-proof and approximately efficient combinatorial auction mechanism for heterogeneous channel redistribution in next section.

4. HETEROGENEOUS CHANNEL REDISTRIBUTION

As shown in Section 3.2, finding the optimal auction decision is computationally intractable. In this section, we present SMASHER, which is a strategy-proof and approximately efficient combinatorial auction mechanism for heterogenous channel redistribution.

4.1 Design of SMASHER

SMASHER consists of the following three major components: virtual channel generation, winner determination, and clearing price calculation.

4.1.1 Virtual Channel Generation

The process of virtual channel generation is the same as the method discussed in Section 3.1, except that we add one more virtual channel vc_i with unit quantity to each requested bundle of buyer $i \in N$. Virtual channel vc_i is used to ensure that at most one of the requested bundles from the buyer i can be granted.

$$S_i'^j = S_i'^j \cup \{vc_i\}, i \in N, 1 \le j \le K,$$

where $S_i'^j$ is updated bundle with virtual channels.

4.1.2 Winner Determination

Before presenting the approximation algorithm for winner determination, we introduce *virtual bid*. The uniform virtual bid $\tilde{b}_i$ over any of requested bundles from the buyer i is defined as

$$\tilde{b}_i \triangleq \frac{b_i}{\max\limits_{1 \le l \le K} \left(\sqrt{\left|S_i'^l\right|}\right)}. \tag{7}$$

Algorithm 1: Approximation Algorithm for Winner Determination

Input: Vector of updated channel requests $\vec{\mathbb{R}}'$, vector of bids $\vec{\mathbb{B}}$.

Output: A pair of sets of winning buyers and allocated bundles of channels $(\mathbb{W}, \mathbb{S})$.

$(\mathbb{W}, \mathbb{S}) \leftarrow (\varnothing, \varnothing)$; $\mathcal{V} \leftarrow \varnothing$;
foreach $i \in \mathbb{N}$ **do**
  $\tilde{b}_i \leftarrow b_i / \max_{1 \le l \le K} \left(\sqrt{\left|S_i'^l\right|} \right)$;
end
Sort $\tilde{b}_i$ in non-increasing order: $\mathbb{L}_1 : \tilde{b}_1 \ge \tilde{b}_2 \ge \ldots \ge \tilde{b}_n$;
for $i = 1, \ldots, n$ **do**
  Sort $S_i'^j$ in non-decreasing order of bundle size: $\mathbb{L}_2 : \left|S_i'^1\right| \le \left|S_i'^2\right| \le \ldots \le \left|S_i'^K\right|$;
  for $j = 1, \ldots, K$ **do**
    if $S_i'^j \cap \mathcal{V} = \varnothing$ **then**
      Add virtual channels in $S_i'^j$ to $\mathcal{V}$;
      $(\mathbb{W}, \mathbb{S}) \leftarrow \left(\mathbb{W} \cup \{i\}, \mathbb{S} \cup \left\{S_i'^j\right\}\right)$;
      break;
    end
  end
end
return $(\mathbb{W}, \mathbb{S})$;

SMASHER sorts all the buyers according to their virtual bids in non-increasing order:

$$\mathbb{L}_1 : \tilde{b}_1 \ge \tilde{b}_2 \ge \ldots \ge \tilde{b}_n.$$

In case of a tie, SMASHER breaks the tie following a bid-independent rule, such as lexicographic order of buyers' ID and channel number.

Following the order in $\mathbb{L}_1$, SMASHER greedily grants the smallest channel bundle, in which no virtual channel has already been allocated, to each buyer.

Algorithm 1 shows the pseudo-code of above winner determination process. In practice, the number of buyers n is much larger than K, thus the time complexity of Algorithm 1 is $\mathcal{O}(n \log n)$.

4.1.3 Clearing Price Calculation

The clearing price is calculated based on *critical virtual bid*.

Definition 3 (Critical Virtual Bid). *The critical virtual bid $cr(i) \in \mathbb{L}_1$ of buyer $i \in \mathbb{N}$ is the minimum virtual bid that the buyer i must exceed to be allocated one of her channel bundles.*

We note that according to the definition of critical virtual bid, no matter which request of the buyer i is granted in the auction, the critical virtual bid $cr(i)$ is always the same.

The critical virtual bid of the buyer $i \in \mathbb{N}$ can be calculated by the following procedure. Given other buyers' requests and bids $\left(\vec{\mathbb{R}}'_{-i}, \vec{\mathbb{B}}_{-i}\right)$, we greedily select virtual bid by rerunning Algorithm 1 until none of the buyer i's requests can be satisfied. The threshold virtual bid $cr(i)$ we select finally is regarded as the critical virtual bid of the buyer i. We now show the method of calculating the clearing price of the buyer i by distinguishing two cases:

1. If the buyer i loses the auction or $cr(i)$ does not exist (denoted by $cr(i) = 0$), then her clearing price is 0.
2. If the buyer i is granted channel bundle $\hat{S}_i'^j$ and there exists a critical virtual bid $cr(i)$, the clearing price p_i of the buyer i is set to

$$p_i \triangleq cr(i) \times \max_{1 \le l \le K} \left(\sqrt{\left|S_i'^l\right|} \right). \quad (8)$$

4.2 Analysis

We prove the strategy-proofness and analyze the approximation ratio of SMASHER in this section.

Theorem 1. *SMASHER is a strategy-proof combinatorial auction mechanism for heterogeneous indivisible channel redistribution.*

Theorem 2. *The approximation ratio of SMASHER is $\mathcal{O}(n\sqrt{m})$, where n is the number of buyers, m is the number of channels.*

We leave the detailed proofs in our technical report [7].

5. CONCLUSION

In this paper, we have made an in-depth study on channel redistribution problem by jointly considering the five design challenges. We have presented a strategy-proof combinatorial auction mechanism for dynamic heterogeneous channel redistribution, namely SMASHER. Our analyses show that SMASHER achieves strategy-proofness and approximately efficient social welfare.

6. REFERENCES

[1] M. Dong, G. Sun, X. Wang, and Q. Zhang. Combinatorial auction with time-frequency flexibility in cognitive radio networks. In *INFOCOM*, 2012.

[2] X. Feng, Y. Chen, J. Zhang, Q. Zhang, and B. Li. TAHES: Truthful double auction for heterogeneous spectrums. In *INFOCOM*, 2012.

[3] J. Huang, R. A. Berry, and M. L. Honig. Auction-based spectrum sharing. *Mobile Networks and Applications*, 11(3):405–418, 2006.

[4] J. Jia, Q. Zhang, Q. Zhang, and M. Liu. Revenue generation for truthful spectrum auction in dynamic spectrum access. In *MobiHoc*, 2009.

[5] F. Wu and N. Vaidya. SMALL: A strategy-proof mechanism for radio spectrum allocation. In *INFOCOM*, 2011.

[6] P. Xu, S. Wang, and X.-Y. Li. SALSA: Strategyproof online spectrum admissions for wireless networks. *IEEE Transactions on Computers*, 59(12):1691 –1702, 2010.

[7] Z. Zheng, F. Wu, and G. Chen. SMASHER: Strategy-proof combinatorial auction mechanisms for heterogeneous channel redistribution. Technical report, available at `http://www.cs.sjtu.edu.cn/~fwu/res/Paper/ZWC12TRSMASHER.pdf`, 2012.

[8] X. Zhou, S. Gandhi, S. Suri, and H. Zheng. eBay in the sky: Strategy-proof wireless spectrum auctions. In *MobiCom*, 2008.

[9] X. Zhou and H. Zheng. TRUST: A general framework for truthful double spectrum auctions. In *INFOCOM*, 2009.

[10] X. Zhou and H. Zheng. Breaking bidder collusion in large-scale spectrum auctions. In *MobiHoc*, 2010.