Incentive-compatible adaptive-width channel allocation for non-cooperative wireless networks

Fan Wu*, Zuying Wei, Chunyang Wu and Guihai Chen

Shanghai Key Laboratory of Scalable Computing and Systems,
Department of Computer Science and Engineering,
Shanghai Jiao Tong University,
Email: wu-fan@sjtu.edu.cn
Email: zu_ying_hi@sjtu.edu.cn
Email: chunyang506@sjtu.edu.cn
Email: gchen@sjtu.edu.cn
*Corresponding author

Bo Sheng

Department of Computer Science,
University of Massachusetts, Boston
Email: shengbo@cs.umb.edu AQ1

Abstract: Due to the limitation of radio spectrum resource and fast deployment of wireless devices, careful channel allocation is of great importance for mitigating the performance degradation caused by interference among different users in wireless networks. Most of existing work focused on fixed-width channel allocation. However, latest researches have demonstrated that it is possible to combine contiguous channels for better utilising the available channels. In this paper, we study the problem of adaptive-width channel allocation in multi-hop, non-cooperative wireless networks from a game-theoretic point of view. We first present a strategic game model and demonstrate the existence of Nash equilibrium (NE). Since a NE is not an ideal solution, we then propose adaptive-width channel allocation in multi-hop, non-cooperative wireless networks (AMPLE), which is a novel incentive approach to guarantee the system performance. Numerical results verify that AMPLE does prevent nodes' misbehaviour, and achieves much higher average system throughputs than anarchical NEs.

Keywords: channel allocation; wireless network; NE; Nash equilibrium; DSE; dominant strategy equilibrium.

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Biographical notes: Fan Wu is an Associate Professor in the Department of Computer Science and Engineering, Shanghai Jiao Tong University. He received his BS in Computer Science from Nanjing University in 2004, and PhD in Computer Science and Engineering from the State University of New York at Buffalo in 2009. He has visited the University of Illinois at Urbana-Champaign (UIUC) as a Post Doc Research Associate. His research interests include wireless networking and mobile computing, algorithmic network economics, and privacy preservation. He is a receipt of China National Natural Science Fund for Outstanding Young Scientists and Pujiang Scholar.

Zuying Wei is a master student from the Department of Computer Science and Engineering at Shanghai Jiao Tong University, China. Her research interests lie in algorithmic game theory, wireless networking and mobile computing.

Chunyang Wu is an undergraduate student from the Department of Computer Science and Engineering at Shanghai Jiao Tong University, China. His research interests lie in algorithmic game theory and wireless networking.

Guihai Chen earned his BS from Nanjing University in 1984, ME from Southeast University in 1987, and PhD from the University of Hong Kong in 1997. He is a distinguished Professor of Shanghai Jiao Tong University, China. He had been invited as a Visiting Professor by many universities including Kyushu Institute of Technology, University of Queensland, and Wayne State University. He has a wide range of research interests with focus on sensor networks, peer-to-peer computing, high-performance computer architecture and combinatorics. He has published more than 250 peer-reviewed papers in well-archived international journals and well-known conference proceedings.

Bo Sheng is currently an Assistant Professor in Computer Science Department at University of Massachusetts Boston. He received his PhD in Computer Science from the College of William and Mary in 2010, and his BS in Computer Science from Nanjing University in 200. His research interests include mobile computing, cloud computing, security and privacy, and wireless networks.

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1 Introduction

Due to historical reasons, radio spectrum is manually divided into communication channels, and each channel is assigned to a specific application in a geographic area. For instance, the commonly used IEEE 802.11 standard specifies several orthogonal channels (e.g., 3 in IEEE 802.11b/g and 12 in IEEE 802.11a). Such static channelisation prevents the limited radio spectrum from being used efficiently (Gummadi and Balakrishnan, 2008; Moscibroda et al., 2008; Rahul et al., 2009). Furthermore, the USA has completed its transition to fully digital television broadcasting on 12 June, 2009, and opened up unlicensed use of TV whitespaces that span 100–250 MHz of spectrum (Second Rep. and Order and Memorandum Opinion and Order, n.d.). This raises the need for dynamic spectrum allocation.

Chandra et al. (2008) proposed that the width of IEEE 802.11-based communication channels can be changed adaptively in software by using commodity Wi-Fi hardware. For example, two contiguous 20 MHz channels can be combined into a 40 MHz channel to provide higher bit-rate. Furthermore, the emergence of cognitive radio makes it more convenient to adaptively utilise available radio spectrum. Although the problem of channel allocation has been extensively studied in the literature, the feature of adaptive-width channel has not been fully considered (Wu et al., 2011).

Since nodes equipped with cognitive radio can easily adapt themselves to operate in any part of radio spectrum spaces, we can no longer assume that the nodes in the network would follow the prescribed spectrum allocation protocol faithfully. The most rational strategy for an individual node is to tune its wireless interface to the available spectrum (channel), in which it can get the best payoff. However, such selfish behaviour may degrade the network's performance, due to inefficient channel allocation. In this paper, we consider the problem of adaptive-width channel allocation in non-cooperative wireless networks, where the participating nodes are always selfish and pursue their own objectives. Wu et al. (2011) presented an incentive scheme to guarantee the system to converge to a state, in which system-wide throughput is optimised. However, their work only applies to a single-hop network, wherein all transmissions on the same channel will collide with each other. This limits the practical usage of the proposed incentive scheme, because spatially well separated transmissions can work on the same channel simultaneously. For example, in a large building, two well separated access points (ACs) can serve wireless users using the same channel. Therefore, we will study the problem of adaptive-width channel allocation in multi-hop, non-cooperative wireless networks, and propose our strong and practical solution.

To understand the impact of participating nodes' selfish behaviour, we first model the problem of adaptive-width channel allocation as a strategic game, and study the Nash equilibrium (NE) the system converge to, when there is no exogenous factor to influence the nodes' behaviour. We introduce a simple algorithm to simulate selfish nodes' behaviours, and to compute a NE the system may converge to. Although the algorithm cannot enumerate all the possible NEs, its outputs provide us the following understanding of the NE:

- NE is not a strong equilibrium for all the players to comply with. In a NE scenario, only under the assumption that all other players kept their equilibrium strategies would a player of the game have incentives to keep its equilibrium strategy. Thus NE does not provide strong incentives for the game player.
- NE is usually not globally efficient, which means that the maximised system-wide performance is not always achieved. So, even if the system converged to one of the NEs, some player might benefit at the cost of system-wide performance degradation.
- Although our algorithm finishes in $O(nc)$ steps, where n is the number of nodes in the network and c is the number of available channels, the convergence may take extremely long time in practice.

Therefore, NE is not an ideal solution to the problem of adaptive-width channel allocation, and we need to seek stronger solutions that can guarantee the system performance at high level.

To achieve strong incentives and to maintain high system performance, we propose an incentive scheme, namely AMPLE, that can guarantee the system converging to a dominant strategy equilibrium (DSE), a novel incentive approach to adaptive-width channel allocation in multi-hop, non-cooperative wireless networks (AMPLE). In game theory, DSE is a solution much stronger than NE. For each node, instead of going through a complicated decision process, simply picking its corresponding strategy in the DSE is the best strategy, regardless of the others' strategies. In the meanwhile, the system-wide performance achieved in the DSE is guaranteed to be high.

The major contributions of this paper are as follows:

- First, to our knowledge, we are the first to study the problem of adaptive-width channel allocation in multi-hop, non-cooperative wireless networks. Our solution is strong and practical.
- Second, we present an algorithm to simulate the selfish behaviour of the nodes. The results of the algorithm show that there exist multiple NEs the system may converge to. More importantly, NE is not a perfect solution concept to the problem studied in this paper.
- Third, we propose an incentive scheme that can guarantee the convergence of the system to a DSE, in which the system-wide performance achieved in the DSE is guaranteed to be high.

The rest of the paper is organised as follows. In Section 2, we present our system model, game model and some necessary concepts. In Section 3, we show the existence of NE in anarchy. In Section 4, we propose AMPLE, as our solution to the problem. In Section 5, we report the evaluation results. In Section 6, we give a brief review of the related work. Finally, in Section 7, we conclude this paper and put forward potential future work.

2 Preliminaries

2.1 System model

In this paper we consider a static wireless network with some access points. Each access point is equipped with a radio interface and can provide data service within its coverage area. Define $N \triangleq \{1, 2, 3, ..., n\}$. Figure 1(a) illustrates a proper example. There are three access points (AC) A, B and C. The dotted circles are the coverage areas of those ACs. In this scenario, A conflicts with C while not with B.

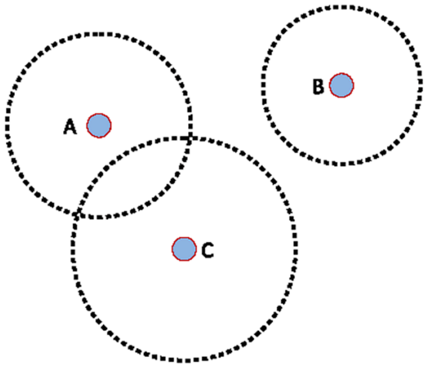
Given a set of channels donated by $C \triangleq \{1, 2, \ldots, c\}$, we assume that the channels are contiguous, orthogonal (non-interfering), and homogenous. Since the ACs need the channels to provide services to their customers, we want to efficiently allocate the channels to the ACs. Due to service quality requirement, we require that the ACs do not have any channel conflict with each other. By treating each AC as a node in the graph, we set up a conflict graph $G \triangleq (N, E)$, where E represents the conflict edge set, $e = (i, j) \in E$ means that j and i conflict with each other.

For any AC $i \in N$, We denote its allocated channel set by $\mathbb{C}(i)$.

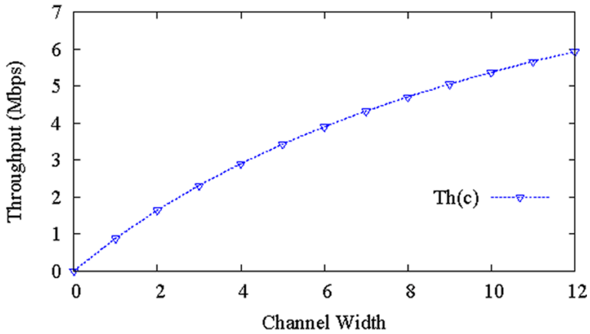
We require that the channels allocated to an AC must be contiguous. An AC can obtain a higher throughput by combining contiguous channels into a wider on Let $Th(c)$ represent the effective aggregated throughput of a channel with the bandwidth of c original channels. As shown in Bianchi (2000), $Th(c)$ is a concave non-decreasing function of c. Figure 1(b) illustrates these properties of $Th(c)$.

For a particular AC, it is able to combine contiguous channels, which are not conflicting with its neighbours. Thus its throughput is that of the combined channel.

Figure 1 (a) An example showing confliction between access points and (b) properties of the effective aggregate throughput $Th(c)$ (see online version for colours)



(a)



(b)

Longest contiguous segment (LCS): Given an integer set A, a contiguous segment is subset that requires the elements are contiguous. We define $LCS(A)$ as the longest contiguous segment (LCA) in A.

Based on this definition, we formulate the throughput of AC i as

$$T_i(s) = Th(|\text{LCS}(\mathbb{C}[i])|).$$

2.2 Game model

We model the adaptive-width channel allocation as a strategic game. In this game, we treat the ACs as players. We assume the players are rational and do not collude or cooperate with each other. The strategy of player $i \in N$ is its allocated channel set:

$$s_i \triangleq \mathbb{C}(i).$$

In the rest of this paper, we use s_i and $\mathbb{C}(i)$ interchangeably.

The strategy profile s is a vector composed of all the players' strategies,

$$s \triangleq (s_1, s_2, ..., s_n)^T.$$

Conventionally, s_{-i} represents the strategy profile of the other players except player i.

For a strategy profile s, let's denote the throughput of player i by $T_i(s)$. As mentioned previously, the allocated channels of one player should be contiguous. If it selects some separated channels that cannot be combined, it can not fully utilise them.

We then define a player's utility. As in the literature (e.g., Zhong et al. (2003), Zhong et al. (2005), Eidenbenz et al. (2005), Wang et al. (2006) and Zhong and Wu (2007)), we assume that there exists some kind of virtual currency in the system. In this paper, we define the utility of player i as

$$u_i(s) \triangleq \alpha T_i(s) - \mathcal{P}_i(s), \tag{1}$$

where α is a coefficient and $\mathcal{P}_i(s)$ represents the charge to player i for using channels. Since a player cannot guarantee the quality of the service provided to its customers, we let $u_i(s) \triangleq -\mathcal{P}_i(s)$, when the player collide with one of its conflicting neighbours.

We then review some solution concepts from game theory used in this paper.

Nash equilibrium (Osborne and Rubenstein, 1994)**:** A strategy profile s^* is a Nash Equilibrium of a strategic game, if for any player $i \in N$ and for any strategy $s_i \neq s_i^*$,

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*). \tag{2}$$

Dominant strategy equilibrium (Osborne and Rubenstein, 1994; Fudenberg and Tirole, 1991)**:** A strategy profile s^* is a DSE of a strategy game, if for any player $i \in N$, any strategy $s \neq s^*$ and any strategy profile of the other players s_{-i},

$$u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i}). \tag{3}$$

3 Anarchical Nash equilibrium

In this section, we show the existence of Nash equilibrium (NE) in anarchy, when there is no external factor to influence the players' behaviours. Each player's objective is to maximise its own throughput and hence the utility of the player is

$$u_i(s) = \alpha T_i(s), \tag{4}$$

if it does not collide with its conflicting neighbours; otherwise

$$u_i(s) = 0. \tag{5}$$

3.1 Computing NE

The pseudo-code for computing a NE is showed in Algorithm 1. We first successively allocate each node a random available channel (Lines 1–7). We denote the set of adjacent nodes to a node as

$$Adj[i] \triangleq \{j|(i,j) \in E\}.$$

Then, we check each node and update its allocated channel(s) if it can get its throughput improved with the new allocation (Lines 8–12). We repeat the above process until no node can improve its throughput by jumping to another set of contiguous channels. We denote the set of adjacent nodes to a node as

$$Adj[i] \triangleq \{j|(i,j) \in E\}.$$

Algorithm 1 Computing a NE

Require: A conflict graph $G = (N, E)$, a set of channels $C = \{0, 1, \ldots, c-1\}$.
Ensure: Channel allocation $\mathbb{C}[i]$ for any node i in N.
1: $\forall i \in N, \mathbb{C}[i] = \Phi$
2: **for** $i \in N$ **do**
3: **if** $C \setminus \bigcup_{j \in Adj[i]} \mathbb{C}[j] \neq \Phi$ **then**
4: x := a random channel in $C \setminus \bigcup_{j \in Adj[i]} \mathbb{C}[j]$
5: $\mathbb{C}[i] := \{x\}$
6: **end if**
7: **end for**
8: **repeat**
9: **for** $i \in N$ **do**
10: $\mathbb{C}[i] := LCS\left(C \setminus \bigcup_{j \in Adj[i]} \mathbb{C}[j]\right)$
11: **end for**
12: **until** No $\mathbb{C}[i]$ can be changed.
13: **return** $\mathbb{C}[i], i \in N$

3.2 Analysis

We prove the channel allocation strategy profile s^* determined by $\mathbb{C}[i], i \in N$, which is computed by Algorithm 1, is a NE.

Theorem 1: *The channel allocation strategy profile $s^\star$ computed by Algorithm 1 is a NE.*

Proof: Since conflicting ACs can not share any channel, the throughput of each conflicting ones will be zero if they share some channels. Hence for any strategy profile s and any player i in N, $u_i(s) = 0$ if $\mathbb{C}[i] \cap \bigcup_{j \in Adj[j]} \mathbb{C}[j] \neq \Phi$.

If for a node i, we choose another $\mathbb{C}'[i]$. Denote this new strategy by s_i. Let $s = (s_i, s_{-i}^*)$. We distinguish two cases:

- $\mathbb{C}'[i] \cap \bigcup_{j \in Adj[j]} \mathbb{C}[j] \neq \Phi$. This happens when Player i collides with its neighbours.
  In this case, it is not able to utilise the channel, so $u_i(s) = 0 \leq u_i(s^*)$.
- $\mathbb{C}'[i] \cap \bigcup_{j \in Adj[j]} \mathbb{C}[j] = \Phi$. This means that Player i combines another set of channels.
  Let $D = C \setminus \bigcup_{j \in Adj[i]} \mathbb{C}[j]$. On one hand, Algorithm 1 ensures $\mathbb{C}[i] = LCS(D)$. On the other hand, $\mathbb{C}'[i] \subseteq D$. So, $|LCS(\mathbb{C}'[i])| \leq |LCS(D)| = |\mathbb{C}[i]|$. Therefore $u_i(s) \leq u_i(s^*)$.

We can conclude that for any player i and for any strategy profile $s = (s_i, s_{-i}^*)$,

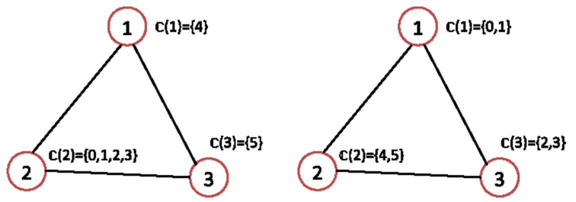
$$u_i(s) \leq u_i(s^*).$$

The result computed by Algorithm 1 is a NE. □

4 Design of AMPLE

NE is not an ideal solution concept. As we have mentioned, given a particular network topology, there may exist many NEs and the global performance might vary in a wide range in different NEs. Figure 2 illustrates an example of comparing two NEs.

Figure 2 An example of comparing two NEs. The right one gives a better global throughput than the left one, when there are six channels and three access points (see online version for colours)



In this section, to cope with the weakness of NE, we propose our carefully designed incentive scheme AMPLE. AMPLE consists of two components. One is an approximate algorithm that gives an allocation with good system performance. To stimulate the ACs to follow this allocation, the other part is a charging scheme to guarantee that following the computed channel allocation is the dominant strategy of each AC. Thus AMPLE guarantees the system to converge to a DSE.

4.1 Channel allocation

We now introduce our channel allocation algorithm. Our procedure can be divided into two phases. The first phase (Algorithm 2) converts the original conflict graph $G \triangleq (N, E)$ into a directed acyclic graph $\tilde{G} \triangleq (N, \tilde{E})$. Based on $\tilde{G}$, the second phase (Algorithm 3) specifies the channels allocated to each node.

4.1.1 Constructing $\tilde{G}$

In this phase, we convert G into a directed acyclic graph $\tilde{G} = (N, \tilde{E})$. The pseudo-code is listed in Algorithm 2.

Based on nodes' degrees, we divide N into several independent sets and give each node an order to represent which set it is in (Lines 1–9). Let $Ord(i)$ represent the order of node i. The loop iteratively finds a node i of maximal degree, remove i and its edges. If i's degree is equal to that of last node j, indicating that i and j do not share an edge (or j is not with maximal degree), then let $Ord(i) := Ord(j)$. Otherwise, let $Ord(i) := Ord(j) + 1$. We record the maximal order as $\mathscr{O}$. Next we construct the directed acyclic graph $\tilde{G} = (N, \tilde{E})$ based on the nodes' orders (Lines 10–17). For any edge (i, j) in E, if $Ord(i) < Ord(j)$ then we add $\langle j, i\rangle$ to $\tilde{E}$; otherwise we add $\langle i, j\rangle$ to $\tilde{E}$.

4.1.2 Allocating channels

We then show the details in Algorithm 3, which computes the channel allocation.

Algorithm 2 Converting the original conflict graph G into a directed acyclic graph $\tilde{G}$

Require: $G = (N, E)$
Ensure: $\tilde{G} = (N, \tilde{E})$, $\mathscr{O}$, $\{Ord(i)|i \in N\}$
1: $\mathscr{O} := 0$
2: **while** $G \neq \Phi$ **do**
3: $\quad\mathscr{O} := \mathscr{O} + 1$
4: $\quad d :=$ the degree of G
5: $\quad$**while** G contains a node ω of degree d **do**
6: $\quad\quad Ord(\omega) = \mathscr{O}$
7: $\quad\quad$Remove ω and all the edges linking ω from G
8: $\quad$**end while**
9: **end while**
10: $\tilde{E} := \Phi$
11: **for** $e := (i, j) \in E$ **do**
12: $\quad$**if** $Ord(i) < Ord(j)$ **then**
13: $\quad\quad\tilde{E} := \tilde{E} \cup \{\langle j, i\rangle\}$
14: $\quad$**else**
15: $\quad\quad\tilde{E} := \tilde{E} \cup \{\langle i, j\rangle\}$
16: $\quad$**end if**
17: **end for**
18: **return** $\tilde{G} = (N, \tilde{E})$, $\mathscr{O}$, $\{Ord(i)|i \in N\}$

Algorithm 3 Computing the allocation

Require: C, $\tilde{G} = (N, \tilde{E})$, $\mathscr{O}$, $\{Ord(i)|i \in N\}$
Ensure: $\{\mathbb{C}(i)|i \in N\}$
$\mathscr{L} := 0$
for $k := \mathscr{O}$ down to 1 **do**
$\quad$**for** $i \in N$ s.t. $Ord(i) = k$ **do**
$\quad\quad \mathcal{L}(i) := \min\left\{N \backslash \bigcup_{j \in prev(i)} \{\mathcal{L}(j)\}\right\}$
$\quad\quad$**if** $\mathcal{L}(i) > \mathscr{L}$ **then**
$\quad\quad\quad \mathscr{L} := \mathcal{L}(i)$
$\quad\quad$**end if**
$\quad$**end for**
end for
if $|C| < \mathscr{L} + 1$ **then**
$\quad$**for** $i \in N$ s.t. $\mathcal{L}(i) < |C|$ **do**
$\quad\quad \mathbb{C}(i) := \{\mathcal{L}(i)\}$
$\quad$**end for**
else
$\quad$**for** $i \in N$ **do**
$\quad \mathbb{C}(i) := \{\lfloor \frac{\mathcal{L}(i)|C|}{\mathscr{L}+1} \rfloor, \ldots, \lfloor \frac{(\mathcal{L}(i)+1)|C|}{\mathscr{L}+1} \rfloor - 1\}$
$\quad$**end for**
end if
repeat
$\quad$**for** $i := 0$ to $n - 1$ **do**
$\quad\quad \mathbb{C}[i] := LCS(C \backslash \bigcup_{j \in Adj[i]} \mathbb{C}[j])$
$\quad$**end for**
until No $\mathbb{C}[i]$ is changed.
return $\{\mathbb{C}(i)|i \in N\}$

For any node $i \in N$, we define

$$prev(i) \triangleq \{s \in \tilde{N} | \langle s, i\rangle \in \tilde{E}\}. \quad (6)$$

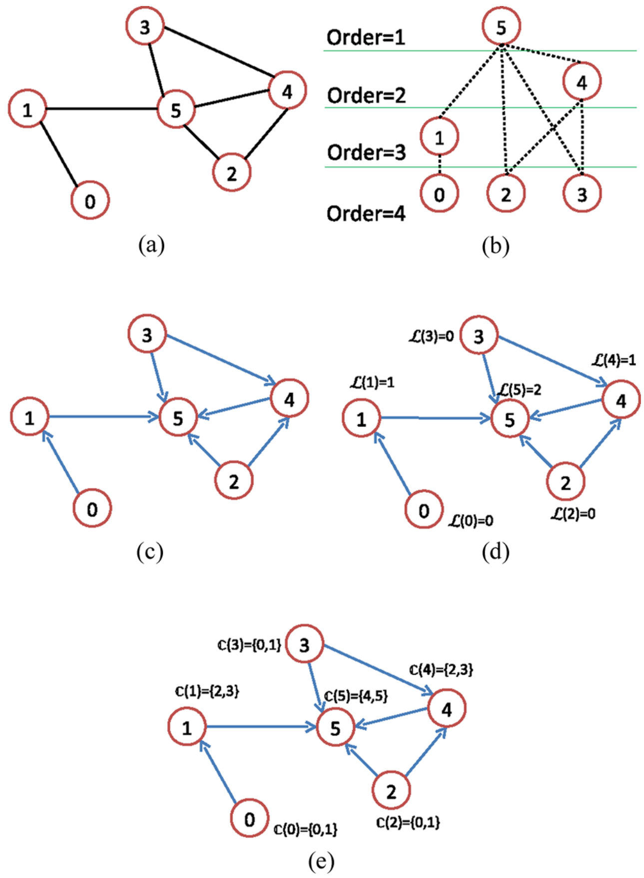
We first initialise $\mathcal{L}(i)$ (Lines 1–9). Here $\mathcal{L}(i)$ is a label which we subsequently use to identify its channels. We give each node a label as the minimal element in $N \setminus \bigcup_{j \in prev(i)} \{\mathcal{L}(j)\}$. In the loop, we record the maximal $\mathcal{L}(i)$ as $\mathscr{L}$ (Line 6).

In lines 10–18, we give each node an original set of channels based on $\mathcal{L}(i)$. If $|C| < \mathscr{L} + 1$, we give each node that satisfies $\mathcal{L}(i) < |C|$ one channel (Line 12). Otherwise we give each node $\lfloor \mathcal{L}(i)|C|/(\mathscr{L}+1) \rfloor$ channels (Line 16).

Similar to Algorithm 1, we then amend the allocation and try to broaden the nodes' channels (Lines 19–23).

We show an example of the allocation algorithm in Figure 3.

Figure 3 An example of the allocation algorithm for six access points and six channels. ($N = \{0, 1, 2, 3, 4, 5\}$, $C = \{0, 1, 2, 3, 4, 5\}$): (a) The original conflict graph $G = (N, E)$; (b) divide the nodes into four independent sets. Here dotted lines are the edges in E. (Lines 1–9 in Algorithm 2); (c) construct $\tilde{G} = (N, \tilde{E})$. This step constructs the directed edges from nodes in higher order to nodes in lower order (Lines 10–17 in Algorithm 2); (d) Calculate $\mathcal{L}(i)$ to each node. (Lines 1–9 in Algorithm 3) and (e) calculate $\mathbb{C}(i)$ to each node (Lines 10–23 in Algorithm 3) (see online version for colours)



4.2 Algorithm analysis

In this section, we discuss the bound of approximation ratio and the time complexity of AMPLE.

Lemma 2: *The maximal order $\mathscr{O}$ computed in Algorithm 2 satisfies:*

$$\mathscr{O} \le Q + 1 \tag{7}$$

Here, Q represents the degree of the conflict graph $G = (N, E)$.

Proof: We demonstrated this inequality by induction. Donate the degree of G by $\delta(G)$.

- For any graph G that $\delta(G) = 0$. It is clear that Algorithm 2 set all the nodes the same order. This causes $\mathscr{O} = 1 = \delta(G) + 1$.
- Assuming for any graph G that $\delta(G) = k (k \ge 0)$, we have $\mathscr{O} \le \delta(G) + 1$. For any graph G that $\delta(G) = k + 1$, Algorithm 2 first find a node ω with degree $k + 1$, set $Ord(\omega) = 1$ and remove all ω's edges from G. Donate the remained graph by G'. We repeat this step in G' till $\delta(G') = k' < k + 1$.
  According to the assumption, for G', we have $\mathscr{O}_{G'} \le k' + 1 \le k + 1$. In this way,

  $$\mathscr{O} = \mathscr{O}_{G'} + 1 \le k + 2 = \delta(G) + 1.$$

By the induction principle, $\mathscr{O} \le \delta(G) + 1$. □

4.2.1 Approximate ratio

The objective of this problem is maximising $\sum_{i \in N} |\mathbb{C}(i)|$. Instead of tightly bounding the approximation ratio, we present a loose lower bound and discuss the performance of AMPLE in Section 5. Let

$$r = \frac{(\sum_{i \in N} |\mathbb{C}(i)|)_{AMPLE}}{(\sum_{i \in N} |\mathbb{C}(i)|)_{\text{Optimal}}}. \tag{8}$$

A loose lower bound: $r > 1/(Q + 1)$. *Here Q represents the degree of the conflict graph $G = (N, E)$.*

$$r > 1/(Q + 1). \tag{9}$$

Here Q represents the degree of the conflict graph $G = (N, E)$.

Proof: Donate the answer of Algorithm 3 by $\{\mathbb{C}(i) | i \in N\}$ and the optimal answer by $\{\mathbb{C}_{opt}(i) | i \in N\}$. According to Lemma 2, for Algorithm 3, we have

$$\mathscr{L} \le \mathscr{O} \le Q + 1. \tag{10}$$

Therefore,

$$\sum_{i \in N} |\mathbb{C}(i)| \ge \frac{|N||C|}{\mathscr{L}} \ge \frac{|N||C|}{Q + 1}. \tag{11}$$

We then talk about an upper bound of the optimal solution. We can at most allocate each node with the full channel set C. Therefore, $\{\mathbb{C}_{opt}(i) | i \in N\} \le |C||N|$.

In sum, we have

$$r > \frac{1/(Q+1)|N||C|}{|N||C|} = 1/(Q+1) \tag{12}$$

□

4.2.2 Time complexity

For Algorithm 2, constructing $\tilde{G}$ takes $O(|N||E|)$ time. In Algorithm 3, calculating $\mathcal{L}(i)$ takes $O(|N|)$ time; computing the original channels takes $O(|N|)$ time; in the worst situation, amending the channels takes $O(|N||C|)$ time. In sum the upper bound of the time complexity is $O(|N||E| + |N||C|)$.

4.3 Design of charging scheme

As we have mentioned, NE does not provide a perfect solution to the problem of adaptive width channel allocation. In this section, we propose a charging scheme to make the system converge to an equilibrium state, called dominant strategy equilibrium (DSE). This scheme is proposed for two objectives:

- The charging scheme surely triggers the system's convergence to a DSE, which is a stable state that all the players follows the allocation proposed computed by AMPLE.
- The charge should be rational and as little as possible. This is because a big or even tremendous charge or punishment would lead no player join the scheme. Exactly as a forfeit of one million dollars for a small mistake like not handling the homework on time is not adopted in real life. An unreasonable charge scheme would strip the significance of the scheme.

Donate the strategy profile determined in Algorithm 3 by s^*. We next introduce a charging formula, which is a virtual currency (Zhong et al., 2003, 2005; Eidenbenz et al., 2005; Wang et al., 2006; Zhong and Wu, 2007) to incentive the players' behaviours.

Charging formula: For any player i and any strategy profile $s = (s_i, s_{-i})$, the charge of player i is

$$\mathcal{P}_i(s) \triangleq \alpha(T_i(s_i^*, s_{-i})/2 + Th(|s_i^* \backslash s_i|) + A), \tag{13}$$

where

$$A = \frac{|s_i \backslash s_i^*| Th^2(|s_i|)}{4(T_i(s_i^*, s_{-i}) + Th(|s_i^* \backslash s_i|))}. \tag{14}$$

For $\mathcal{P}_i(s)$, the first term $T_i(s_i^*, s_{-i})/2$ is an essential part of charge. The second term $Th(|s_i^* \backslash s_i|)$ and the third term A together forms an external part of charge. This external part treats as a punishment, an additional charge. When the player obeys s_i^*, this part is zero, which means no penalty is imposed. However, when it does not obey s_i^* and behave s_i, this part varies and increase higher than the additional utility obtained by s_i.

Then, we prove the strategy profile s^* is a DSE.

Theorem 4: *The channel allocation strategy profile s^* computed by Algorithm 3 is a DSE under the charging scheme.*

Proof: For any profile $s = (s_i, s_{-i})$, the utility of player i, $u_i(s_i, s_{-i})$ is

$$u_i(s_i, s_{-i}) = \alpha(T_i(s_i, s_{-i}) - \mathcal{P}_i(s)). \tag{15}$$

If i chooses s_i^* as its strategy,

$$u_i(s_i^*, s_{-i}) = \alpha\left(T_i(s_i^*, s_{-i})/2\right). \tag{16}$$

Omitting the coefficient α,

$$\begin{aligned}
&1/\alpha(u_i(s_i^*, s_{-i}) - u_i(s_i, s_{-i})) \\
=&T_i(s_i^*, s_{-i}) - T_i(s_i, s_{-i}) + Th(|s_i^* \backslash s_i|) + A \\
=&T_i(s_i^*, s_{-i}) - T_i(s_i, s_{-i}) + Th(|s_i^* \backslash s_i|) \\
&+ \frac{|s_i \backslash s_i^*| Th^2(|s_i|)}{4(T_i(s_i^*, s_{-i}) + Th(|s_i^* \backslash s_i|))} \\
\geq& - T_i(s_i, s_{-i}) \\
&+ 2\sqrt{\frac{(T_i(s_i^*, s_{-i}) + Th(|s_i^* \backslash s_i|)) Th^2(|s_i|)}{4(T_i(s_i^*, s_{-i}) + Th(|s_i^* \backslash s_i|))}} \\
=& - T_i(s_i, s_{-i}) + Th(|s_i|) \\
\geq& 0,
\end{aligned} \tag{17}$$

we have

$$u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i}). \tag{18}$$

Therefore we conclude that s^* is DSE. □

5 Numerical results

We implement AMPLE and evaluate its performance using network simulations. The objective of our simulations is twofold. One is to test the performance of our channel allocation algorithm's outputs, which is the system-wide throughput. Since no proper existing works are comparable to our system, this evaluation compares the system-wide throughput achieved by anarchical NE and AMPLE's DSE. The other one is to verify that the system indeed converge to the DSE when AMPLE is used.

5.1 Simulation methodology

In the simulation experiments, we use a basic CSMA/CA protocol with binary slotted exponential back-off as the MAC layer protocol. Following (Wu et al., 2011), the parameters used for the experiments are listed in Table 1.

Metrics: We evaluate two quantitative values as metrics in this paper:

- *Utility*: Utility is the difference between the player's valuation on throughput and charge for using the channels. This metric reflects the impacts of a player's behaviour on its own.

- *System-wide throughput*: It is the sum of all the players' throughputs. This metric is used to measure the effectiveness of our design on the performance of the channel allocation game.

Table 1 Parameters used to obtain numerical results

PHY&MAC header	50 bytes
ACK packet size	30 bytes
Minimum contention window	32
Number of backoff stages	5
Original channel bit rate	1 Mbps
Propagation delay	1 μs
Slot time	50 μs
SIFS	28 μs
DIFS	128 μs
ACK timeout	300 μs

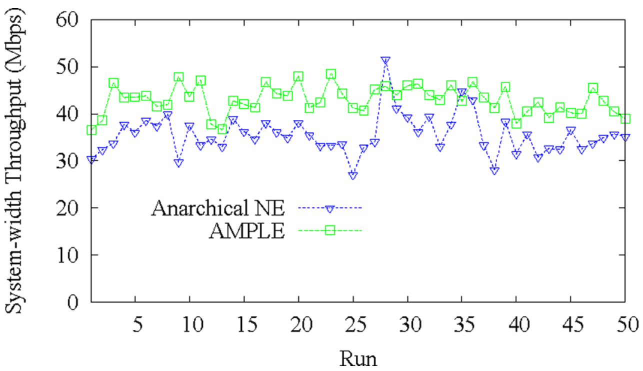
5.2 *Performance*

In this set of simulations, we evaluate the system-wide throughput of AMPLE and anarchical NE.

In the first simulation, we assume there are 20 ACs and 12 channels. We set the degree of each point ranges from 3 to 5 and obeys a binomial distribution ($\sim Bin(20, 0.2)$), in which the average degree is 4. The first simulation is repeated 10^4 times. In each run, we generate a conflict graph, execute and record the system-wide throughputs of AMPLE and anarchical NE. Due to the limitation of space, we show the results of the first 50 runs in Figure 4.

From Figure 4, we can observe that AMPLE gives relatively higher throughput than anarchical NE. Although anarchical NE gets higher throughput some times (almost twice every 25 runs), the average system-wide throughput of AMPLE is better than that of the anarchical NE. From this evaluation, the average ratio of the system-wide throughputs between AMPLE and anarchical NE is 1.1457, showing that AMPLE achieves an average of 15% higher throughput than that of anarchical NE.

Figure 4 The result of the first 50 runs of the simulation measuring the system-wide throughputs of AMPLE and anarchical NE. In each run, there are 20 access points and 12 channels (see online version for colours)



In the second evaluation, we fix the number of ACs at 20, and vary the number of channels among 3, 6, 8, and 12. Other settings are the same as the first evaluation. In this evaluation, we repeat each simulation until the convergence level 10^{-4} is reached.

Figure 5(a) illustrates the results. We can see that AMPLE always achieves higher system-wide throughput than anarchical NE does. At the same time, the standard deviations of AMPLE's results are also relatively smaller, which shows that the performance of AMPLE is more stable.

In the third evaluation, we vary the number of ACs, while fixing the number of channels at 12. We simulate the number of ACs from 5 to 50. Other settings are the same as the first simulation. In this evaluation, we also repeat each simulation until the convergence level 10^{-4} is reached.

Figure 5(b) shows that both the system-wide throughput of AMPLE and anarchical NE increase with the number of ACs. However, AMPLE's throughput is larger than that of anarchical NE, and the gap between AMPLE and anarchical NE grows with the number of nodes.

We also record the average utilisation of a channel in the third evaluation. Here, channel utilisation means the average number of ACs allocated to each channel. Figure 5(c) shows that the channel utilisation of AMPLE is always higher than that of anarchical NE.

In the forth evaluation, we vary the average degree of the ACs from 1 to 20, while the other settings are the same as the first simulation. Figure 5(d) shows that the system-wide throughput of both AMPLE and anarchical NE decreases when the network become more and more denser. However, AMPLE still always achieve better average system-wide throughput than anarchical NE.

5.3 *Truthfulness of AMPLE*

In this set of evaluations, we assume that 20% players are not following the channel allocation computed by AMPLE. We assume there are 12 channels, 20 players, and the average degree of the ACs is 4. In each run, we randomly pick four misbehaving ACs and let them deviate from the channel allocation computed by AMPLE. Then we record the utility got by a fifth AC in 50 runs.

Figure 6 shows that the utility of a player when it follows or deviates from the channel allocation computed by AMPLE. We can observe that following the computed channel allocation is always no worse than that of deviating from it. Besides, the utility when following the scheme is always positive while the utility when deviating it is usually negative. So when AMPLE is used, the incentives for following the computed channel allocation is always guaranteed, no matter what the other players do. This result verifies that players cannot benefit by deviating from the channel allocation computed by AMPLE, when our charging scheme is used. Therefore, the convergence to the DSE is guaranteed on our charging scheme.

Figure 5 Experiment results for the first to the forth evaluations: (a) the results of system-wide throughput achieved by AMPLE and anarchical NE when there are 3, 6, 8, and 12 channels and 20 access points. The height of the bar show the average throughput, and the error-bar shows the standard deviation of the measured results; (b) the results of system-wide throughput achieved by AMPLE and anarchical NE when the total of access points varies from 5 to 50. The number of channels is 12; (c) the results of channel utilisation achieved by AMPLE and anarchical NE when the total of access points varies from 5 to 50. The number of channels is 12 and (d) the results of system-wide throughput achieved by AMPLE and anarchical NE when node degree varies from 1 to 20 (see online version for colours)

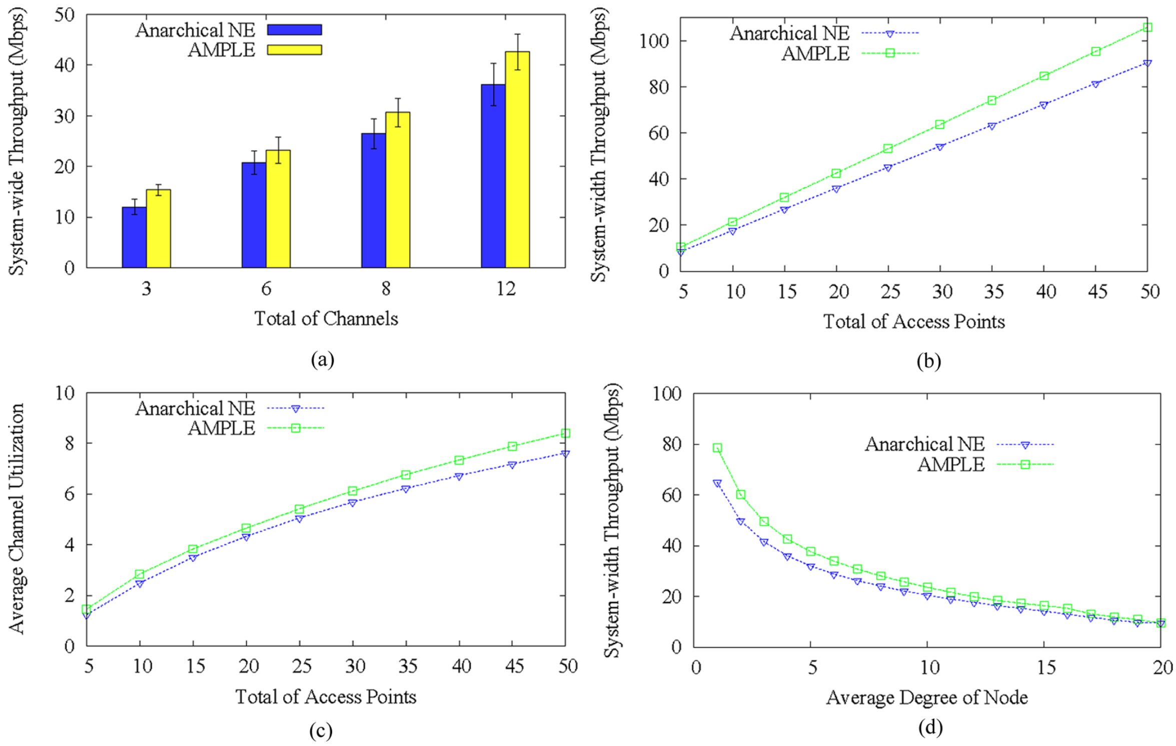
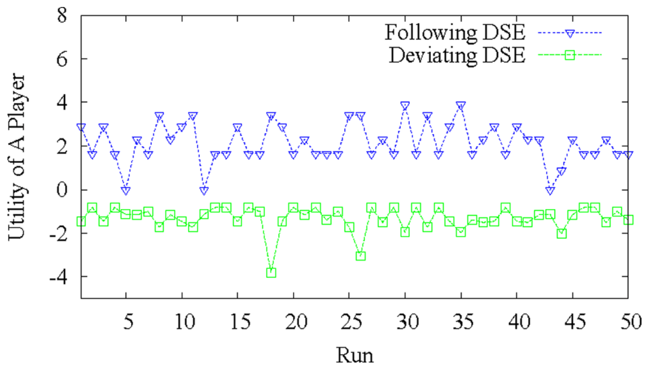


Figure 6 The utility of a player when it follows or deviates from the channel allocation computed by AMPLE (see online version for colours)



6 Related work

In this section, we review the related work in this field. Félegyházi et al. (2007) first proposed a game model for the static multi-radio multi-channel allocation. Wu et al. (2008) later put forward a mechanism to converge the multi-radio multi-channel allocation game to the strongly dominant strategy equilibrium (SDSE). They both only considered the problem in a single collision domain which is different from the scenario we consider in this paper. Recently, a number of strategy-proof auction-based spectrum allocation mechanisms (e.g., TRUST (Zhou and Zheng, 2009), SMALL (Wu and Vaidya, 2010), and VERITAS (Zhou et al., 2008)) have been proposed to solve the problem in multiple collision domain. An important relevant work on channel allocation game is Halldórsson et al. (2004), in which the authors modelled it as a graph colouring problem and discussed the price of anarchical state under various topology conditions. However, none the above work considers adaptive-width channels. A latest work by Wu et al. (2011) discussed the allocation in adaptive-width channels. However, it is only valid in a single collision domain as well.

In wireless networks, the game theory is also applied to study problems such as media access. For example, MacKenzie and Wicker (2003) studied the behaviours of selfish nodes in Aloha networks. Later, Čagalj et al. (2005) and Konorski (2002) utilised game-theoretic approaches to investigate the media access problem of selfish behaviour in CSMA/CA networks. Nie and Comaniciu (2005) proposed a game theoretic framework to study the behaviour of cognitive radios for distributed adaptive spectrum allocation in cognitive radio networks. Some other relevant works on incentive-compatibility in wireless networks are Anderegg and Eidenbenz (2003), Srinivasan et al. (2003), Wang et al. (2004), Zhong et al. (2005), Zhong et al. (2003), Wang et al. (2006), Ben Salem et al. (2003), Eidenbenz et al. (2005), Zhong and Wu (2007), Xu et al. (2011) and Deek et al. (2011).

7 Conclusion and future work

In this paper, we proposed an approach for adaptive-width channel allocation in multi-hop, non-cooperative wireless networks. We first gave an algorithm to compute an efficient channel allocation, and then presented a charging scheme to guarantee that it is to the best interest of each player to follow the computed channel allocation. Evaluation results showed that our approach achieved good performance. As for future work, there can be several potential directions. One of the possible direction is to consider the case, in which the s can be carefully to partially overlapping channels.

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