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RESEARCH ARTICLE

TRADE: A truthful online combinatorial auction for spectrum allocation in cognitive radio networks

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ABSTRACT

Auctions have been shown to be able to tackle the problem of spectrum scarcity effectively, but most of existing works only focus on static scenarios. They cannot deal with the requests of spectrum users as they arrive and leave dynamically. Bidders can either cheat by bidding untruthfully or cheat about the arrival and departure time. In this paper, we model the radio spectrum allocation problem as a sealed-bid online combinatorial auction and propose a truthful mechanism called TRADE. TRADE is a truthful and an individual rational mechanism with polynomial time complexity. It can prevent bidders from cheating in the auction while achieving good bidder satisfaction, spectrum utilization, and social welfare.

KEYWORDS
mechanism design; online mechanism; combinatorial auction; spectrum allocation; cognitive radio networks

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1. INTRODUCTION

With the fast growth of wireless technology in recent years, the demand for spectrum increases dramatically. Although governments, such as Federal Communications Commission in the United States, serve as regulators to allocate radio spectrum, large chunks of radio spectrum are still left idle most of the time [1]. Cognitive radio networks were proposed to solve the low-efficiency in channel utilization [2]. In the cognitive radio networks, the primary licensed operators are called primary users (PUs) and the users who want to exploit the spectrum opportunity are called secondary users (SUs). To provide enough incentives for PUs to release their spectrum opportunity, auction is implemented as a fair, open, and efficient method [3].

However, designing a practical spectrum auction mechanism has two major challenges. One design challenge is that finding the optimal spectrum allocation is computationally hard (non-deterministic polynomial complete) even for some simple cases [4,5]. Another design challenge is SUs' strategic behavior. Because SUs are selfish and always want to maximize their objectives, truthfully behaving SUs can be discouraged from participating in the auction if the auction cannot guarantee truthfulness (see Section 2.2 for the definiton of truthfulness) [3].

Recent works on spectrum auction is started by Zhou *et al.* with an eBay-like dynamic spectrum market model, called Veritas [6]. This mechanism outperforms conventional spectrum auction by improving spectrum utilization up to 200%. Although the auction can be periodically run, bidders (SUs) still suffer from great inconvenience or even less utility when their job lengths vary.

Online auction [4] is an effective method to overcome the aforementioned problem and appears to be attractive in practice. In online auctions, bidders submit their requests at any time. Topaz [7] is the first truthful model to solve the online spectrum auction and model that problem as a three-dimensional bin packing problem. It compares the performance between different pre-emption factors. However, raising bids exponentially with job progress can decrease spectrum utilization and bidder satisfaction, because some bids can be enlarged dramatically after continuous winning for slots to the extent that can be used to complete job several times. Furthermore, the allocation method that numbers the channels and allocates them with the smallest index to winners is not an optimal way for maximizing utilization when spectrum is insufficient for bidders.

Previous auction [8] should decide the bandwidth of an auction because an inappropriate bandwidth would reduce the participation of spectrum users. An auction mechanism for spectrum allocation would be considered time-frequency flexible when it allows bidders to bid for their desirable frequency and preferable usage time sections. To cope with time-frequency flexibility, Dong *et al.* [9] proposed a combinatorial auction that achieves a social welfare larger than worst-case approximation ratio. But it does not consider spatial reusability of spectrum. Moreover, ordering bids with the size of requested slots may degrade spectrum utilization.

Despite all the previous works, designing a practical spectrum auction mechanism has not been fully studied, so we model the problem of spectrum allocation as an online and time-frequency flexible auction. In this paper, we propose TRADE—a TRuthful online combinatorial Auction for spectrum allocation in cognitive raDio nEtworks, to address the aforementioned challenges. In our model, PUs release usage opportunity to the auctioneer. The auctioneer divides usage time into small slots and segments frequency into channels. We regard each slot with every channel as a good to be sold. Each bidder submit her bid as a bundle of channels, the maximal price she is willing to pay, her arrival and departure time, and her job length to the auctioneer. By applying our mechanism TRADE, the auctioneer can determine winners and their payments.

We make the following contributions in this paper:

- To the best of our knowledge, TRADE is the first online combinatorial auction mechanism for spectrum allocation.
- We theoretically prove that TRADE satisfies truthfulness and individual rationality.
- To achieve high bidder satisfaction ratio, we design virtual bids to give higher priorities to bidders whose job has been started.
- We have implemented TRADE and evaluated its performance numerically. Our evaluation results show that TRADE achieves good performance in bidder satisfaction, spectrum utilization, and social welfare.

The rest of this paper is organized as follows. Section 2 presents our auction model and design objective. In Section 3, we illustrate the design challenges in detail. TRADE is introduced in Section 4 with proof of truthfulness and computational complexity. We show how we evaluate our work in Section 5. In Section 6, we review related works. In Section 7, we draw our conclusions.

2. AUCTION MODEL AND DESIGN OBJECTIVE

We first briefly introduce our online combinatorial spectrum auction model and show our design objective together with economic properties.

2.1. Auction model

We concentrate on the scenario where auctioneer sells acquired spectrum to SUs. A channel can be leased to multiple SUs if they can transmit and receive signals simultaneously with an adequate signal to interference and noise ratio. Buyers (SUs) submit their requests to the auctioneer privately. The auctioneer divides spectrum opportunity by time and frequency dimensions. Our auction model is a sealed-bid one. We denote m idle and identical channels by $C = \{c_1, c_2, \ldots, c_m\}$ and time slots by $T = \{t_1, t_2, \ldots\}$. The number of channels is constant over time. To simplify the problem, we denote the set of buyers by $N = \{1, 2, \ldots, n\}$. Let $B = \{B_1, B_2, \ldots, B_n\}$ denote the set of request types. Each type B_i can be denoted by $\{a_i, d_i, l_i, C_i, b_i\}$, which consists of arrival time, departure time, job length, requested channel set ($C_i \subseteq C$), and the maximal price the bidder would like to pay. Bidder i's true valuation is denoted as v_i. Payment set P consists of each bidder's payment , $\{p_1, p_2, \ldots, p_n\}$ and the interfering neighbors of bidder i are in the interfering neighbor set I_i. Because conflict among PUs has been solved by auction performers such as the Federal Communications Commission, in our model, we only consider conflict between SUs that depends on their locations, time-periods, and requirements of channels. As mentioned in protocol interference model [10], bidders under the interference range of each other cannot share the same channel. We define bidder i's interfering neighbor as some bidders within bidder i's interference range and request for the same channel with i. It is calculated on the basis of conflict graph $G^\dagger$. As Figure 1 shows, each node represents a bidder and an edge between two nodes represents that the two are conflicted with each other. They cannot share a channel. The utility of bidder i is defined as follows:

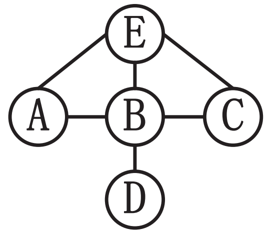

Figure 1. Example of conflict graph.

$$u_i = \begin{cases} v_i - p_i & \text{if bidder } i \text{ wins,} \\ 0 & \text{otherwise.} \end{cases}$$

Let $U = \{u_1, u_2, \ldots, u_m\}$ denote the set of utilities of bidders. We also denote the set of allocation for each channels as $A = \{A_{c_1}, A_{c_2}, \ldots, A_{c_m}\}$. For any channel c_j, A_{c_j} consists of the bidders who obtain it. Usage array $S_{t_\tau}(i)$ keeps the number of winning slots of bidder i before

†The auctioneer can identify all the bidders when they register to participate, and using existing methods (*e.g.*, [11,12]) to calculate $\boldsymbol{G}$

Table I. Notations in our model for bidder i.

a_i	Arrival time	b_i	Bid
C_i	Set of request channels	d_i	Departure time
I_i	Set of interfering neighbors	l_i	Job length
p_i	Payment	v_i	Valuation of request channels
u_i	Utility	$\hat{b}_i$	Virtual bid

time $t_\tau \in T$. Set W consists of bidders who have won for enough time slots for their jobs. Detailed list of notation can be seen in Table I.

We assume that bidders cannot come later or depart earlier than they advocate, which means that they are present during the periods they submit. Same as existing works, we also assume that bidders truthfully submit their requested channel sets and job length. Requests rejected by the auctioneer can still be considered in the following time slots.

In combinatorial auction [4], bidders submit multiple bidding tuples to the auctioneer. Every bidding tuple consists of a bundle of goods and the maximal price the bidder is willing to pay. No good is assigned to more than one bidder and no bidder receives more than a bundle of goods. In combinatorial spectrum auction, goods are channels, which can be spatially reused. Different from general case, we assume that each buyer can only submit one bundle of channels.

Toy Example. Let us consider a toy example with five bidders competing for six channels as shown in Figure 1 at time t_6. Their requests are shown as follows. $B_A = \{1, 6, 2, \{c_1, c_4\}, 10\}$, $B_B = \{5, 9, 3, \{c_1, c_6\}, 20\}$, $B_C = \{4, 8, 2, \{c_5, c_6\}, 15\}$, $B_D = \{6, 9, 2, \{c_2, c_3\}, 10\}$, and $B_E = \{3, 7, 2, \{c_4, c_5\}, 20\}$. We can see that (A, B), (A, E), (B, C), and (C, E) are interfering pairs. Although pair (B, D) and pair (B, E) are in the interference range of each other, their requested channel sets do not contain a same element. Hence, they are not interfering pairs.

After receiving all bids for next time slot, the auctioneer should decide whether to allocate channels to a bidder or to pre-empt a bidder with a lower bid on-the-fly. When each winner gets available channels, her usage is not mandatory. In an online auction, the auctioneer must decide winners and redistribute the spectrum to the buyers at each time slot and without knowledge of who will subsequently arrive.

In order to handle price difference, our mechanism must be able to handle pre-emption. To state it differently, a bidder who submits her request with a higher value, can pre-empt other winners in former time slots with a lower value.

Payment is calculated when one bidder finishes her job before departure. In the example of Figure 1, bidders' payments are all 0 because they all have conflict-free slots to do their works. For example, A used $[1, 2]$, B used $[7, 9]$, C used $[5, 6]$, D used $[6, 7]$ and E used $[3, 4]$. In this scheduling manner, all the bidders can finish their jobs in their present time and no pre-emption occurred. We will give a detailed design of payment in Section 4.

2.2. Design objective

Our design objective is to guarantee truthfulness and individual rationality. Our mechanism can effectively discourage bidders from cheating by enforcing the following property.

Definition 1 (Truthfulness [3,7]). *Let a_i, d_i, and v_i represent bidder i's true arrival time, departure time, and true valuation. An online auction is bounded (a,d,v)-truthful if and only if no bidder i can improve her utility by bidding $b_i \neq v_i$, or falsely reporting her arrival time $a'_i > a_i$, or deadline $d'_i < d_i$, or any combination of them.*

The next property *individual rationality* provides bidders incentives to participate by guaranteeing non-negative utilities when bid truthfully.

Definition 2 (Individual rationality [3]). *An auction is individually rational if no winning bidder pays more than her bid ($p_i \leq b_i$).*

Finally, our goal is to achieve as good spectrum utilization and bidder satisfaction as possible without sacrificing social welfare. Their detailed definitions can be found in Section 5. To evaluate the worst-case performance of our mechanism against optimal off-line solution, competitive analysis [13,14] is adopted.

Definition 3 (Competitive ratio on social welfare [4]). *An auction mechanism Λ is c-competitive with respect to the social welfare if for every bidding sequence z, $E_\Lambda(z) \geq E_{OPT}(z)/c$, where E indicates the function of social welfare. Here, c is the competitive ratio of mechanism Λ.*

We will give the detailed explanation of social welfare in Section 5.2 and the detailed competitive analysis in Section 5.

3. DESIGN CHALLENGES OF ONLINE COMBINATORIAL AUCTION

3.1. Online decisions

The major design challenge is online decisions. Online auction is a special kind of auction. Bidders will arrive from time to time. The auctioneer does not know who will come in the future. What is more, pre-emption will lead to reduction in spectrum utilization because partial usage will not be charged and damage the auction's reliability. Also, the pre-empted bidder will be discouraged from participating in later auctions. It is challenging to design decision process to cope with such uncertainty and pre-emption effect.

3.2. Spectrum distribution

Distribution is challenging to the auctioneer because she needs to allocate channels in hybrid domains of time, space, and frequency. Unlike conventional auction (*e.g.*, [6,7]), which only considers interference geographically, combinatorial auction's interference constraints of frequency depend on bidders' requests. Such complex conflict format also makes decision process challenging.

3.3. Resist bid-cheating and time-cheating

To ensure truthfulness, auction must resist bid-cheating and time-cheating because bidders are selfish. If no one can improve their utilities by misreporting their requests, then bidders will report their actual requests. Existing online mechanisms (*e.g.*, [14–16]) make reasonable assumptions to restrict bidders' misreport patterns. We also assume that a bidder has little incentive to misreport her requested channels and job length. But designing an auction to resist bid-cheating and time-cheating is still challenging.

4. TRADE

In this section, we present the design of TRADE as an online combinatorial auction and prove its truthfulness.

4.1. Auction procedure

4.1.1. Bid transformation.

We raise the winners' bids to protect them from being pre-empted. The pre-emption factor is defined as $1+\theta_i$. For bidder i who has been winning for continuous $S_{t_{\tau-1}}(i)$ time slots before time $t_{\tau-1}$ (allocation at time t_τ is not determined), $\theta_i = S_{t_{\tau-1}}(i)/l_i$ represents her job progress before time $t_{\tau-1}$. Also, in our design, we must lower some bidders' bids who have more interfering neighbors.

Hence in TRADE, a bidder who has almost finished her job will have a higher priority and who has a larger set of interfering neighbors will have a lower priority. Virtual bid is denoted by $\hat{b}_i$, defined as follow:

$$\hat{b}_i = \frac{b_i}{(|I_i|+1)^\alpha} \cdot (1+\theta_i)$$

where $\alpha \geq 0$.

Algorithm 1 calculates virtual bid set. In the first step, we calculate interfering neighbor set I of all bidders by enumerating conflict graph G at time t_τ. As time goes on, G changes at different time slots. So I is calculated at each time slot. After that, each element in virtual bid set $\hat{B}$ can be generated depending on actual bids, usage array, and interfering neighbor set. At last, we sort $\hat{B}$ by elements' value in non-increasing order. The first parameter τ of Algorithm 1 indicates that our algorithm must run at each time slot. Bidders participate at each time slot before departure time and their bids will be considered at each round.

Algorithm 1 $Bid-Trans(\tau, B, G, S, \hat{B})$

Input: a) critical time t_τ; b) bid set B; c) conflict graph G; d) usage array S;
Output: Virtual bid set $\hat{B}$;
for $i \in N$ **do**
  $I_i = \emptyset$;
  for $j \in N \setminus \{i\}$ **do**
    if edge $(i,j) \in G_{t_\tau}$ **and** $C_i \cap C_j \neq \emptyset$ **then**
      $I_i = I_i \cup \{j\}$;
    end if
  end for
end for
$\hat{B} = \emptyset$;
for $b_i \in B_{t_\tau}$ **do**
  $\theta_i = S_{t_{\tau-1}}(i)/l_i$;
  $\hat{b}_i = \frac{b_i}{(|I_i|+1)^\alpha} \cdot (1+\theta_i)$;
  $\hat{B} = \hat{B} \cup \{\hat{b}_i\}$;
end for
Sort $\hat{B}$ by elements' value in non-increasing order
Return $\hat{B}$;

4.1.2. Winner determination.

We allocate the channels to the bidders with the highest virtual bid first. The function $Top(\hat{B})$ returns the bidder who ranks highest in $\hat{B}$. We suppose that bidder i has the highest virtual bid. The auctioneer judges whether any bidder i's requested channels has been allocated to her interfering neighbors by judging the following equation:

$$\bigcup_{c_j \in C_i} A_{c_j} \cap I_i = \emptyset$$

If it is true, then the auctioneer can allocate the channels to bidder i. The usage array of time t_τ is based on last time $t_{\tau-1}$. Otherwise, bidder i's usage of channels cannot be fulfilled at this time slot and she is pre-empted if she won at former time $t_{\tau-1}$. $S_{t_\tau(i)}$ is set to zero. If bidder i's usage array at time t_τ is larger or equal to her job length l_i, then auctioneer adds bidder i to W. Also, the auctioneer adds bidder i to each of her requested channels' allocation set. Detailed procedure can be seen in Algorithm 2.

4.1.3. Payment calculation.

Payment of bidder i is calculated before she leaves. In order to resist bid-cheating, we need to find the critical bid for bidder i to finish her job during her presence in the auction. First, we calculate $\lambda_i(t_g)$ for each t_g ($g \in [a_i + l_i - 1, d_i)$). It is the minimum bid for bidder i to win for slots $[t_{g-l_i+1}, t_g]$. The formula of p_i also resists time-cheating by removing the payment's time dependency.

$$p_i = \min_{g \in [a_i+l_i-1, d_i)} \lambda_i(t_g)$$

Algorithm 2 $Cha-Alloc(\tau, \hat{B}, S)$

Input: a) critical time t_τ; b) virtual bids set $\hat{B}$;
Output: Usage array S;
$A = \emptyset$;
while $\hat{B} \neq \emptyset$ **do**
  $i = Top(\hat{B})$;
  if $\bigcup_{c_j \in C_i} A_{c_j} \cap I_i = \emptyset$ **then**
    $S_{t_\tau}(i) = S_{t_{\tau-1}}(i) + 1$;
    if $S_{t_\tau}(i) \geq l_i$ **and** $i \notin W$ **then**
      $W = W \cup \{i\}$;
    end if
    for $c_j \in C_i$ **do**
      $A_{c_j} = A_{c_j} \cup \{i\}$;
    end for
  else
    $S_{t_\tau}(i) = 0$;
  end if
  $\hat{B} = \hat{B} \backslash \{\hat{b}_i\}$;
end while
Return S;

Algorithm 3 determines the price that all winners need to pay when they finish their jobs before departure time. The function $CriValWin(\tau, i, \hat{B})$ returns the minimum virtual bid for bidder i to win in the time t_τ given virtual bid set $\hat{B}$. That is, to find a lower virtual bid given by one of bidder i's interfering neighbors, denoted j, bidder j's request would be granted if bidder i does not submit her bid. If no such bidder j exists, then *CriValWin* returns 0. The first parameter of Algorithm 3 indicates that this algorithm is not run at each time slot but just before bidders' departure. It is just run for once.

Algorithm 3 $Pay-Calc(i, B, G, S, P)$

Input: a) bidder i; b) bids set B; c) conflict graph G; d) usage array S;
Output: Payment set P;
for $g \in [a_i + l_i - 1, d_i)$ **do**
  if $S_{t_g}(i) \geq l_i$ **then**
    for $\tau \in [g - l_i + 1, g]$ **do**
      $Bid-Trans(\tau, B, G, \hat{B})$;
      $\hat{B} = \hat{B} \backslash \{\hat{b}_i\}$;
      $\eta_i(t_\tau) = CriValWin(\tau, i, \hat{B})$;
      $\theta_i = S_{t_{\tau-1}}(i)/l_i$;
      $k_i(t_\tau) = \frac{\eta_i(t_\tau) \cdot (|I_i|+1)^\alpha}{1+\theta_i}$;
    end for
    $\lambda_i(t_g) = \max_{\tau \in [g-l_i+1, g]} k_i(t_\tau)$;
  end if
end for
$p_i = \min_{g \in [a_i+l_i-1, d_i)} \lambda_i(t_g)$;
$P = P \cup p_i$;
Return P;

4.2. Truthfulness of TRADE

We give a detailed proof to show that TRADE is truthful, which means that no bidder can improve her utility by bidding a different price, arrival, or departure time.

Our proof consists of three steps: (i) monotonic; (ii) critical; and (iii) truthful.

4.2.1. Monotonic.

Lemma 1. *If bidder i wins at t_τ when bids b_i, then she will also win when bids $b_i' > b_i$, assuming that all other requests remain the same.*

Proof. As the Algorithm 1 shows, if b_i is increased to b_i', $\hat{b}_i < \hat{b}_i'$. Bidder i's virtual bid will rank higher in $\hat{B}$. Hence, he or she will still win. □

4.2.2. Critical.

Lemma 2. *Any winning bidder i is not able to finish her job by bidding a lower value, so there exists a critical value h_i. Bidder i will finish her job when bids $b_i \geq h_i$, and on the contrary when bids $b_i < h_i$.*

Proof. Bidder i wins more than l_i continuous slots during her arrival and departure time if she finishes her job. The lowest bid that can win for l_i continuous slots in the auction is h_i. With Lemma 1, this lemma is proved. □

Lemma 3. *Algorithm 3 charges any winning bidder i by her critical value $p_i = h_i$.*

Proof. We can see in Algorithm 3 that $\lambda_i(t_g)$ is a critical bid for winning slots $[t_{g-l_i+1}, t_g]$, h_i is the minimal $\lambda_i(t_g)$ of slots $[t_{a_i+l_i-1}, t_{d_i})$. That is the definition of p_i, so $p_i = h_i$. □

Lemma 4. *TRADE is individually rational if any bidder i bids truthfully $v_i = b_i$, e.g., $u_i \geq 0$.*

Proof. Bidder's utility u_i is 0 or $v_i - p_i$. We will show that the latter one is always greater than or equal to 0. From Algorithm 3, we can see that

$$\begin{aligned} p_i &= \min_{g \in [a_i+l_i-1, d_i)} \lambda_i(t_g) \\ u_i = v_i - p_i &= b_i - \min_{g \in [a_i+l_i-1, d_i)} \lambda_i(t_g) \\ &= b_i - \max_{\tau \in [g-l_i+1, g]} k_i(t_\tau)(\exists g) \\ &= b_i - \frac{\eta_i(t_\tau) \cdot (|I_i|+1)^\alpha}{1+\theta_i}(\exists \tau) \\ &\geq b_i - \frac{\hat{b}_i \cdot (|I_i|+1)^\alpha}{1+\theta_i} = b_i - b_i = 0 \end{aligned}$$

$\eta_i(t_\tau)$ is equal to some bidder j's virtual bid $\hat{b}_j$ and $\hat{b}_j \leq \hat{b}_i$ at time t_τ. □

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4.2.3. Truthful.

Theorem 1. *Online spectrum auction TRADE is bounded (a,d,v)-truthful.*

Proof. First, we show that any bidder i cannot improve her utility by bidding a different bid b_i'.

There are four possible cases of cheating in the auction as shown in Table II. When bidder i truthfully bids $b_i = v_i$, her utility is u_i. Otherwise, when bids $b_i' \neq v_i$, her utility is u_i'. We will prove that $u_i \geq u_i'$ for all the cases.

- Case 1: Bidder i is rejected by truthfully bidding b_i, so $u_i = 0$. But bidder i wins by bidding b_i'. From Lemma 2, we can conclude that $b_i' > b_i$. Now we consider the utility of bidder i. Because $b_i < h_i \leq b_i'$, $u_i' = v_i - p_i = b_i - h_i < 0$.
- Case 2: Bidder i wins when bids truthfully, so by Lemma 4, $u_i \geq 0$. When bidder i cheats by bidding b_i', she loses. From Lemma 1 we can conclude that $b_i' < b_i$ and $u_i' = 0$, so $u_i \geq u_i'$ is also satisfied.
- Case 3: In this case, bidder i loses with both bids. She will not be charged, hence $u_i = u_i' = 0$.
- Case 4: Bidder i wins with both bids. By Lemma 3, $u_i = u_i' = v_i - p_i$.

Second, we will show that bidder i cannot improve her utility by arriving earlier or departing later. Let a_i' and d_i' denote i's reported arrival and departure time, $a_i < a_i'$ and $d_i > d_i'$. There are four possible cases.

- Case 1: Bidder i loses when arrives and departs in time, but wins when cheats. Because $[a_i', d_i') \subseteq [a_i, d_i)$, this case cannot happen.
- Case 2: Bidder i wins when arrives and departs in time, but loses when cheats. From Lemma 4, $u_i \geq 0$, while $u_i' = 0$, so $u_i \geq u_i'$
- Case 3: Bidder i loses in both cases. In this case, she will not be charged, $u_i = u_i' = 0$.
- Case 4: Bidder i wins in both cases. From Algorithm 3, we can see that as $[a_i', d_i') \subseteq [a_i, d_i)$, we have $p_i' \geq p_i$. Hence, $u_i = v_i - p_i \geq v_i - p_i' = u_i'$.

From the aforementioned proof, we have shown that bidders' utility cannot be increased by bid-cheating and time-cheating. Therefore, TRADE is bounded (a, d, v)-truthful. □

Table II. Cases of cheating.

Cases	b_i'	b_i	a_i', d_i'	a_i, d_i
1	Win	Lose	Win	Lose
2	Lose	Win	Lose	Win
3	Lose	Lose	Lose	Lose
4	Win	Win	Win	Win

4.3. Computational complexity

We now analyze TRADE's overall computational complexity under the environment of m channels, n bidders, time set T, and conflict graph $G = (V, E)$. Algorithm 1 will take at most $\mathcal{O}(m|E|)$ to calculate interfering neighbor set I, $\mathcal{O}(n)$ to calculate virtual bids, $\mathcal{O}(n \log n)$ to sort the virtual bids. Hence, the overall complexity of Algorithm 1 is less than $\mathcal{O}(m|E| + n \log n)$.

Algorithm 2 will enumerate all bidders in the virtual bid set and check if their requested channels interfere with their neighbors. So allocation will take at most $\mathcal{O}(mn)$ time. Also, recording the allocation set costs at most $\mathcal{O}(m)$. The overall complexity of Algorithm 2 is less than $\mathcal{O}(mn)$.

We search the bidders in W for their departure time in less than $\mathcal{O}(n)$. In Algorithm 3, each bidder runs Algorithm 1 and function $CriValWin$ in each loop, and $CriValWin$ has the same complexity with Algorithm 2. Each loop costs at most $\mathcal{O}(|T|)$ time, so completely less than $\mathcal{O}(|T|^2(m|E| + n \log n + mn))$ time. So overall complexity of Algorithm 3 is less than $\mathcal{O}(n|T|^2(m|E| + n \log n + mn))$. As $|E| \leq \frac{n(n-1)}{2}$, the overall computational complexity of TRADE is less than $\mathcal{O}(mn^3|T|^2)$. We conclude that TRADE runs in polynomial time.

4.4. Performance bound

The performance of TRADE depends on bidders' requests and conflict graph, so we are unable to derive any bound under general conflict graph. Because even in the simple scenario, there is no deterministic truthful mechanism whose performance is constant-competitive with that of off-line Vickrey–Clarke–Groves mechanism [14].

Hence, we restrict bidders' behaviors by assuming that each bidder requests for only one time slot. We also assume that conflict graph is a complete graph and all bidders' requested bundles of channels are conflict.

Theorem 2. *Online spectrum auction TRADE is a $ln(T)$-competitive mechanism.*

Proof. By using the same way of proof in [14], we are able to derive a competitive ratio. That is, to charge the valuation of any winning bidders in an optimal solution OPT to a winner in our mechanism TRADE. Without loss of generality, we suppose that pre-emption is not allowed in OPT. In addition, any winner i in OPT is also a winner in TRADE, then her valuation is charged to herself. Otherwise, for any bidder j_0, if she is a winner in OPT and her allocation is pre-empted by another bidder j_1 in TRADE. In the same way, supposing that j_1 is pre-empted by j_2. Continue this pre-emptive chain until a bidder j_k. She is not pre-empted by any bidders and charge j_0's valuation v_{j_0} to j_k. We now calculate the maximal total value charged to any bidder j_x with valuation v_{j_0} who wins at time slot t_τ. If j_0 wins in OPT, then there is a charge of v_{j_0}. Consider a bidder j_1 in OPT who is charged with v_{j_0}.

Let $t'_\tau = t_{\tau-\delta}$ be the time slot at which j_0 receives an allocation in OPT. It is clear from the algorithm that $\delta \in [0, T]$ and the valuation of j_1 is at most $v_{j_0}/(1+\delta)$. Furthermore, the value of δ for any two such j_x must be apart by at least one. Therefore, the total charge to j_0 is at most $v_{j_0} + \sum_{x=1}^{T} \frac{v_{j_0}}{1+x} \doteq ln(T)v_{j_0}$.

This shows that our algorithm is $ln(T)$-competitive. Because T is a small value, we can conclude that TRADE is a constant competitive algorithm. □

5. EVALUATION

5.1. Methodology

We evaluate TRADE's performance with random bids and time behaviors.

Although we can not compare TRADE with existing works because no prior models have achieved same truthfulness in an online combinatorial spectrum auction setting, we simulate a model extended from [7] called ex-topaz to compare with TRADE. The extension adds a procedure to handle bidder's specific requests of channels.

We implement TRADE and ex-topaz to evaluate their performances. The parameters used in the evaluation are shown in Table III. There are 100 time slots with 10 min each. At most 400 bidders arrive during the auction. Their bid price, arrival time, and departure time are distributed randomly within the given ranges. Job length is randomly taken from range [5, 15] with an average of 10 slots. We run our program 200 times and take the average result under each condition. We focus on how to configure TRADE with different α settings and choose a value of α to evaluate TRADE's performance in details. Then, we compare TRADE with ex-topaz in a specific setting.

5.2. Metrics

We use the following four performance metrics.

- *Bidder satisfaction ratio*: $\frac{|W|}{n}$.
- *Spectrum utilization ratio*: $\frac{\sum_{i \in W} l_i |C_i|}{m|T|}$.
- *Social welfare*: $\sum_{i \in W} v_i$.
- *Revenue*: $\sum_{i \in W} p_i$.

5.3. Results

5.3.1. Prologue.

We compare TRADE's performance with and without pre-emption. Evaluation results show that although forbidding pre-emption can increase spectrum utilization a little bit, social welfare however decrease sharply.

5.3.2. First part.

We study the four performance metrics when the number of bidders is 200 under different values of α as Figure 2 showed. Figure 2(a) examines the bidder satisfaction. At first, bidder satisfaction ratio increases because α reduces interfering frequency between higher bids and improves bidders' chances of satisfying their requests. As α increases further, bidder satisfaction ratio falls because TRADE inclines to allocate by bidders' interfering neighbor sizes. Bidders who have the least number of interfering neighbors will be allocated first. As interfering neighbor changes with time, TRADE cannot guarantee bidders who win at current time can still win at next time, so bidder satisfaction ratio is lower. When $\alpha = 2$, the maximal bidder satisfaction ratio is achieved. Different numbers of auctioned channels have the same trends.

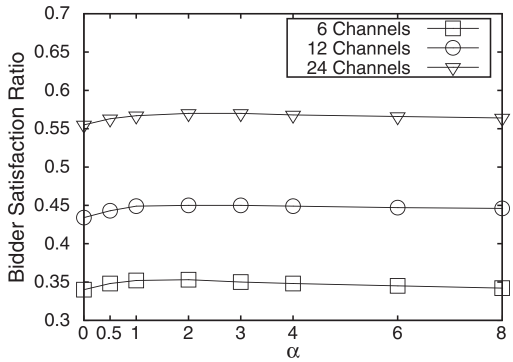
Spectrum utilization ratio, as shown in Figure 2(b), increases when α is less than 0.5 but decreases when $\alpha > 0.5$. Because small α can reduce interfering frequency and increase bidder satisfaction, while large α makes the allocation method less dependent on bidders' requested channels, so spectrum utilization ratio decreases. When the number of auctioned channels is larger, the difference between different α is smaller because each bidder has less interfering channels. Different allocation has almost the same spectrum utilization. Our design of α works well when spectrum is a scarce resource.

When $\alpha \le 1$, social welfare, as shown in Figure 2(c), increases because of the increase of bidder satisfaction ratio. But it falls when $\alpha > 1$, same as bidder satisfaction ratio. Bidders who have less interfering neighbors are more likely to win in each slot, and bidders who have higher bids cannot be guaranteed to finish their jobs, hence, social welfare decreases.

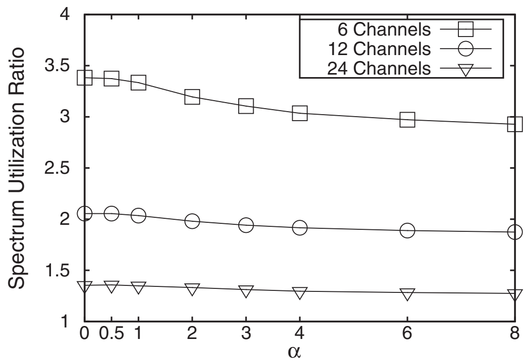
Revenue, as shown in Figure 2(d), tends to decrease as α increases. Comparing different settings of channels, when $\alpha < 5$, the more the number of channels, the lesser the amount of revenue. However, there is an opposite trend when $\alpha > 5$. It is shown that the difference between the revenues of various numbers of channels is relatively smaller than that of social welfare.

Table III. Parameters used in the evaluation.

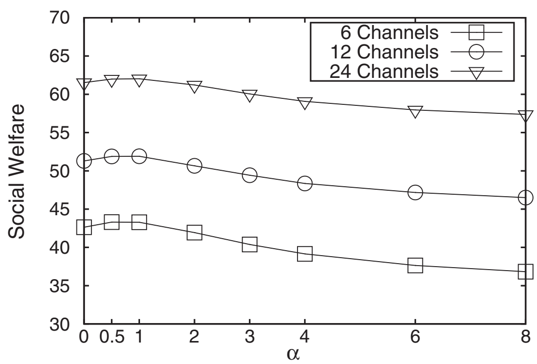
Region size	Conflict distance	Bid valuation	Slots per auction
2km × 2km	425m	(0,1]	100
# of bidders	Slots of per job	# of channels	# of required channels
[100,400]	[5,15]	[6,24]	[1,6]



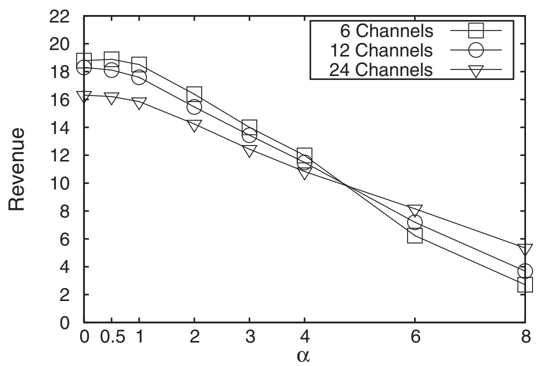
(a) Bidder Satisfaction Ratio.



(b) Spectrum Utilization Ratio.

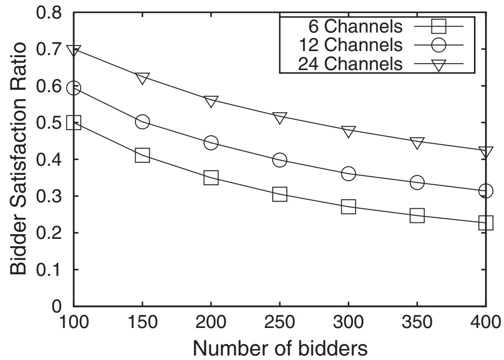


(c) Social Welfare.

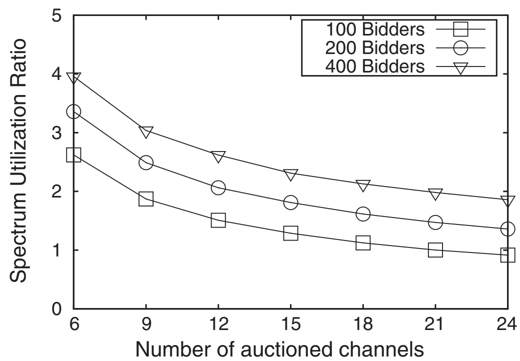


(d) Revenue.

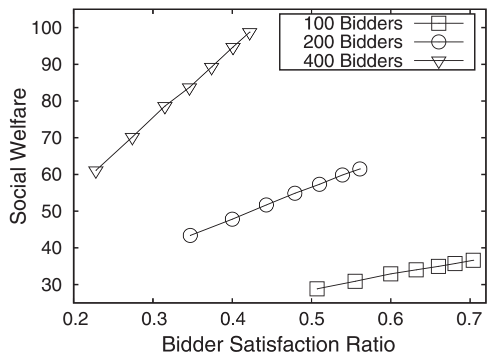
Figure 2. Impact of different value of α on TRADE.



(a) Bidder Satisfaction Ratio.



(b) Spectrum Utilization Ratio.



(c) Social Welfare vs. Bidder Satisfaction Ratio.

Figure 3. Evaluate TRADE when $\alpha = 0.5$.

5.3.3. Second part.

As we want to maximize spectrum utilization, we choose $\alpha = 0.5$ and depict the relation between bidder satisfaction ratio and the number of bidders in Figure 3(a) with 6, 12, and 24 channels, respectively. The results show that bidder satisfaction ratio decreases as the number of bidders increases, because the number of auctioned channels is limited. When the number of auctioned channels gets larger, bidder satisfaction ratio gets larger, too. That is to say, more bidders can finish their jobs.

We show the spectrum utilization ratio with the number of auctioned channels in Figure 3(b). It shows that spectrum utilization ratio decreases as the number of auctioned channels increases, because the number of requested channels is limited. When the number of bidders gets larger, spectrum utilization ratio gets larger, too.

Finally, in Figure 3(c), we depict the relation between social welfare and bidder satisfaction ratio in the same setting with Figure 3(b). The results show that social welfare increases almost linearly with bidder satisfaction, which means that, TRADE does not sacrifice social welfare to achieve a good bidder satisfaction. The ratio between social welfare and bidder satisfaction ratio indicates the level of winners' average valuation. It is almost constant in TRADE and illustrates the high performance of TRADE. $\alpha = 0.5$ is a reasonable value of bidder selection.

5.3.4. Third part.

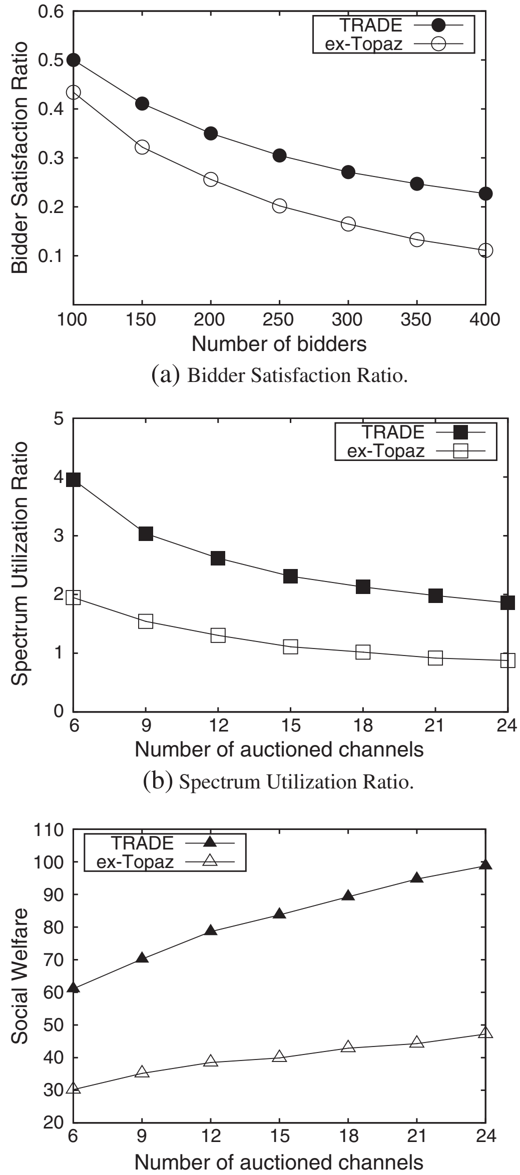
In the third part, we compare TRADE's performance with ex-topaz. We try various configurations, and the results show that TRADE outperforms ex-topaz greatly.

We choose the same as in Section 5.3.3 with six channels to compare bidder satisfaction ratio in Figure 4(a). Bidder satisfaction ratio of ex-topaz decreases greatly compared with TRADE as the number of bidders increases. In other words, TRADE can serve more bidders if channels are relatively scarce.

We also compare their spectrum utilization ratios when the number of bidders is 400. The results are shown in Figure 4(b). As the number of channels increases, spectrum utilization of ex-topaz declines to approximately 1, which means that spectrum reusability is not well exploited. However, our mechanism TRADE can ensure that each channel is used about twice.

We also compare the social welfare between TRADE and ex-topaz. As shown in Figure 4(b) and 4(c), TRADE can achieve a better social welfare than that of ex-topaz.

From the aforementioned discussions, we can draw the conclusion that TRADE is an efficient and truthful online combinatorial auction mechanism.



(a) Bidder Satisfaction Ratio.

(b) Spectrum Utilization Ratio.

(c) Social Welfare.

Figure 4. Compare TRADE with ex-topaz.

6. RELATED WORKS

The first online auction mechanism was proposed by Lavi [13] to solve the problem of electrical commerce, computable wireless resource allocation, and software agent transaction. These problems are all concerned with the characteristic of the variable transaction time, and buyers will not wait for a long time. Mohammad Hajiaghayi *et al.* analyzed the strategy-proof online auction under the situation of limited-supply items [17] and re-usable goods [14]. They are unsuitable for spectrum auction because of spatial reusable characteristic of spectrum. Besides auction, mechanisms have adopted genetic-algorithm based approaches to allocate channels (*e.g.*, [18–20]). They are suitable for use in various wireless networks without strategic behavior.

Spectrum auctions are motivated by Veritas [6], which is the first truthful auction mechanism for spectrum allocation. Sengupta and Chatterjee [21] consider both strategic behavior of bidders and feasibility of implementing fast allocation algorithms. However, it does not take interference issue into consideration. Trust [22] is a double auction mechanism to solve the allocation problem with McAfee mechanism. Wu *et al*. [23] design a truthful, scalable, and collusion-resistant mechanism. P. Xu *et al.* [24] designed efficient methods for dynamic spectrum usage. Truthful double auction mechanism for heterogeneous spectrums [25] first deals with heterogeneous spectrum. Gopinathan and Li [26] studied auctions in a prior-free setting to maximize revenue. Tofu [15] is a semi-truthful online auction with almost optimal outcome while it can not handle spectrum's spatial reusable property and diverse cheating forms. Deek and Zheng [7] proposed an online auction mechanism to solve spectrum allocation as a three-dimensional bin packing problem. The work of Dong *et al.* [9] is the most related work to ours, but it lacks exploration of spectrum reuse.

7. CONCLUSION

In this paper, we have modeled the channel allocation problem as a sealed-bid online combinatorial auction and propose TRADE that can handle buyers' requests for varying frequency. TRADE is truthful and individually rational, runs in polynomial time. TRADE supports spectrum's temporal and spatial reuse.

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